IMPACTO CIENTÍFICO
EDITORA

COMPUTATIONAL FLIGHT PREDICTION FOR MINI PET ROCKETS, APPLIED TO REFORESTATION

10.56238/edimpacto2025.005-008

Christiano Pedro Guirlanda[1], Pedro Calistro Pescatori Dutra[2], Paulo Tibúrcio Pereira[3], Gabriela Luiza Almeida Silva[4], Gabriela Ribeiro dos Anjos[5] and Juan Canellas Bosch Neto[6].

ABSTRACT

This work refers to modeling and computer simulation of PET mini-rockets with air and water propulsion and focuses on the development of a computer program for flight prediction of water- and air-propelled mini-rockets. The specific objectives of this work were: design and construction of a wind tunnel; experimental acquisition of the drag coefficients of mini-rockets in a wind tunnel; construction of launch bases for mini-rockets for the validation of the flight prediction computer program; mini-rocket projects and development of a computer program for mini-rocket flight prediction. The methodology began with the design and assembly of a wind tunnel to determine the drag coefficient of the mini-rockets. A mini-rocket launch base with water and air propulsion was developed, built and used. The thrust values of several mini-rockets were determined experimentally on a bench while keeping the mini-rocket fixed. Image and video analysis were performed in the bench test with the determination of the water flow at the exit of the rocket nozzle and determination of the thrust profile of the mini-rockets (PET 2 liters). The modeling and simulation of the flight prediction were developed from two systems of differential equations, based on the equations of conservation of mass and momentum, which were solved by the

[1] Federal University of São João Del Rey, Department of Chemical Engineering, Ouro Branco Campus
Lattes: http://lattes.cnpq.br/6800107712115887
Orcid : https://orcid.org/0000-0002-7909-6951
E-mail: cpguirlanda@gmail.com

[2] Federal University of São João Del Rey, Department of Chemical Engineering, Ouro Branco Campus
Lattes: http://lattes.cnpq.br/0063417636531775
Orcid: https://orcid.org/0009-0009-7759-1631
E-mail: pedrocalistropd@gmail.com

[3] Federal University of São João Del Rey, Department of Chemical Engineering, Ouro Branco Campus
Lattes: http://lattes.cnpq.br/9589927393728395
Orcid: https://orcid.org/0000-0003-2344-5777
E-mail: paulotiburcio@ufsj.edu.br

[4] Federal University of São João Del Rey, Department of Chemical Engineering, Ouro Branco Campus
Lattes: http://lattes.cnpq.br/6594736090145487
Orcid: https://orcid.org/0009-0002-6834-6887
Email: gabi-lsilva@hotmail.com

[5] Federal University of São João Del Rey, Department of Chemical Engineering, Ouro Branco Campus
Lattes: https://lattes.cnpq.br/1931654776334838
Orcid: https://ORCiD: 0009-0006-7063-8431
E-mail: gabrielaribeeiro@yahoo.com

[6] Federal University of São João Del Rey, Department of Chemical Engineering, Ouro Branco Campus
Lattes: http://lattes.cnpq.br/0529973259046894
Orcid: https://orcid.org/0000-0002-2719-7689
Email: jboschbr@ufsj.edu.br

Runge Kutta numerical method. It was necessary to solve two systems of nonlinear differential equations, one for the Propulsion Phase and the other for the oblique launch. The computer program has data input defined by the user and that generates the various outputs in matrix format or in the format of figures. The simulation of the mini-rocket's flight prediction resulted in figures containing the most important flight parameters. The results of the computer program were validated through the field launches of the mini-rockets and it was found that it is an important tool for the design of mini-rockets intended for the reforestation of degraded areas.

Keywords: Mini rockets. Wind tunnel. Flight prediction program. Reforestation.

INTRODUCTION

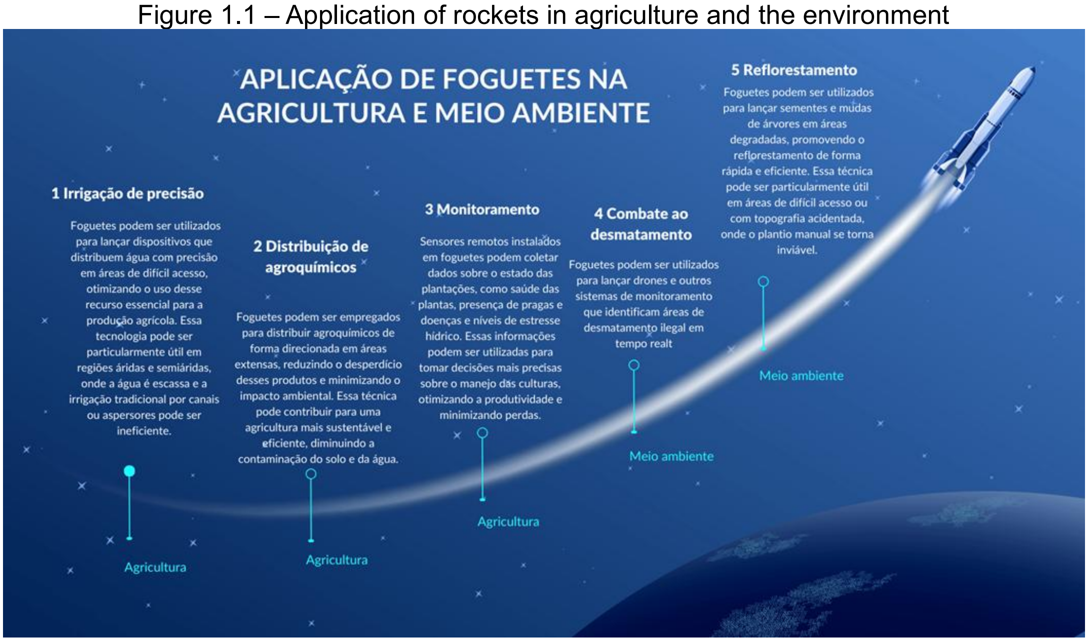
The ecological restoration of degraded areas, an urgent global challenge, faces multiple obstacles that demand innovative solutions. Habitat fragmentation, resource scarcity, and the complexity of degradation processes are just some of the challenges to be overcome [[1]].

Reports have pointed out that approximately one-third of the planet's arable land is already degraded, indicating the urgent need to recover or remedy the damage caused by poor agricultural practices and other factors [[2]].

The use of rockets for ecological restoration purposes presents unique challenges. The precision in seed deposition, the selection of the most suitable species for each environment and the evaluation of long-term environmental impacts are issues that require in-depth research. In addition, the costs associated with the development and operation of seed launching systems can be high, limiting their large-scale application.

The use of rockets in agriculture and in the environmental area, although still under development, opens up a range of innovative possibilities for the management of natural resources and environmental preservation. This technology, traditionally associated with space exploration, has the potential to optimize agricultural practices, monitor the environment, and combat challenges such as deforestation and desertification, as shown in Figure 1.1.

Figure 1.1 – Application of rockets in agriculture and the environment



Source: the authors

The specific objectives of this work were: design and construction of a wind tunnel; experimental acquisition of the drag coefficients of mini-rockets in a wind tunnel;

construction of launch bases for mini-rockets for the validation of the flight prediction computer program; design of mini-rockets and the development of a computer program for mini-rocket flight prediction.

LITERATURE REVIEW

Aerial seeding is a technique for direct transmission of seeds by the use of aerial vehicles, such as drones, airplanes or helicopters [3]. The method of launching the seeds chosen (aerial, manual or mechanized) influences the costs, with methods by planes generally more expensive.

The aerial seeding technique for reforestation has been employed since the 1950s [4]. In industrialized countries, aerial seeding is already considered a practical reforestation technique with considerable success reported in the United States of America (USA), Canada, Australia, Russia, India and New Zealand [5].

