REDISCOVERING THE PYTHAGORIANS

REDESCOBRINDO OS PITAGÓRIOS

REDESCUBRIENDO A LOS PITAGÓRICOS

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ABSTRACT
The research will explore the Pythagoreans, an ancient philosophical and religious society led by Pythagoras. The work will focus on their belief that the universe can be explained through mathematics and numbers. We will analyze how their philosophy linked the study of numbers with musical harmony, astronomy, and ethics, arguing that numerical relationships are the key to understanding the structure of the cosmos. The investigation will reveal how the Pythagoreans considered mathematics not just as a calculation tool, but as a path to achieve the purification of the soul and a connection to the divine order of the universe, thus laying the foundation for a profound relationship between science and spirituality that has influenced Western thought.

Keywords: Pythagoreans. Mysticism. Harmony. Number Theory. Tetractys.

RESUMEN
La investigación explorará a los Pitagóricos, una antigua sociedad filosófica y religiosa liderada por Pitágoras. El trabajo se centrará en su creencia de que el universo puede ser explicado a través de las matemáticas y los números. Esta propuesta revela cómo los pitagóricos consideraban la matemática no solo como una herramienta de cálculo, sino como un camino para alcanzar la purificación del alma y la conexión con el orden divino del universo, sentando así las bases de una profunda relación entre ciencia y espiritualidad que ha influido en el pensamiento occidental.

Palabras clave: Pitagóricos. Misticismo. Armonía. Teoría de Números. Tetractys.

RESUMO
A pesquisa explorará os pitagóricos, uma antiga sociedade filosófica e religiosa liderada por Pitágoras. O trabalho se concentrará na crença deles de que o universo pode ser explicado por meio da matemática e dos números. Esta proposta revela como os pitagóricos viam a matemática não apenas como uma ferramenta de cálculo, mas como um caminho para alcançar a purificação da alma e a conexão com a ordem divina do universo, lançando assim as bases para uma profunda relação entre ciência e espiritualidade que influenciou o pensamento ocidental.

Palavras-chave: Pitagóricos. Misticismo. Harmonia. Teoria dos Números. Tetractys.

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1 INTRODUCTION

For the Pythagoreans, mathematics was not just a set of rules to calculate; It was a key to deciphering the underlying order of the universe. They believed that numbers were the essence of all things and that the cosmos was governed by proportions and numerical harmony. This holistic view, which united science with philosophy and spirituality, laid the foundation for the systematic study of the universe.

This research is crucial because it explores how this ancient school transformed mathematics from a practical tool to an abstract and theoretical discipline. By studying their contributions, not only do we appreciate the origins of concepts such as irrational numbers and musical proportions, but we also reveal the deep relationship between the pursuit of scientific knowledge and the understanding of reality. The work of the Pythagoreans influenced later thinkers such as Plato and Euclid, and their legacy lives on to this day in the way we conceive of the cosmos as something rational and orderly. Therefore, rediscovering the Pythagoreans is essential to understanding how mathematics became the fundamental language of science.

2 METHODOLOGY

This study was based on a bibliographic documentary research, using a qualitative approach. The methodology consisted of an exhaustive analysis of secondary sources, such as books, research articles, encyclopedias and academic texts, with the aim of understanding the worldview and contributions of the Pythagoreans.

The research process was divided into the following phases:

1. **Review and collection of information:** A systematic search of academic databases and digital libraries was carried out for relevant materials on Pythagoras and the Pythagorean school. The selection focused on texts that addressed both his mathematical contributions and his philosophy, cosmology, and spiritual beliefs.
2. **Analysis of sources:** The information collected was critically analyzed. The main focus was to identify and synthesize the key concepts of the Pythagorean doctrine, paying particular attention to the interconnection between mathematics and its religious and philosophical perspective.
3. **Synthesis and writing:** The findings of the analysis were organized and structured to develop a coherent narrative. This phase included the writing of each section of the

research, from the problem statement to the conclusions, ensuring that each argument was supported by the bibliographic sources reviewed.

