SIMILARITY IN DISPLACEMENT OF THE BAR WITH CIRCULAR SECTION ON THE INCLINED AXIS OF THE PARALYMPIC WHEELCHAIR

10.56238/edimpacto2025.005-004

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ABSTRACT

The analytical solution of several practical problems in engineering is difficult to perform, and must be solved numerically or experimentally. Therefore, a great solution is experimental modeling. Based on dimensional analysis, which is based on the methods of measurement and analysis of physical problems, similarity is developed. This work aimed to find a predictive equation that makes it possible to determine which would be the deformations suffered by the axis that could influence the performance of the person with disabilities during their commuting. To validate the process, a finite element software was used to simulate the chair and change all the variables involved. Finally, a general predictive equation was raised and made it possible to predict behaviors that could be corrected with small adjustments by simple calculations.

Keywords: Similarity. Finite Elements. Biomechanics. Mechanics of Solids.

1 INTRODUCTION

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Sports activities for people with disabilities have been a reality since the mid-twentieth century. And it has been promisingly, since then, adding value and scientific techniques to its context. In Brazil, Paralympic sports have been progressively gaining ground in the last ten years. Practically all sports are already present in the country, with structured technical teams (Torres, 2015). In 2016 there will be the Olympics and Paralympics in Rio de Janeiro, stimulating various economic sectors, para-athletes and researchers in order to develop new knowledge and technologies. Thus, it is important to know the specificities of the aforementioned relationship – chair/athlete. However, few changes have been proposed regarding the dimensions of the structural components of the equipment. If equipment of this nature has the role of advising the disabled user in order to guarantee him maximum freedom of movement and stability, then it is not justified to disregard a continuous diversity of anthropometric dimensions by generalizing them in models manufactured in series (Cardoso, 2015). As in other cases of accessibility accessories (crutches, boots, supports, gloves, protective equipment), wheelchairs were adopted by their users for extra-conventional activities, such as sports activities. And for this, these accessories and equipment were applied in their natural form, that is, without adequate adaptations. This work addresses the modeling behavior of a circular bar that is a component of the rear axle of a Paralympic wheelchair, so that the camber of the propulsion wheel can be adjusted - it is how much it can lean in relation to the ground. Generally, wheelchair sports unanimously use camber other than zero degrees – wheel perpendicular to ground level. Camber develops more stability to rotation and agile movements for wheelchairs. However, they also imply an increase in propulsion resistance. Camber also performs a safety function – as it is a contact sport, it keeps the athletes' hands on the rims, during propulsion, away from one athlete to another, during shocks or collisions (Cardoso, 2015).

2 METHODOLOGY

Based on dimensional analysis, which is based on the methods of measurement and analysis of physical problems, similarity is developed. Once the variables affect the phenomenon have been identified, these variables can be represented in dimensionless groups or π-terms. This representation, known as Buckingham's π-term theorem, makes it possible to determine which dimensionless groups are important to the problem and to predict the functional relationship between them (Murphy, 1950). The number of π terms needed to express the phenomenon according to Buckingham's Pi-terms theorem is:

$$s=n-b \qquad (1)$$

Where, s is the number of π-terms, n is the number of variables involved, and b is the number of basic dimensions involved.

The π-terms must be dimensionless and linearly independent, in addition, a new π-terms can be obtained by combining other π-terms, through mutual divisions or multiplications, which allows for greater simplification. The π terms can be expressed as follows:

$$\pi_1=F(\pi_2,\pi_3,\pi_4,...,\pi_s) \quad (2)$$

Therefore, the methodology for an approximate model is presented where five variables that directly affect the deflection dependent variable were considered, which are:

a) The applied force (F);
b) The length of the bar (C);
c) The diameter of the circular bar (Ø);
d) Displacement (δ);
e) The modulus of elasticity (E).

3 IDENTIFICATION OF DIMENSIONLESS π-TERMS

To determine the dimensionless terms involved in the problems, the methodology described by Murphy (1950) was adopted, that is, once the variables involved are raised, a matrix is set up with the exponents of the dimension involved for each variable (Carneiro, 1996).

To generate the matrix, the variables involved in the process, which were described above, are first written in a horizontal line. Then, in a vertical line to the left of the horizontal line, the relevant dimensions are denoted. The relevant dimensions are the unit of force (F) and the unit of length (L).

Thus, generating the matrix of dimensional variables:

$$\begin{array}{c|ccccc} & \delta & F & \emptyset & E & C \\ \hline F & & & & & \\ L & & & \text{Dimensões} & & \end{array} \quad (3)$$

For each variable, the following dimensions are established:

Table 1. Dimensions of the variables.