One promising approach to accelerate the reforestation of degraded areas is rocket seed dropping. This technique, still under development, offers the possibility of reaching areas that are difficult to access, such as steep slopes and remote regions, facilitating the dispersal of seeds of native species and the restoration of vegetation cover.

Ecological applications of rockets have been proposed [6], aiming to restore degraded areas in the caatinga biome of Brazil. Rockets made of biodegradable cassava starch plastic demonstrate the potential of rocket seed launching to restore degraded areas in different biomes. However, it is critical that this technology is developed sustainably and integrated with other ecological restoration practices.

A more cost-effective alternative may be PET water rockets, also known as PET bottle rockets, which were conceived from the idea of American engineer and physicist Robert Goddard [7,8]. PET Rockets are a pedagogical, functional and sustainable tool that combines scientific principles with creativity and ingenuity [6]. Built with simple materials and launched using only water, these rockets provide a hands-on and engaging experience for students of all ages, sparking curiosity about concepts such as propulsion, aerodynamics, and physics.

The fundamental principle behind the flight of a PET rocket into water is Newton's third law, which states that for every action, there is an equal and opposite reaction. The force exerted by the pressurized water on the bottle generates a reaction force of equal intensity, but in the opposite direction, which propels the bottle upwards. This interaction between the action of water on the bottle and the reaction of the bottle on water is a classic example of Newton's third law [9,10].

In addition to Newton's third law, several other quantities related to aerodynamics influence the flight of a PET rocket, areas of knowledge such as Aerospace Sciences, Physics, Mathematics and Chemistry are related to the functionality and flight behavior of the rocket as well as the design of the structure, which is divided into a hood, body and fins [11]. The aerodynamic shape of the PET bottle, with its conical base and narrow nozzle, contributes to the stability and range of the rocket. Air flows more easily over the curved surface of the bottle, creating an area of low pressure at the top, while high pressure at the bottom propels the rocket upwards [12].

The use of computer simulations for research and teaching provides students and researchers with an interactive and dynamic experience of knowledge [13]. By manipulating variables and collecting data in virtual environments, the construction of knowledge about physical phenomena develops essential skills such as data analysis and problem solving. This pedagogical approach, which explores the potential of digital technologies, has been shown to be effective in making knowledge of the physical sciences more engaging and meaningful [14,15].

The complexity of the phenomena involved in space launches requires the use of differential equations to accurately describe the relevant variables and predict the behavior of the system [16]. They allow modeling and analyzing the behavior of dynamic systems, such as mathematically describing the forces acting on a rocket during launch, such as buoyant force, gravitational force, air resistance, and centripetal force. Some rocket parameters, such as mass, speed and altitude, vary continuously during launch. The differential equations make it possible to model these variations over time, optimize optimal trajectories for the rocket, maximizing the payload or minimizing fuel consumption, analyze the stability of the rocket's flight, ensuring that it does not go into rotation or oscillate in an uncontrolled way.

The differential equation systems for modeling mini-rocket launches are based on the equations of conservation of mass and momentum, exemplified in the nonlinear differential equations 2.1 to 2.3, generating a system of differential equations, which can be solved by the Runge-Kutta numerical method.

A momentum conservation balance can be performed in the mini-rocket flight prediction modeling considering the forces acting on the system: gravitational and nonlinear drag force (Equations 1, 2 and 3). A system of nonlinear differential equations can be set up for the mini-rocket's propulsion step. Differential equations 2 and 3 are relative to the conservation of momentum in y and x, considering the gravitational force and drag forces acting on the minirocket [17].

$$\Sigma\vec{F}saem - \Sigma\vec{F}entram + \overrightarrow{\Sigma F}acúmulo = \overrightarrow{\Sigma F}externas \quad (1)$$

$$-\dot{m}.v_y.\cos(\theta) - 0 + \frac{d(m.v_y)}{dt} = 0 \quad - 0.5.Cd.\rho \quad (2)$$

$$-\dot{m}.v_x.\mathrm{sen}(\theta) - 0 + \frac{d(m.v_x)}{dt} = 0 \quad - 0.5.Cd.\rho.A.v_x{}^2 \quad (3)$$

Where (v_y) is the velocity in the direction of the y-axis, where) is the velocity in the direction of the x-axis, launch angle(θ), drag coefficient(Cd), cross-sectional area of the mini-rocket(A), mass of the mini-rocket(m), air density (ρ), gravitational acceleration(g), and water flow rate at the outlet of the mini-rocket's nozzle(.$(v_x \dot{m})$

In the literature, it is addressed in several works [16], the resolution of these equations by algebraic methods, using simplifications such as: linearized drag force equation, constant mass system and a smaller number of dynamic variables, generating more imprecise results and with a smaller number of flight parameters.

The difference of this work in relation to those found in the literature is that in this one we addressed the dynamic modeling for flight prediction [17], using two systems of nonlinear equations with twenty-two dynamic variables, one system for the propulsion stage and the other for the oblique launch. The conservation equations were solved by the Runge-Kutta method, finding results for various parameters of the mini-rocket's flight.

METHODOLOGY

MATERIALS

Dynamometer

The AMF 5 tensile and compression force meter measures up to 5 N, with an accuracy of 0.01 N was used to predict the drag forces of the mini-rockets and later determine the drag coefficient.

Fog Machine

The smoke generator device has a mini vacuum pump that produces a pressure of 0.9 bar and a flow rate of 13 liters per minute, a steel cabinet with rubber feet and a handle, a long silicone hose with 2m and a conical smoke outlet nozzle with diameters from 6 to 15 mm and a hole with a diameter of 4 mm. The smoke machine had the purpose of visualizing the flow of air on the surface of the mini-rockets.

Winding tunnel

The wind tunnel was designed, assembled and built aiming at determining the drag coefficients of the mini-rockets and visualizing the flow profile.

Anemometer

With the CB-8909 anemometer, the air speed in the wind tunnel was measured. The anemometer probe has height adjustment up to 40 cm in length, allowing the user to position the instrument in the desired location, in order to obtain air speed measurements at various points in the wind tunnel.

Mini-rocket launch base, water and air propulsion.

The mini-rocket launch base was designed and built in stainless steel, contains a trigger for releasing the rocket at a certain pressure, was used to carry out several experimental mini-rocket launches

METHODOLOGY

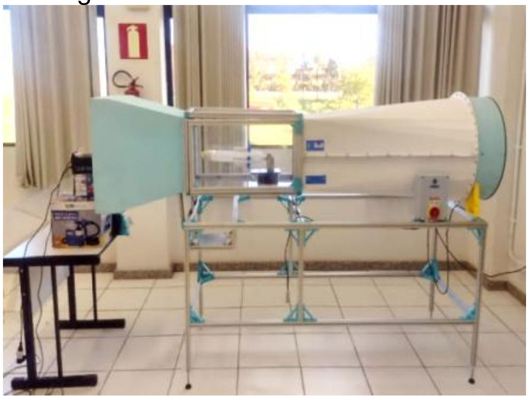
Construction of the wind tunnel

The wind tunnel was designed, assembled and built by the research group, with support from FAPEMIG and is available for use in the Integrated Engineering Laboratory of CAP-UFSJ, as shown in Figures 3.1 and 3.2.

Figure 3.1 – Wind tunnel in the process of assembly

Source: the authors

Figure 3.7 – Assembled vent tunnel



Source: the authors

The wind tunnel structure was assembled by the research group and has an aluminum profile suitable for operations and movements in research laboratories. The wind tunnel has an air inlet with honeycomb structures to reduce turbulence and transform the airflow to the laminar regime. Dimensions: viewing window, sectional area of 400mmx400mm and length of 400mm, followed by suction area with fan. The industrial fan has a power of 3/4 Hp with a diameter of 60 cm, coupled to the tunnel outlet. At the top of the viewing window, there are sampling points for air velocity measurements, through the insertion of a hot wire anemometer and points for sampling the drag force through a dynamometer.