The use of secondary sources was essential for this study, as it allowed us to reconstruct a comprehensive view of the Pythagoreans based on expert interpretation and the synthesis of multiple perspectives. This approach ensured the academic rigor of the work without the need to access primary sources, many of which are fragmentary or have been lost over time.

2.1 STATEMENT OF THE PROBLEM

Despite their profound influence on Western thought, understanding of the Pythagoreans has focused primarily on their mathematical contributions, such as the famous Pythagorean theorem. However, this view is incomplete, as it omits the philosophical and spiritual core of his doctrine.

Pythagorean society did not conceive of mathematics as a simple calculation tool, but as a key to deciphering the cosmic order and achieving the purification of the soul.

The central problem of this research lies in the historical disconnect between the scientific and the spiritual facets of the Pythagoreans. There is a tendency to separate their mathematical work from their religious and philosophical beliefs, ignoring that for them they were two sides of the same coin.

This disconnect raises the following questions:

How exactly did the Pythagoreans use mathematics to explore the universe?

How did this fusion of science and spirituality lay the foundation for the relationship that has endured in Western thought?

By answering these questions, this research seeks to rediscover the Pythagoreans' holistic view and show how their approach, which united rational knowledge with mystical belief, remains relevant to understanding the history of science and philosophy.

2.2 JUSTIFICATION

This research is theoretically relevant because it seeks to fill a gap in philosophical and scientific historiography. The predominant view of the Pythagoreans has focused on their contribution to mathematics (the Pythagorean Theorem, the irrationality of the root of 2), leaving aside their holistic worldview.

By studying his belief that the universe could be explained through numbers, not only as a calculation tool but as a means for purification of the soul and connection to the divine order, this research challenges the conventional narrative.

A new reading is proposed that integrates his scientific ideas with his spiritual practices, demonstrating that the modern separation between science and mysticism did not exist in his thought.

2.3 OBJECTIVES

- Analyze primary and secondary sources to identify the fundamental principles of the Pythagorean worldview.
- To determine the connection between the religious and philosophical practices of the Pythagoreans and their mathematical thinking.
- To evaluate the influence of the fusion between science and Pythagorean spirituality on the development of Western thought.

2.4 CONCEPTUAL ELEMENTS

Number theory and number relations:

The Pythagoreans made fundamental contributions to number theory that have endured throughout history. One of their most fascinating discoveries was the concept of irrational numbers, in which they challenged the notion that everything could be expressed as a fraction.

This finding opened new doors in mathematical understanding, which allowed them to define the behavior of some numbers, making various classifications and coining numerous names for the various types of numbers.

An early written Greek testimony to the existence of irrational numbers was published in a Dialogue of Plato, Theaetetes, or Science. Here Theodore of Cyrene shows that the square roots of 3,5,7...,17 are irrational. (Ruíz, 2003, p. 34)

It is assumed that these immeasurable reasons were discovered by Hippasus of Metapontum, who paid his life for the discovery. According to Aristotle, the Pythagoreans provided proof that the square root of two is incommensurable, using a method called reductio ad absurdum: an indirect logical method. (Ruíz, 2003, p. 34)

It is relevant to note that the Babylonians already used incommensurable ratios, that is, irrational numbers, and approximated them in their calculations.

However, both they and the Egyptians seemed not to have recognized the distinct nature of these types of numbers, despite incorporating them into their practice. Instead, the Greeks, especially the Pythagoreans, always discerned that irrational numbers were different from other numbers.

One of the consequences of the Pythagoreans' refusal to accept irrational numbers was the loss of the connection between numbers and geometry.

In the field of geometry, lengths, areas and different ratios could be considered, but when establishing numerical relationships, only those that were commensurable were accepted. This limited the possibilities for development in geometry, arithmetic, and algebra, resulting in Greek geometry that was not really metrical.