VARIABLES (n)	DIMENSIONS (b)
Deflection (δ)	L
The applied force (F)	F
Diameter of Circular Bar (Ø)	L
The modulus of elasticity (E)	FL-2
The length of the Bar (C)	L

Given the dimensions of each variable, the matrix is filled:

$$\begin{array}{c|ccccc} & \delta & \varnothing & E & F & C \\ \hline F & 0 & 0 & 1 & 1 & 0 \\ L & 1 & 1 & -2 & 0 & 1 \end{array} \quad (4)$$

Expressed by "A" the matrix in the last two columns and by "B" the matrix formed by the rest of the elements of the original matrix, it is given that:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (5)$$

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & -2 \end{bmatrix} \quad (6)$$

This determinant must be non-zero, if it is equal to zero, the rows and columns of the matrix must be reordered so that the determinant is non-zero.

$$det[A] = det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 \quad (7)$$

From matrices A and B, a third matrix C is generated:

$$C = [A^{-1}\, x\, B]^T \quad (8)$$

Therefore:

$$C = \begin{bmatrix} 0 & -1 \\ 0 & -1 \\ -1 & 2 \end{bmatrix} \quad (9)$$

Finally, a final matrix is generated with the π-terms using the matrices A, B, C, and D, where D is an identity matrix of the order of the number of π terms, as follows (Murphy, 1950):

Number of π-terms;

s=5-2=3 (10)

Shape of the final die:

$$\begin{array}{c|ccc|cc} . & \delta & \emptyset & E & F & C \\ \hline F & & & & & \\ L & & B & & A & \\ \hline \pi_1 & & & & & \\ \pi_2 & & D & & C & \\ \pi_3 & & & & & \end{array} \tag{11}$$

Evidently:

$$\begin{array}{c|ccc|cc} . & \delta & \emptyset & E & F & C \\ \hline F & 0 & 0 & 1 & 1 & 0 \\ L & 1 & 1 & -2 & 0 & 1 \\ \pi_1 & 1 & 0 & 0 & 0 & -1 \\ \pi_2 & 0 & 1 & 0 & 0 & -1 \\ \pi_3 & 0 & 0 & 1 & -1 & 2 \end{array}$$

Consequently:

$$\pi_1 = \frac{\delta}{C} \tag{12}$$

$$\pi_2 = \frac{\emptyset}{C} \tag{13}$$

$$\pi_3 = \frac{EC^2}{F} \tag{14}$$

Therefore:

$$\pi_1 = F(\pi_2, \pi_3) \tag{15}$$

$$\frac{\delta}{C} = F\left(\frac{\emptyset}{C}, \frac{EC^2}{F}\right) \tag{16}$$

To arrive at a final equation as a function of the π-terms, a combination obtained by multiplying the component equations of "s" Pi-terms is used (Sedov, 1986), that is:

$$\pi_1 = \frac{F(\pi_2, \overline{\pi_3})F(\overline{\pi_2}, \pi_3)}{F(\overline{\pi_2}, \overline{\pi_3})} \tag{17}$$

To obtain the component equations of each of the Pi-terms, a relation is written where the dependent Pi-term will be a function only of any other Pi-term, while all the other parameters will be kept constant.

The functions chosen to perform the simulations were potential functions of the type , and thus combined by the product function shown above.$\pi_1 = c_1 {\pi_i}^{c_2}$

Like this:

$$F(\pi_2, \overline{\pi_3}) = (\pi_1)_{\overline{\pi_3}} = G{\pi_2}^m \quad (18)$$

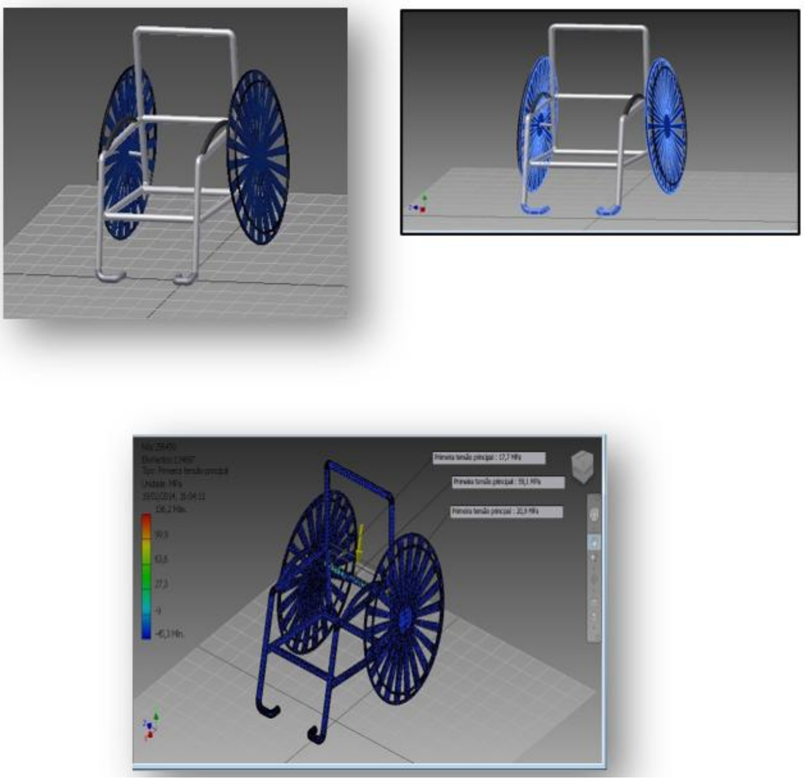
$$F(\overline{\pi_2}, \pi_3) = (\pi_1)_{\overline{\pi_2}} = H{\pi_3}^n \quad (19)$$