Tests to determine the drag coefficient of mini-rockets

The project began with the tests to determine the drag coefficient of the mini-rockets in the wind tunnel.

Equation 4 relates the drag force with the drag coefficient, cross-sectional area of flow, air flow velocity in the wind tunnel, it was possible to calculate the drag coefficient of the mini-rockets.

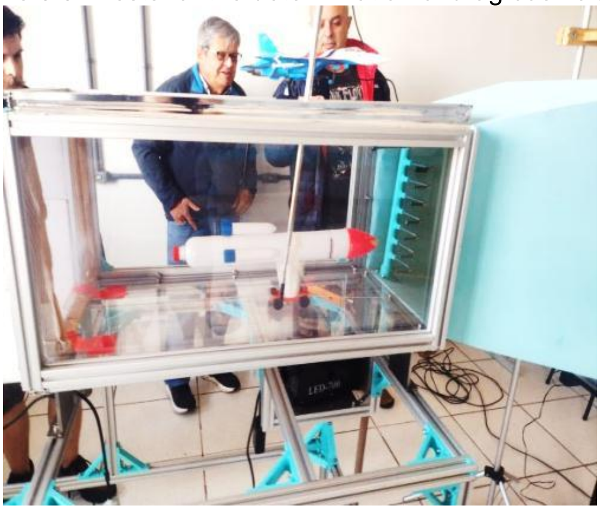
$$F_d = 0.5 \, . \, C_d . \, \rho . \, A . \, v^2 \qquad (4)$$

Where: (F_d) is the drag force [N], C_d is the drag coefficient [-], (ρ) air density [kg/m3], (A) transverse flow area [m2] and v is the air velocity [m/s].

The industrial fan attached to the wind tunnel has a rotation adjustment system that allowed the air velocity to be set to 10 m/s, with the aid of an anemometer. An assembly was carried out for the measurements in the wind tunnel that contained a cart and a *nylon line* coupled to a dynamometer. The drag forces of the mini-rockets were obtained by the difference in forces read between the mini-rocket with the cart and with the cart alone. The cross-sectional areas of the mini-rockets were calculated and the air density was determined under the atmospheric conditions of the test.

Figure 3.3 shows a photo of a test to determine the drag coefficients in the research group's laboratory.

Figure 3.3 - Tests for the determination of drag coefficients



Source: the authors

From the results of the experimental tests of item 3.2.2 and through Equation (4) it was possible to determine the drag coefficients (Cd) for each minirocket,

Construction of mini-rocket launch bases

Two mini-rocket launch bases with water and air propulsion were developed and built. The bases contain a trigger for release at a certain pressure, as shown in Figure 3.4.

Figure 3.4. The mini-rocket launch base with water and air propulsion

Source: the authors

With the launch base built, experimental launches of biodegradable rockets can be carried out for validation of the flight prediction program.

The material used in the mini-rockets was polyethylene (PET). The biodegradable material for the mini-rockets is in the testing phase and will possibly be made of (PLA), which is a biodegradable polymer and will be molded in a 3D printer.

Determination of mini-rocket thrust

The thrusts of the mini-rockets were determined through experimental tests on a bench keeping the rockets fixed. A content of thirty percent water was inserted into the mini-rocket (PET), an optimal value for propulsion. The value of the release pressure was recorded. From the analysis of images in video editing software, it was possible to outline the water flow profile at the outlet of the nozzle over time.

The thrusts of the mini-rockets were determined by Equations 5 and 6 from the equations of conservation of mass and momentum.

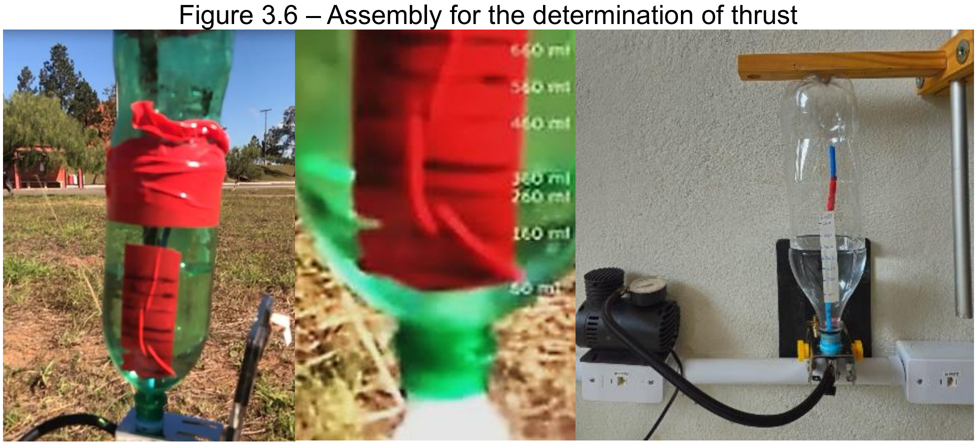
$$E = \dot{m} . v \tag{5}$$

$$v = \dot{m} / (\rho . A) \tag{6}$$

Where: E is the thrust of the mini-rocket [N], the mass flow rate of water at the outlet of the nozzle [kg/s], v velocity of output at the nozzle of the rocket [m/s], the transverse flow area of the rocket outlet nozzle [m3], and $\dot{m}\rho$ the density of the water [kg/m3].

Figure 3.6 shows assembly for the determination of thrust

Figure 3.6 – Assembly for the determination of thrust



Development of the computer program for flight prediction

The modeling and simulation of the flight prediction were carried out through the development of two systems of differential equations based on the equations of conservation of mass and momentum, which were solved by the Runge Kutta fourth-order numerical method using *OCTAVE GNU software*. The thrust values of the mini-rockets, determined in item 3.2.4, were used. By analyzing the results of the experimental tests, it was considered that the thrust is generated by the expulsion of water from the system, since the air mass of the system is insignificant in relation to the mass of water, which is ejected from the system and which generates the reaction force responsible for the propulsion of the system. After the short propulsion period (Phase 1), the mini-rocket moves to (Phase 2), equivalent to an oblique launch. Thus, it was necessary to develop two systems of nonlinear differential equations, one for each phase. The two systems of differential equations were solved in series.

Equation of the system

Phase 1 – Propulsion

A momentum conservation balance (Equation 7) was used to assemble the system of nonlinear differential equations for Phase 1. The first two differential equations for propulsion, Phase 1, are relative to the conservation of momentum in x and y (Equations 8 and 9), considering the gravitational force and drag forces.

$$\Sigma\vec{F}entram - \Sigma\vec{F}saem + \overrightarrow{\Sigma F}acumulo = \overrightarrow{\Sigma F}externas \quad (7)$$

$$0 \qquad - (-E.\cos(\theta)) + \frac{d(m.v_y)}{dt} = -m.g - 0.5.Cd.\rho.A.{v_y}^2 \quad (8)$$

$$0 \qquad -(-E.sen(\theta)) + \frac{d(m.v_x)}{dt} = 0 \quad - 0.5.Cd.\rho.A.{v_x}^2 \quad (9)$$

The dynamic variables chosen for the assembly of the differential equation system were: velocity in y), velocity in x), displacement in y (Y), displacement in x (X), launch angle (θ), drag coefficient (Cd), cross-sectional area of the mini-rocket (A), propulsion time (Temprop), mass of the mini-rocket (m), air density (ρ), mass of the rocket (m), thrust (E) and flow rate (.(v_y (v_x $\dot{m}$)

$$\frac{dy}{dt} = v_y \tag{10}$$

$$\frac{dx}{dt} = v_x \tag{11}$$

$$\frac{d\theta}{dt} = 0 \tag{12}$$

$$\frac{dCd}{dt} = 0 \tag{13}$$

$$\frac{dA}{dt} = 0 \tag{14}$$

$$\frac{dTemprop}{dt} = 0 \tag{15}$$

$$\frac{dm}{dt} = -\dot{m} \tag{16}$$

$$\frac{d\dot{m}}{dt} = 0 \tag{17}$$

$$\frac{dE}{dt} = 0 \tag{18}$$

$$\frac{d\rho}{dt} = 0 \tag{19}$$

The system of nonlinear differential equations of Phase 1 had a total of twelve dynamic variables considered, Equations 8 to 19 with their respective initial conditions were solved by the Runge Kutta fourth-order numerical method. The mini-rocket thrust value, determined in item 3.2.4, drag coefficients determined in item 3.2.2 and the initial conditions necessary for each differential equation were used. Figure 3.7 shows the FUNCTION subroutine of Phase 1 propulsion, which is called by the main program.