The Pythagoreans made advances in the understanding of triangles, parallel lines, polygons, circles, spheres and polyhedra, and, of course, in the Pythagorean theorem, in addition, it is known that they knew that the sum of the angles of a triangle is 180 degrees.

Historians believe that the concept of mathematical proof in the Pythagorean tradition began to develop towards the end of this society, around 400 BC, whereas in its beginnings there was no such notion. From a broader point of view, an element related to cosmology can be added, as Pythagoras and his followers managed to link astronomy with arithmetic and geometry, even considering music as a discipline.

Although at that time there was no quantitative astronomy capable of accurately predicting celestial movements, the importance given to mathematics was very different from that of the Babylonians and Egyptians.

For the latter, mathematics was mainly used to make measurements useful for agriculture and navigation, without there being a connection between the structure of the world and mathematics. In other words, the world was thought to follow the whim of the gods, rather than the properties of numbers and figures.

The number theory developed by the Pythagoreans had a significant impact on the advancement of mathematics. In addition, by accepting the idea of an orderly and symmetrical universe, they also influenced cosmology, as they sought to understand the shape of the Earth and the movement of the stars.

But because of his mystical procedure, many of his definitions are quite difficult to understand, so that it is sometimes convenient to have recourse to the preliminaries of Book VII of Euclid's Elements, where a large part of them are collected, in the easy-to-understand and rigorous language, characteristic of the great compiler of elementary Greek mathematics.

Even and odd numbers.

Odd and even numbers are subdivided into four classes:

(González, 1991)

1) ***Parmente even***: when their half is even, they are of the form: 2n·[2k+1], n>1).
2) ***Oddly even***: when their half is odd (they are of the form 2·[2k+1], n>1).
3) ***Odd parment***: when divided by an odd number gives an even number (they are of the form 2n·[2k+1]·p, n>1).
4) ***Odd odd***: when it has only odd divisors.

Prime and compound numbers were studied in their most rudimentary form by the Pythagoreans, who had a deep interest in the nature of numbers and their properties. However, Euclid, in his work *The Elements*, made a more systematic and rigorous study on these issues.

***Linear, flat,* and *solid numbers*.**

1) ***Linear***: it is the one that does not have divisors (i.e., the primes).
2) ***Plane***: it is the product of two numbers that are its sides (*Euclid,* D.VII.16).
3) ***Solid***: it is the product of three numbers that are its sides (*Euclid,* D.VII.17).
4) ***Square***: it is the product of a number by itself (*Euclid,* D.VII.18).
5) ***Cubic***: it is the product of a number by itself twice (*Euclid,* D.VII.19).

***Perfect numbers, deficient*, *abundant* and friendly numbers.**

1) ***Deficient***: it is a number that is less than the sum of its aliquot parts (positive eigendivisors of that number).

For example:

a) The number 8, since its proper divisors are 1,2 and 4, therefore the sum of its aliquots is 1+2+4=7, which is less than 8
b) The number 10, since its proper divisors are 1,2 and 5, therefore the sum of its aliquots is 1+2+5=8, which is less than 10

2) ***Abundant***: it is a number that is greater than the sum of its aliquot parts.

For example:

a) The number 18, since its proper divisors are 1, 2, 3, 6 and 9. Therefore the sum is 1 + 2 + 3 + 6 + 9 = 21, which is greater than 18.
b) The number 20, since its proper divisors are 1, 2, 4, 5 and 10. Therefore the sum is 1 + 2 + 4 + 5 + 10 = 22, which is greater than 20

3) ***Perfect*****:** it is a number that is equal to the sum of its aliquot parts.

For example:

a) The number 6, since its proper divisors are 1, 2 and 3. Therefore, the sum is 1 + 2 + 3 = 6.

b) The number 28, since its proper divisors are 1, 2, 4, 7 and 14. Therefore the sum is 1 + 2 + 4 + 7 + 14 = 28.

4) ***Friendly numbers*****:** are numbers in which each is equal to the sum of the divisors of the other.

For example:

a) The numbers 220 and 284, because the proper divisors of 220 are: 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, in addition the sum is 1 + 2 + 4 + 4 + 5 + 10 + 11 + 20 + 22 + 44 + 55 + 110 = 284. On the other hand, the proper divisors of 284 are: 1, 2, 4, 71, 142 and the sum is 1 + 2 + 4 + 71 + 142 = 220.

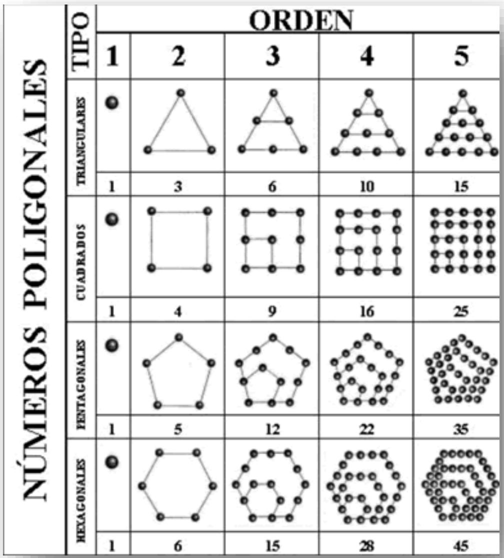
b) The numbers 1184 and 1210, because the proper divisors of 1184 are: 1, 2, 4, 8, 16, 37, 74, 148, 296 and 592 and the sum is 1 + 2 + 4 + 8 + 16 + 37 + 74 + 148 + 296 + 592 = 1210. When identifying the proper divisors of 1210 are: 1, 2, 5, 10, 11, 22, 55, 121 and 242. The sum is 1 + 2 + 5 + 10 + 11 + 22 + 55 + 121 + 242 = 1184.

Perfect numbers and friendly numbers have always caused great fascination, which is why the search for perfect numbers and friends has deployed a waste of mathematical ink from the early Pythagorean times to the present day, in which powerful computing instruments are applied. The early Pythagoreans knew only the perfect numbers 6 and 8.

Figure 1

Polygonal Numbers, Mathematical Biography: Pythagoras (11 of 18) Pythagoras: Polygonal Numbers. (n.d.).

The Pythagoreans used to represent numbers by dots on parchment or pebbles in the sand and classified them according to the polygonal shapes of these point distributions, that is, they associated numbers with geometric figures obtained by the regular arrangement of points, the sum of which determines the number represented.



Polygonal numbers, which represent geometric figures such as triangles and squares, were also part of his exploration.

The numbers 1,3,6 and 10 are called triangular because they could be arranged in the shape of triangles. Numbers 4 and 10 were favorites. The numbers 1,4,9,16,etc.

They were called squares because they could be arranged in such a way that they formed squares. Non-prime numbers that could not be perfect squares were called oblongs. (Ruíz, 2003, p. 32)

It is interesting to note that some properties could be extracted from the way the numbers were arranged: for example, that the sum of 2 consecutive triangular numbers is a square number. (Ruíz, 2003, p. 32)

But beyond their discoveries, the Pythagoreans shared a profound vision: they believed that everything in the universe could be explained through numbers and the relationships that existed between them.

For them, numbers were not just symbols, but the key to unraveling the mysteries of the cosmos, making mathematics a way of understanding the world around us. This perspective revealed a connection between mathematics and philosophy, bringing science and spirituality closer together in a quest for knowledge.