FIGURE 3.7 - FUNCTION of Phase 1 propulsion, which is called by the main program.

```
function f=propulsao(t,Y)
% Vy     Vx    y    x    angulo  Cd    Areapet  Tempopropulsao  massa  Vazao  Empuxo  Densar
% Y(1)  Y(2)  Y(3)  Y(4)  Y(5)   Y(6)   Y(7)     Y(8)           Y(9)   Y(10)  Y(11)   Y(12)

 g=9.81;

 % Sistema de EDos propuilsão
 f(1,1)=-g+((sind(Y(5))*Y(11))/Y(9))-(Y(1)/Y(9))*Y(10)-(0.5*Y(6)*Y(12)*Y(7)*(Y(1)^2))/Y(9);
 f(2,1)=   ((cosd(Y(5))*Y(11))/Y(9))-(Y(2)/Y(9))*Y(10)-(0.5*Y(6)*Y(12)*Y(7)*(Y(2)^2))/Y(9);
 f(3,1)=Y(1);
 f(4,1)=Y(2);
 f(5,1)=0;
 f(6,1)=0;
 f(7,1)=0;
 f(8,1)=0;
 f(9,1)=-Y(10);
 f(10,1)=0;
 f(11,1)=0;
 f(12,1)=0;
  % [tempo1,s]=ode45('propulsao',[0 tempoprop],[0 0 0 0 angulo cd areapet tempoprop massatotal vazao E densar])
```

Source: the authors

Phase 2 - Oblique Launch

For the oblique launch (Phase 2) nonlinear differential equations were used, similar to those of Phase 1, but without the buoyant force. The first two differential equations for Phase 2 are relative to the conservation of momentum in x and y (Equations 20 and 21), considering the gravitational force and drag forces.

$$\frac{d(m.v_y)}{dt} = -mg - 0.5.Cd.\rho.A.{v_y}^2 \quad (20)$$

$$\frac{d\,(m.v_x)}{dt} = -0.5.Cd.\rho.A.{v_x}^2 \quad (21)$$

The dynamic variables chosen for the assembly of the Differential Equations system were: velocity in y), velocity in x), displacement in y (Y), displacement in x (X), launch angle (θ), drag coefficient (Cd), cross-sectional area of the mini-rocket (A), oblique flight time in Phase 2 (tempodevoooblique), mass of the mini-rocket (m) and air density (ρ).$(v_y$ $(v_x$

$$\frac{dy}{dt} = v_y \quad (22)$$

$$\frac{dx}{dt} = v_x \quad (23)$$

$$\frac{d\theta}{dt} = 0 \quad (24)$$

$$\frac{dCd}{dt} = 0 \quad (25)$$

$$\frac{dA}{dt} = 0 \quad (26)$$

$$\frac{dTempovooobliquo}{dt} = 0 \quad (27)$$

$$\frac{dm}{dt} = 0 \quad (28)$$

$$\frac{d\rho}{dt} = 0 \quad (29)$$

The system of nonlinear differential equations of Phase 2 had a total of ten dynamic variables considered, Equations 20 to 29, with their respective initial conditions and which was solved by the Runge Kutta fourth-order numerical method. Figure 3.8 shows the FUNCTION subroutine of Phase 2, oblique throw, which is called by the main program.

FIGURE 3.8 - *FUNCTION* of Phase 2, oblique throw.

```
function f=obliquo(t,Y)
% Vy   Vx    y   x    angulo  Cd    Areapet Tempo max  m    densar
% Y(1)  Y(2)  Y(3)  Y(4)   Y(5)  Y(6)   Y(7)    Y(8)  Y(9)   Y(10)

 g=9.81;
 f(1,1)=-g-((0.5*Y(6)*Y(10)*Y(7)*(Y(1)^2)))/Y(9);
 f(2,1)=(-0.5*Y(6)*Y(10)*Y(7)*(Y(2)^2))/Y(9);
 f(3,1)=Y(1);
 f(4,1)=Y(2);
 f(5,1)=0;
 f(6,1)=0;
 f(7,1)=0;
 f(8,1)=0;
 f(9,1)=0;
 f(10,1)=0;

%  [tempo,s2]=ode45('obliquo',[tempoprop tob],[s(ul,1) s(ul,2) s(ul,3) s(ul,4) angulo cd areapet tempoprop (massa+semente) densar])
```

Source: the authors

With the two subroutines ready, the main program was prepared that unites Phase 1 and 2 in series (propulsion and oblique launch). The computer program has user-defined data inputs that generate the various outputs in matrix format or in the form of figures. The program was used to predict several launches with loads and with different angles of launches.

The inputs from the computer program are requested as shown in Figure 3.9

FIGURE 3.9 – Computer program inputs

```
pressao=input('Entre com a Pressão de lançamento em PSI, sugestão de 60 a 140 psi: ')
massa=input('Entre com Massa do Foguete vazio (kg)')
angulo=input('Angulo de lançamento (graus) de 10 a 90 graus: ')
liquido=input('Massa de água no foguete(kg): ')
semente=input('Massa de sementes (kg): ')
cd=input('Entre com o Cd do foguete: ')
areapet=input('Entre com a área transversal do PET : ')
densar=input('Entre com a densidade do ar (kg/m3): ')
```

Source: the authors

The validation was carried out by comparing the data from the computer simulation with the data from the real experimental launches, carried out with the launch base and the mini-rockets.

RESULTS

RESULTS FOR DRAG COEFFICIENTS FOR MINI-ROCKETS

The results for the drag coefficients for the mini-rockets were obtained according to the methodology described in item 3.2.2. Table 1 shows the values of the drag coefficients of the mini-rockets for different design configurations.

Table 1 - Values of the drag coefficients of the mini-rockets for different configurations.

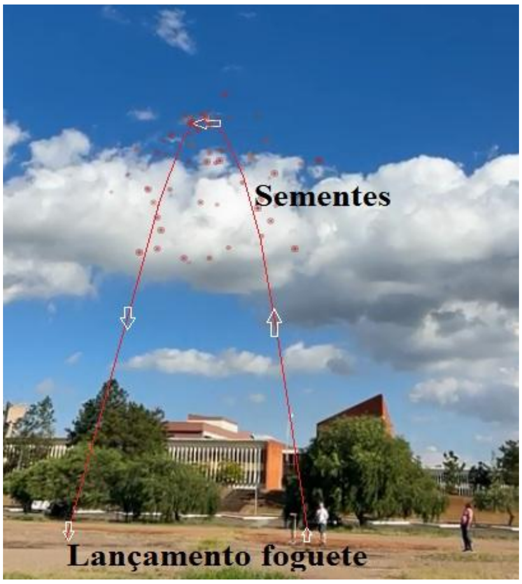
Mini Fog (Type)	Sweeping coefficient (Cd)	Measurement error
Minifoguete 1	0,13	+/- 0,01
Minifoguete 2	0,28	+/- 0,01
Minifoguete 3	0,20	+/- 0,01
Minifoguete 4	0,38	+/- 0,01
Minifoguete 5	0,51	+/- 0,01

A significant difference was observed in the values of the drag coefficients, because the mini-rockets have very different design configurations, impacting the aerodynamic profile and the response of the drag coefficient.