3 GEOMETRY AND THE PYTHAGOREAN THEOREM

3.1 PYTHAGOREAN THEOREM

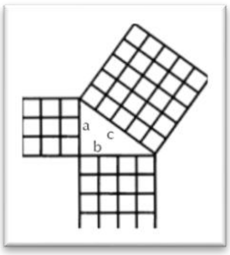
The Pythagorean theorem was first known in ancient Babylon and Egypt (early 1900 B.C.). The relationship was demonstrated in a 4000-year-old Babylonian tablet, now known as Plimpton 322. However, the relationship was not disclosed until Pythagoras explicitly stated it. This theorem states that, in a right triangle, the hypotenuse is equal to the square root of the sum of the squares of the legs.

Figure 2

Right Triangle in Babylon, Marisofi, R. G. (2015, February 9)

Figure 3

Right Triangle in Egypt, Aguilera, C. (2024, February 5)



The Pythagorean Theorem was developed by several of the pre-Hellenic Eastern civilizations (Babylon, Egypt, India and China) to later enter the Greek world through Pythagoras and cross it with Plato and Euclid.

The various demonstrations show us the great mathematical development of that time, and the most important thing is to see the wealth of ideas and that mathematics is not as structured as it is supposed to be, that there is creativity and imagination.

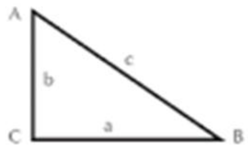
The Egyptians knew and used the fact that the triangle of sides 3, 4 and 5 (or proportional to these numbers), called the "Egyptian Triangle", is a rectangle, to draw a line perpendicular to another, like a "carpenter's square", which was a common practice of official surveyors to recover the borders of the boundaries of the lands after the periodic landslides produced by the floods of the Nile River. In ancient Egypt the Egyptian Triangle was also called the Triangle of Isis and had a certain sacred character, because the number three represented Osiris, four Isis and five Horus. In India they arose as a result of the planning of temples and the construction of altars, between the eighth and second centuries B.C., in India arithmetic-geometric, practical and primitive knowledge related to the Pythagorean Theorem was developed.

The Hindus used the rope not only for measuring, but also for drawing perpendicular lines, by means of string triples whose lengths constitute Pythagorean triples such as 3,4,5; 5,12,13; 8,15,17; 7,24,25. The Pythagorean triples of the Hindus are classified as follows:

Figure 4

Pythagorean Hindu Terns, (n.d.). History of Mathematics

c - b = 1			c - b = 2			c - b = 3		
a	b	c	a	b	c	a	b	c
3	4	5	8	15	17	15	36	39
5	12	13	12	35	37			
7	24	25						



The Pythagorean Theorem was discovered in approximately 500 BC. It bears this name because its discovery falls on the Pythagorean school. Although the Pythagoreans did not discover this theorem, they were the first to find a formal proof of the theorem.

3.2 CONSTRUCTION OF PERFECT SOLIDS

The Pythagoreans geometrically constructed the first perfect solids, in the context of their mathematical and aesthetic philosophy. They identified five solids that are known as Plato's solids. These are: Tetrahedron, a solid with 4 triangular faces, Hexahedron containing 6 square faces, Octahedron which is characterized by having 8 triangular faces, the Dodecahedron, solid with 12 pentagonal faces and the Icosahedron which has 20 triangular faces.

Each of these solids has special geometric properties and has been the subject of study in both mathematics and philosophy. The Pythagoreans found a connection between these solids and the elements of nature, as well as a symbolism that gave them a sense of harmony and beauty in the universe.

The Pythagoreans believed that these solids corresponded to the fundamental elements of the universe.

- The tetrahedron was associated with fire.
- The cube was linked to the earth.
- The octahedron was related to air.
- The dodecahedron symbolized the ether or the cosmos.
- The icosahedron was associated with water.

This connection reflected his vision of the world as a harmonious and orderly whole. In addition, they were pioneers in the study of geometry. Plato's solids were examples of regular shapes that helped to understand geometric properties, such as symmetry, proportion, and the relationship between dimensions. This contributed to the development of Euclidean geometry later on. In addition, they considered beauty and harmony to be deeply rooted in mathematics. This allowed them to find that these solids were ideal examples of symmetry and regularity, representing universal harmony.