TESTS WITH THE LAUNCH PAD

Figure 3.5 shows the launch of a mini-rocket with water and air propulsion using the launch base for the validation of the flight prediction computer program and analysis of the rocket designs. Launch pressure and maximum range for the mini-rockets with 2-liter PET boosters were noted and compared with the results of the flight prediction computer program.

Figure 3.5 - Launch of a mini-rocket with water and air propulsion

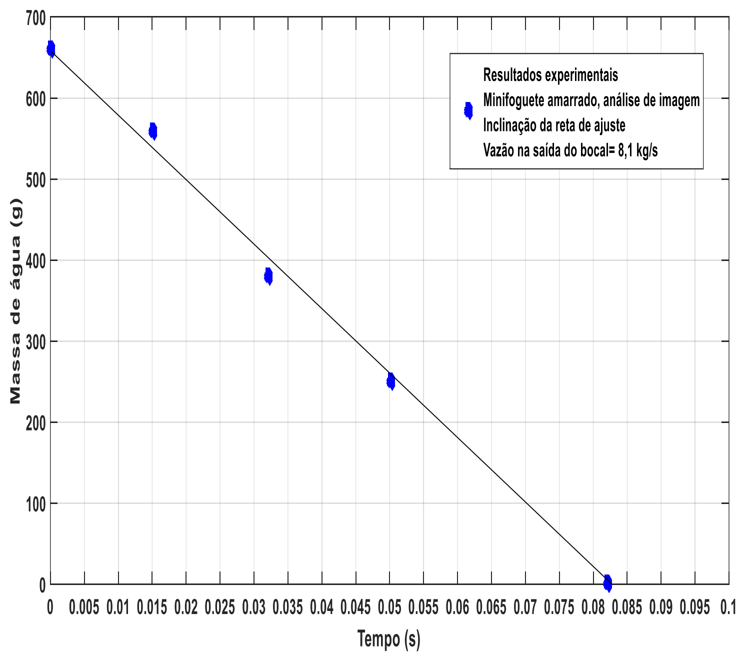


Source: the authors

RESULTS FOR MINI-ROCKET THRUST

The thrust of the mini-rockets was obtained according to the methodology described in item 3.2.3. Figure 4.1 shows the thrust profile of a mini-rocket obtained experimentally at a launch pressure of 70 Psi.

Figure 4.1 - Thrust profile of an experimentally obtained mini-rocket.



It can be seen in Figure 4.1 that the flow rate at the outlet of the nozzles of the mini-rockets powered by water and air has a constant value of 8.1 kg/s, consequently the thrust is constant and is generated by the expulsion of water from the system, resulting in a reactional force, according to Newton's third law. The thrust of this mini-rocket was calculated by Equations 3.2 and 3.3 and resulted in a value of 145 Newtons.

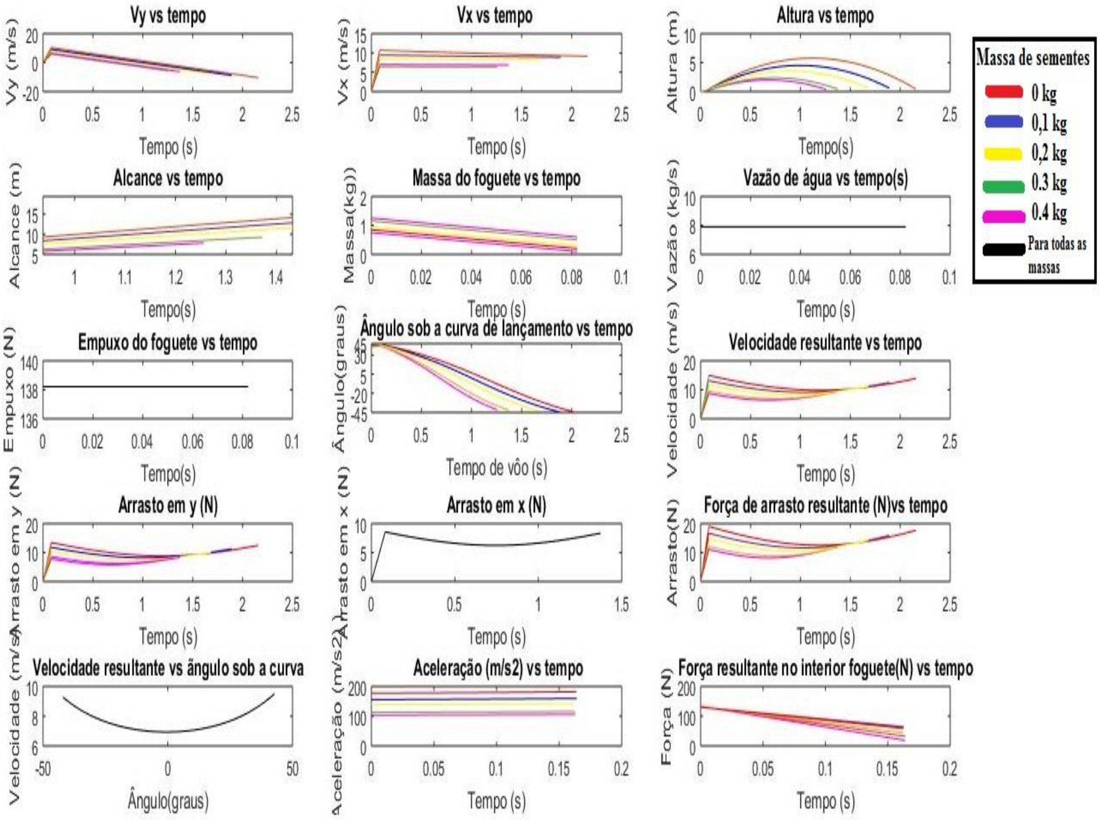
$$E = \dot{m}.(\dot{m}/\rho.A) = 8.1\ x\ 8.1/(1000\ x\ \ 0.0004523) = 143\ N$$

RESULTS OF THE COMPUTER PROGRAM

The system of differential equations with twelve dynamic variables (Phase 1 Propulsion) and the system of differential equations with ten dynamic variables (Phase 2 Oblique Throw) were solved in series by the RUNGE-KUTTA numerical method, generating Figures 4.2 to 4.6 of this item. The results of the computer program for the flight prediction of the minirockets are shown in Figures 4.2, 4.3, 4.4, 4.5 and 4.6.

Figure 4.2 shows the simulation results obtained in the flight computer program for different seed loads carried by the mini-rockets, 0 kg, 0.1 kg, 0.2 kg, 0.3 kg and 0.4 kg. The launch pressure was 70 psi, launch angle forty-five degrees, percentage of water in the mini-rocket thirty percent, and drag coefficient 0.2.

Figure 4.2 - Results of the computer program for different seed masses

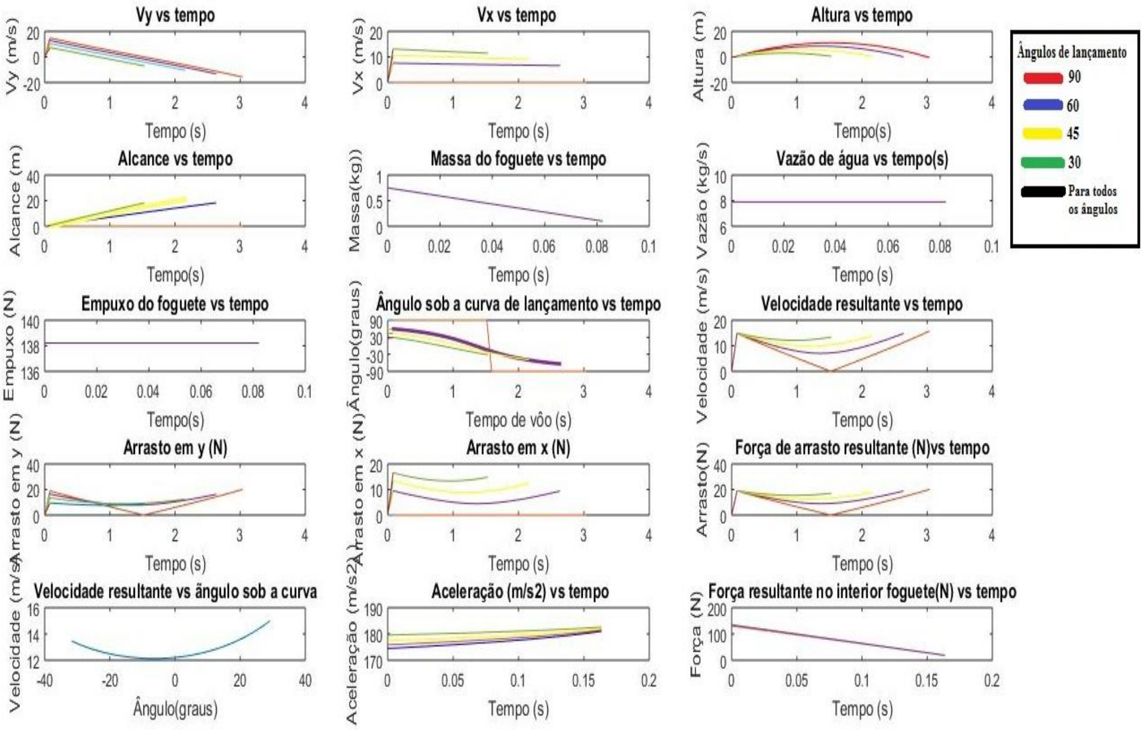


Source: the authors

A decrease in the mini-rocket range and height profile was observed, with the increase in the load of the mini-rockets, being a critical factor for the system (load).

Figure 4.3 shows the simulation results obtained using the flight computer program for different launch angles 0°, 30°, 45°, 60° and 90°. The launch pressure used was 70 psi, the percentage of water in the minirocket was 30% and the drag coefficient was 0.2 [-].

Figure 4.3 - Results of the computer program for different launch angles

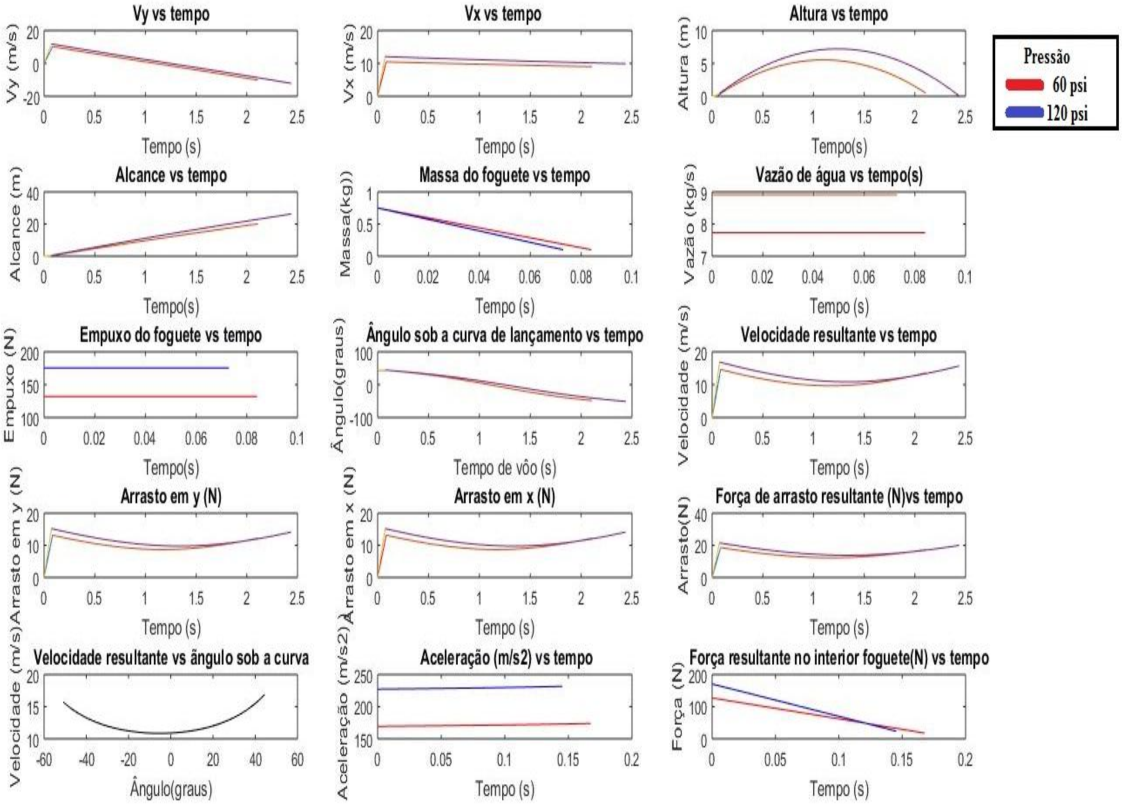


Source: the authors

It was observed that for the angles selected in the simulation: 0o, 30o, 45o, 60o and 90o , the angle that maximized the range was forty-five degrees.

Figure 4.4 shows the results of the flight prediction computer program, with launch pressures of 60 psi and 120 psi. It was observed from the results that the higher the launch pressure, the greater the range of the 2L PET mini-rocket.

Figure 4.4 - Flight forecast, with launch pressures of 60 psi and 120 psi for the 2L PET mini-rocket

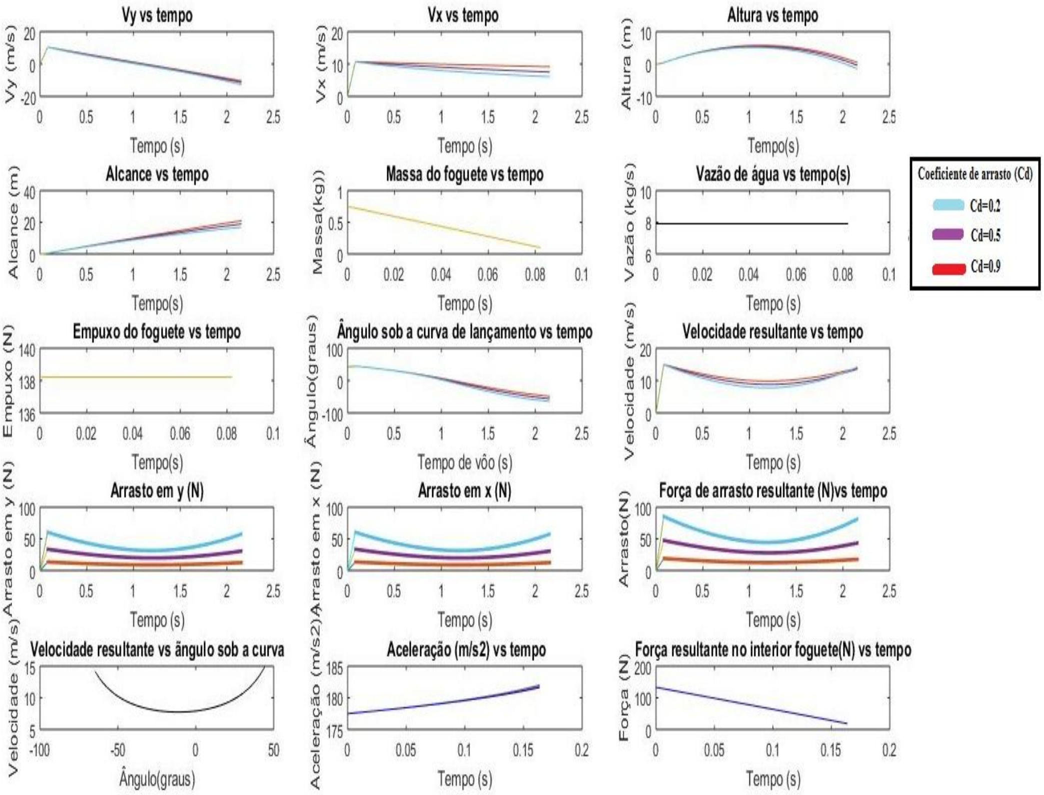


Source: the authors

It was observed from the results that the higher the launch pressure, the greater the range of the mini-rocket.