The Pythagoreans also believed that there was an intrinsic relationship between mathematics and music, where numerical proportions determine harmonic relationships. Plato's solids embodied this idea of harmony in the universe, and finally, they also considered that studying these solids was part of their search for truth and understanding of the cosmos. They believed that, by exploring these forms, they could come closer to a deeper understanding of nature and the laws that govern the universe.

Taken together, the perfect solids were not only objects of mathematical study, but they were also symbols of the Pythagoreans' philosophical and spiritual beliefs about order and harmony in the universe.

It is believed that Pythagoras knew how to build the first three, but it was Hippasus of Metapontus who discovered the dodecahedron.

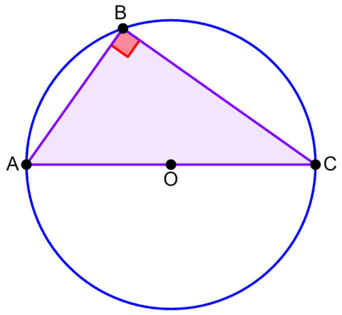
3.3 GEOMETRY OF THE TRIANGLE

The Pythagoreans showed that a triangle inscribed in a semicircle is a right triangle. This theorem states that a triangle inscribed in a semicircle is a right triangle is one of the most important contributions of the Pythagoreans and is commonly associated with the famous theorem of Thales. Although the theorem itself is attributed to Thales, who is contemporary with or earlier than the Pythagoreans, the Pythagorean community deepened its study. Thales is known for his contributions to geometry, and is credited with the first theorem relating geometry to circumference

The statement of the theorem states that any triangle that is inscribed in a semicircle has one of its angles as a right angle (90 degrees). This means that if the diameter of the semicircle is one side of the triangle, the vertex opposite the diameter will form a right angle.

Figure 5

Thales' theorem, taken from Guzmán, J. H. (2021, December 29)



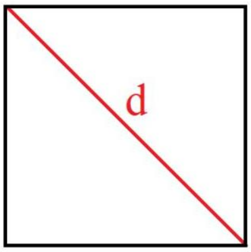
3.4 SQUARE ROOT

The Pythagoreans discovered irrational numbers, such as the square root of two. The Pythagoreans, followers of Pythagoras in ancient Greece, believed that everything in the universe could be explained by rational numbers, that is, those that can be expressed as the quotient of two integers. However, they discovered that the diagonal of a square could not be expressed in this way, which led to the discovery of irrational numbers. These discoveries have transformed our understanding of the world, leading to a crisis in ancient Greece. The Pythagoreans considered that numbers were the essence of the universe and that everything could be explained by them, favoring rational numbers. However, when trying to calculate the diagonal of a square, they realized that the square root of 2 could not be expressed as a fraction, which challenged their worldview.

Hippasus showed that the diagonal of a square could not be expressed as a fraction of two integers, meaning that it could not be expressed as a rational number. This finding came as a major shock to the Pythagoreans, who believed that all numbers could be expressed as ratios of integers.

Figure 6

Diagonal de Cuadrado, Àngels. (2015, July 15)



Hippasus' discovery of the irrational challenged the fundamental beliefs of the Pythagorean school, which valued the idea that everything in the universe could be explained through integers and rational relationships. It is said that the Pythagoreans, seeing themselves threatened by this new idea, regarded Hippasus' discovery as heresy.

History indicates that Hippasus paid a high price for his discovery; Some versions say that he was expelled from the Pythagorean community or even suffered a more tragic fate. However, his discovery laid the foundation for the further development of number theory and the understanding of the properties of irrational numbers.

The discovery of irrational numbers led to a fracture in his philosophy, resulting in the condemnation of Hippasus of Metapontum for revealing such a finding. Initially, the Pythagoreans tried to deny and conceal the existence of these numbers, but eventually, mathematicians such as Euclid formalized their existence, signaling an advance of mathematical thought over the ideological limitations of the sect. This represented a liberation of mathematics from the philosophical constraints imposed by the Pythagoreans.