Figure 4.5 shows the results of the flight prediction computer program, for drag coefficients (Cd) with values: 0.2 [-], 0.5 [-] and 0.9 [-].

Figure 4.5 - Results of the flight prediction computer program for different drag coefficients (Cd).

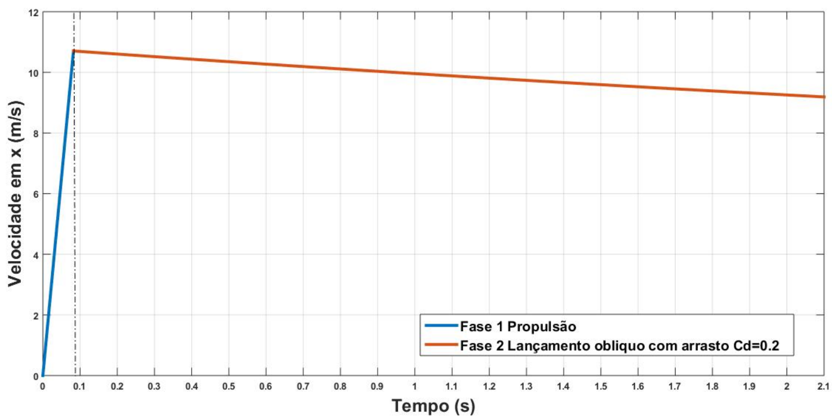


Source: the authors

Figure 4.5 shows that the higher the drag coefficient Cd[], the higher the value of the net drag force (N) on the minirockets.

Figure 4.6 shows one of the results of the flight prediction computer program describing Phase 1 (propulsion) and Phase 2 (oblique launch).

FIGURE 4.6 - Phase 1 (propulsion) and Phase 2 (oblique launch).

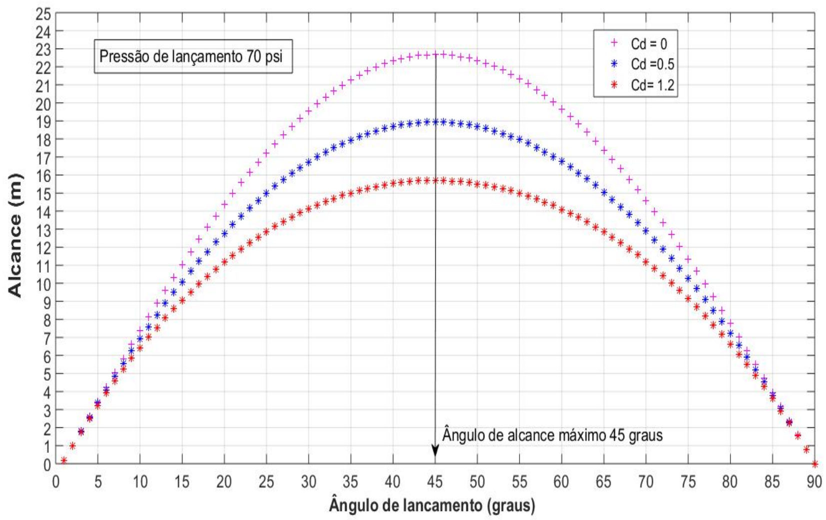


Source: the authors

Figure 4.6 shows the results of the two systems of differential equations for the propulsion and oblique launch steps.

Figure 4.7 shows the profile of the mini-rocket range curve for different launch angles considering the drag force.

Figure 4.7 - Mini-rocket range curve profile for different launch angles

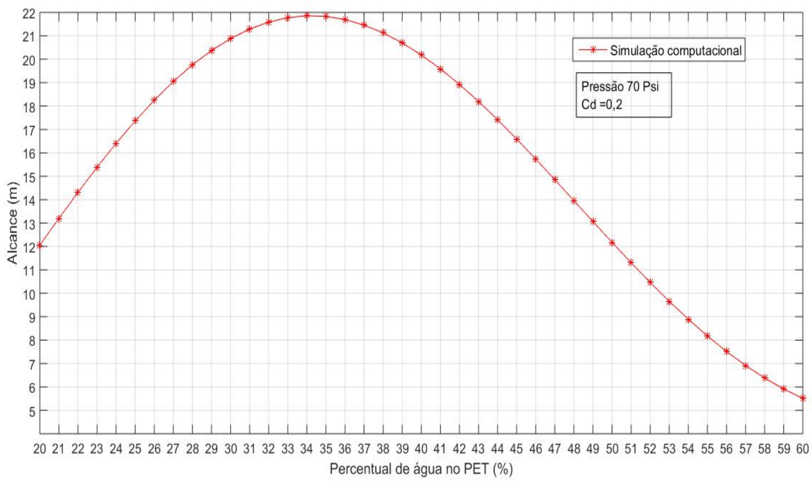


Source: the authors

It was observed by computer simulation that the launch angle that maximizes the range of the mini-rocket is forty-five degrees.

Figure 4.8 shows the profile of the mini-rocket range curve for different percentages of water in the mini-rocket's propulsion system.

Figure 4.8 - Profile of the mini-rocket range curve for different percentages of water in the propulsion system of the mini-rocket for 2 L PET.



Source: the authors

It was observed by computer simulation that the percentage of water in the mini-rocket's propulsion system that maximizes range is 34 percent, approximately 1/3 of the volume of the propulsion system.

Figure 4.9 shows the profile of the mini-rocket's maximum range curve for different launch pressures.

Figure 4.9 - Profile of the mini-rocket maximum range curve for different launch pressures.

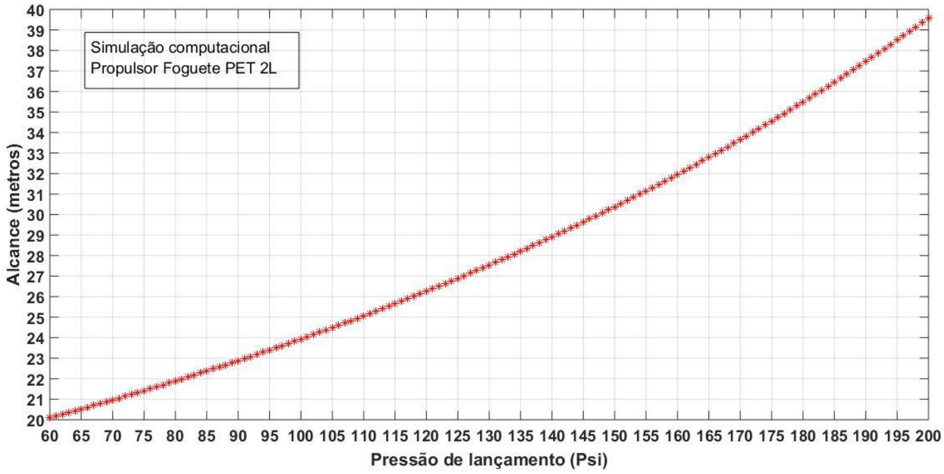
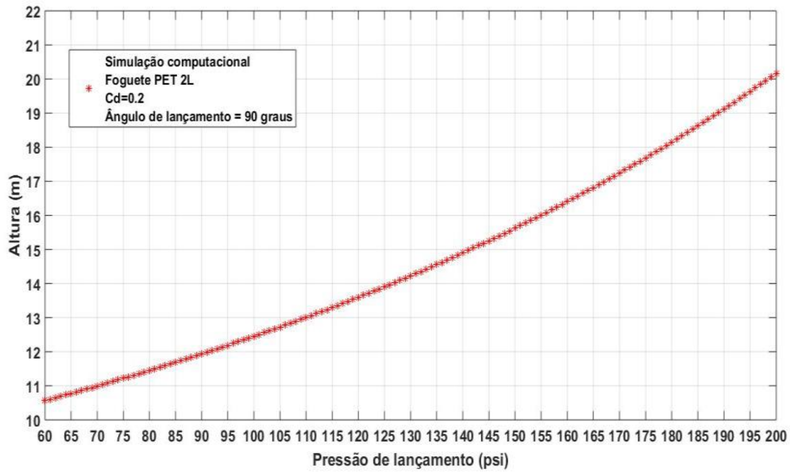


Figure 4.10 shows the maximum height profile reached for different launch pressures of the PET 2L mini-rocket.

Figure 4.10 - Maximum height profile of the mini-rocket for different launch pressures.



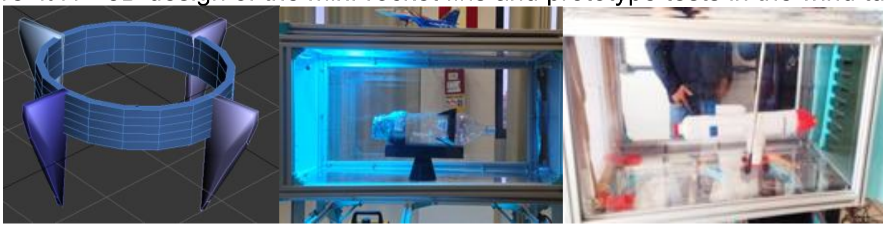
VALIDATION OF THE COMPUTER SIMULATION FROM FIELD LAUNCHES

The validations of the computer simulations were performed by comparing the parameters of several experimental launches, resulting in values close to and with errors of less than ten percent. The discrepancies between simulated and experimental values can be explained by local variations in wind speed and variations in the stability of the mini-rocket (center of mass). The validation of the computer program was carried out by comparing the results of the simulation with the data from the real launches of the mini-rockets.

DESIGN OF THE MINI-ROCKET FINS

Figure 7 shows a three-dimensional design of a fin for mini-rockets made with biodegradable material (PLA) and printed on a 3D printer. The project was carried out in a computer environment.

Figure 4.11 - 3D design of the mini-rocket fins and prototype tests in the wind tunnel



CONCLUSION

The designed and built wind tunnel was very useful for determining the drag coefficients of the mini-rockets.

The assembly for the determination of the output flow rate in the nozzle of the minirocket was necessary for the calculation of the thrust of the minirocket. The thrust of a mini-rocket has been determined and is constant throughout the Propulsion Phase.

The construction of the mini-rocket launch base was very important for carrying out the experimental launches, mini-rocket designs and validation of the flight prediction computer program.

The two computational routines developed contain two systems of nonlinear differential equations, with 22 dynamic variables, representing the Propulsion Phase, ~~and~~ Oblique Launch and resulted in figures that contain the most important parameters such as: the range of the mini-rocket, flight time, angle under the curve, speed in *x* and *y*, resulting velocity, launch angle, resulting acceleration, thrust and height as a function of time (transient regime).

In the literature, the resolution of differential equation systems by algebraic methods is addressed in several works, using simplifications such as: linearized drag force equation, constant mass system and a smaller number of dynamic variables, generating more imprecise results and with a smaller number of flight parameters. The difference of this work in relation to those found in the literature is that in this one it was addressed the dynamic modeling for flight prediction, using two systems of nonlinear equations with twenty-two dynamic variables, one system for the propulsion stage and the other for the oblique launch. The conservation equations were solved by the Runge-Kutta method, finding results for various parameters of the mini-rocket's flight.

The computer program developed is an important tool to assist in the design of mini-rockets for the reforestation of degraded areas.

REFERENCES

1. Salomão, P. E. A., Barbosa, L. C., & Cordeiro, I. J. M. (2020). Recuperação de áreas degradadas por pastagem: uma breve revisão. Research, Society and Development, 9(2), e57922057. https://doi.org/10.33448/rsd-v9i2.2057. Accessed on: September 26, 2024.

2. Food and Agriculture Organization (FAO). (2022). Healthy soils for a healthy people and planet: FAO calls for reversal of soil degradation. Rome, Italy: Food and Agriculture Organization. https://www.fao.org/director-general/news/details/Healthy-soils-for-a-healthy-people-and-planet-FAO-calls-for-reversal-of-soil-degradation/en. Accessed on: September 26, 2024.

3. Buters, T. M. (2019). Drone-based remote sensing as a novel tool to assess restoration trajectory at fine-scale by identifying and monitoring seedling emergence and performance. (Doctoral dissertation, Curtin University). http://hdl.handle.net/20.500.11937/78329. Accessed on: September 15, 2024.

4. Kiama, S. M., et al. (2023). Review of the application of aerial seeding technology in restoration of degraded forests. Authorea Preprints. https://doi.org/10.22541/au.168606342.23089041/v1. Accessed on: September 26, 2024.

5. Joshi, S. C. (1986). Aerial seeding for environmental conservation. Indian Forester, 112(1), 1–5. https://www.indianforester.co.in/index.php/indianforester/article/view/9501. Accessed on: September 26, 2024.

6. Nunes, C., Aparecida, M., Nunes, Z. C., & Emanuel, A. A. V. (2018). Prática pedagógica: uso do veículo lançador de sementes como ferramenta de aprendizagem e recuperação de áreas degradadas. https://encuentros.virtualeduca.red/storage/ponencias/bahia2018/S2NuUy3a55eW1oxHb30LOuhKRcRZdQ3F6hNfM8QD.pdf. Accessed on: September 25, 2024.

7. Marchetti, M. (2024). The history of the Italian astronautical sector and the AIR, The Italian Rocket Association. Acta Astronautica. https://doi.org/10.1016/j.actaastro.2024.07.012. Accessed on: September 26, 2024.

8. Nikkisha, S., et al. (2024). Review on past, present and future rocket propulsion technologies.

9. Sharma, P., et al. (2024). Rocket science unveiled: A differential equation exploration of motion. Alifmatika: Jurnal Pendidikan dan Pembelajaran Matematika, 6(1), 42–50. https://doi.org/10.35316/alifmatika.2024.v6i1.42-50. Accessed on: September 26, 2024.

10. National Aeronautics and Space Administration (NASA). (2018). Newton's Third Law: Rocket Racers. STEMonstrations and Classroom Connections. https://www.nasa.gov/wp-content/uploads/2018/03/stemonstrations_newtons-third-law.pdf. Accessed on: September 26, 2024.

11. Sousa, I. G. G., & Silva, E. A. da. (2024). Variáveis influenciadoras nos voos de foguetes artesanais: uma alternativa para o ensino. RCT-Revista de Ciência e Tecnologia, 10.

12. Souza, J. A. (2007). Um foguete de garrafas PET. Física na Escola, 8(2), 4–11.

13. Liu, H. C., Andre, T., & Greenbowe, T. J. (2008). Journal of Science Education and Technology, 17, 466.

14. Sengel, E., & Ozden, M. Y. (2010). Eurasian Journal of Education Research, 39, 191.

15. Jaime, D. M., & Leonel, A. A. (2024). Uso de simulações: um estudo sobre potencialidades e desafios apresentados pelas pesquisas da área de ensino de física. Revista Brasileira de Ensino de Física, 46, e20230309.

16. Alves, L. A. (2021). Movimento vertical de minifoguetes: equações de trajetórias e análises gráficas. Revista Brasileira de Ensino de Física, 43, e20200479. https://doi.org/10.1590/1806-9126-RBEF-2020-0479. Accessed on: September 26, 2024.

17. Bosch Neto, J. C. (2019). Modelagem e simulação de processos dinâmicos aplicados às engenharias: química, de bioprocessos, elétrica, mecânica, de controle, aeroespacial e fluidodinâmica computacional. São Paulo: Appris.