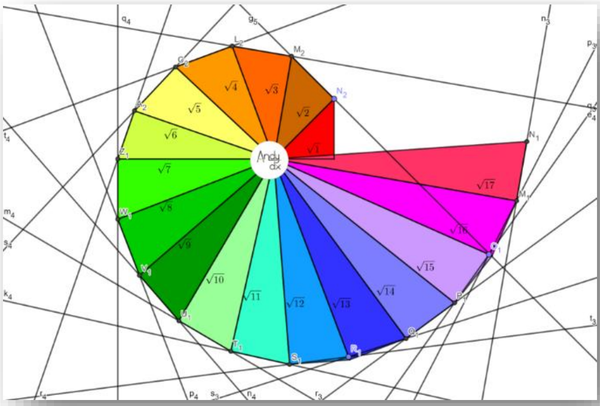
3.5 PYTHAGOREAN SNAIL

The Pythagorean snail is a tool for geometrically visualizing the square roots of consecutive integers.

The Pythagorean snail method, also known as Theodore's spiral or Pythagorean conch, is a geometric construction that illustrates the relationship between natural numbers and their square roots by means of a sequence of contiguous right triangles. Although attributed to Theodore of Cyrene, a disciple of Pythagoras, the method reflects the deep connection between geometry and number theory in ancient Greece.

Figure 7

Pythagorean snail, devours pereiro. (2020,w October 25)



Spiral Construction:

1. **First triangle:** Start with an isosceles right triangle whose legs measure 1 unit. Applying the Pythagorean theorem, the hypotenuse turns out to be . $\sqrt{2}$
2. **Successive triangles:** From the hypotenuse of the previous triangle, a new right triangle is constructed by adding a leg of 1 unit perpendicularly. The hypotenuse of this new triangle is calculated again using the Pythagorean theorem, obtaining √3. This process repeats itself, generating a sequence of triangles whose hypotenuses correspond to √2, √3, √4, √5, and so on.

Featured Properties:

Hypotenuse: The hypotenuses of constructed triangles form the sequence of square roots of consecutive natural numbers: √2, √3, √4, √5, etc.

Approximation of π: As more triangles are added to the spiral, the distance between the consecutive arms approaches the value of π, demonstrating a geometric relationship between the spiral and this fundamental mathematical constant.

Theodore's spiral is not only a visual demonstration of the irrationality of certain square roots, but it also highlights the interconnectedness between different areas of mathematics, such as geometry and number theory. This landmark method continues to be a valuable educational tool for illustrating fundamental mathematical concepts in tangible and accessible ways.

4 DISCUSSION

Research on the Pythagoreans, through the analysis of secondary sources, has revealed that their legacy cannot be reduced to a simple mathematical theorem. The study has made it possible to rediscover the holistic nature of his thought, where mathematics and spirituality were not separate disciplines, but parts of the same philosophical system. This fusion is the key to understanding their worldview and the impact they had on the development of Western thought.

We have seen that, for the Pythagoreans, numbers were not mere abstractions, but entities that possessed a mystical meaning and ordered reality. Music, for example, was not considered an art, but an audible manifestation of perfect numerical proportions, a reflection of cosmic harmony. Research shows that this perspective influenced later philosophers such as Plato, who adopted the idea that fundamental reality resides in ideal shapes and numbers. The modern separation between science (the rational and empirical) and religion (the mystical and spiritual) did not exist in Pythagorean thought. His philosophy is therefore a reminder that the pursuit of knowledge can be a path to a deeper understanding of existence, uniting intellect with contemplation.

5 CONCLUSIONS

Mathematics was not only a calculation tool for the Pythagoreans, but a means of purifying the soul and attaining an understanding of the divine order of the universe.

The research carried out confirms that the Pythagorean school was a pioneer in the fusion of science (mathematics and astronomy) with spirituality, setting a precedent in the history of Western thought.

For them, numbers were the key to deciphering the cosmos. They believed that numerical and geometric relationships underlay all of reality, from music to planetary motions.

The Pythagorean legacy transcended his time and influenced figures such as Plato and later movements such as Neoplatonism, demonstrating the deep mark his thought left on history.

This study rediscovers the Pythagoreans as a philosophical and religious society that had an integral vision of existence, in which rational knowledge was intertwined with the search for a mystical purpose.

REFERENCES

Aguilera, C. (2024, February 5). Pythagorean Theorem: What it is, some proofs and example of practical application. Smartick. https://www.smartick.es/blog/matematicas/geometria/teorema-de-pitagoras/

Àngels. (2015, July 15). How to calculate the diagonal of a square. Mundo Deportivo. https://www.mundodeportivo.com/uncomo/educacion/articulo/como-calcular-la-diagonal-de-un-cuadrado-40008.html

Boyer, C. B., & Merzbach, U. C. (2011). A history of mathematics (3rd ed.). John Wiley & Sons.

deborapereiro. (2020, October 25). Construct a Theodore spiral. GeoGebra. https://www.geogebra.org/m/cj3mjzyf

Euclid. (1991). The Elements (M. T. González de la Cueva, Trans.). Gredos. (Original work published ca. 300 BCE)

Epsilons. (n.d.). Stories: Irrationality of the root of two. Epsilons. Retrieved April 5, 2025, from https://www.epsilones.com/paginas/historias/historias-003-irracionalidad-raizdos.html

García, A. (2023, May 6). Theodore's Spiral: The Pythagorean Theorem Imprinted on the Universe. Andydx. https://andydx.com/objetos/la-espiral-de-teodoro/

Guzmán, J. H. (2021, December 29). Thales' Theorem: Explanation with Proof and Examples. Neurosparks. https://www.neurochispas.com/wiki/teorema-de-tales-explicacion-y-ejemplos/

Blacksmith. (2025, January 27). Teodoro's spiral and irrational numbers. Blogspot. https://herreramaths.blogspot.com/2025/01/la-espiral-de-teodoro-y-los-numeros.html

Marisofi, R. G. (2015, February 9). Origins of the Pythagorean theorem. Mathematics for Business. https://matematicasynegocios.wordpress.com/2015/02/09/origenes-del-teorema-de-pitagoras/

Plofker, K. (2009). Mathematics in India: 500 BCE–1800 CE. Princeton University Press.

Superprof AR. (n.d.). Mathematics: All about Thales and his famous theorem. Superprof AR. Retrieved April 5, 2025, from https://www.superprof.com.ar/blog/historia-tales-conocimiento-matematico/

Pedagogical and Technological University of Colombia. (n.d.). Mathematicians: Pythagoras (8 of 18) Pythagorean classification and naming of numbers. Edu.co. Retrieved February 16, 2025, from https://virtual.uptc.edu.co/ova/estadistica/docs/autores/pag/mat/Pitagoras8.asp.htm

Pedagogical and Technological University of Colombia. (n.d.). Mathematicians Biography: Pythagoras (11 of 18) Polygonal numbers. Edu.co. Retrieved September 18, 2025, from https://virtual.uptc.edu.co/ova/estadistica/docs/autores/pag/mat/Pitagoras11.asp.htm

Vila, V. C. (n.d.). Pythagorean crisis due to irrational numbers. Classes in pajamas. Retrieved April 5, 2025, from https://www.clasesenpijama.com/crisis-pitagorica-por-los-numeros-irracionales/

Wordpress. (n.d.). Image: Pythagorean theorem. Wordpress. Retrieved February 16, 2025, from https://matematicasynegocios.wordpress.com/wp-content/uploads/2015/02/4010d-fot1.gif