To determine the constants G, H, m, n, simulations will be performed by the finite element method using the Autodesk *Inventor Professional 2013 software* (student version).

4 SIMULATION

The model adopted was one-to-one. First, the project represented in the schematic drawing of the Paralympic wheelchair was submitted to stress analysis by finite element method, from the design and simulation software. According to the scheme, the program increased the design with the mesh of the nodal elements.

Figure 1. Mesh Increment: (Elements: 124697 – Knots: 256450).



The constraints or sockets were applied in the rigid regions of the model, that is, those where the support for the requests is greater. In the case of the wheelchair, these regions are represented by the rear and front wheels. In the schematic, the front wheels were not represented, so that their equivalent of setting was the footrest, as being the closest neighborhood. These imposed restrictions are highlighted in neon blue, Fig. 1.

The objective of the simulation was to observe the maximum displacements in the critical region of the model, represented by the coupling axis of the rear wheels. Thus, this component of the system as a whole is the object of studies addressed here, highlighted below, with the respective displacement after the request:

Figure 2. Cargo simulation.

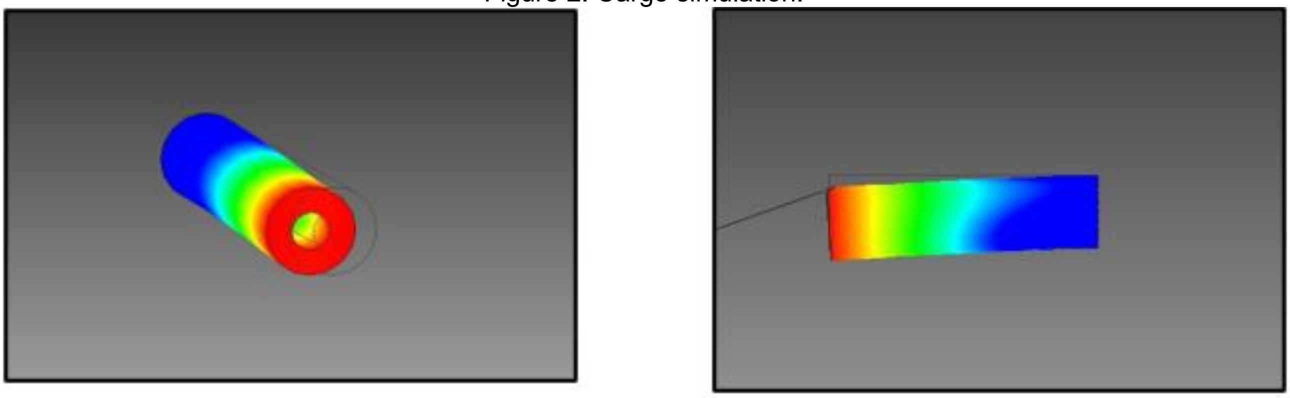


Table 2. Properties of the circular bar on the wheel axle.

Dimensions	
Ø (bar diameter)	0.0172 (m)
C (bar length)	0.0540 (m)
Properties	
E(Aluminum 6061)	7.0 x 1010 (N/m2)
V(Poisson coefficient)	0.33 (u.l.)

5 CONSTANT PARAMETERS

For ··:$\overline{\pi_2}$

Table 3. Constant Variables of $\overline{\pi_2}$.

Constant Variables	Dimensions
Bar Length (C)	0.054 (m)
Applied Force (F)	800.0 (N)

For ··:$\overline{\pi_3}$

Table 4. Constant Variables of $\overline{\pi_3}$.

Constant Variables	Dimensions
Bar Length (C)	0.054 (m)
Bar diameter (Ø)	0.0172 (m)

6 SIMULATIONS

Towards:

$$F(\pi_2, \overline{\pi_3}) = (\pi_1)_{\overline{\pi_3}} \ = G\pi_2{}^m \tag{20}$$

Π 3_constante = (E x C2) ÷ F = (7.0e10 x 0.0542) ÷ 800.0 = 255150
(21)

Table 5. Constant Variables of $\overline{\pi_3}$.

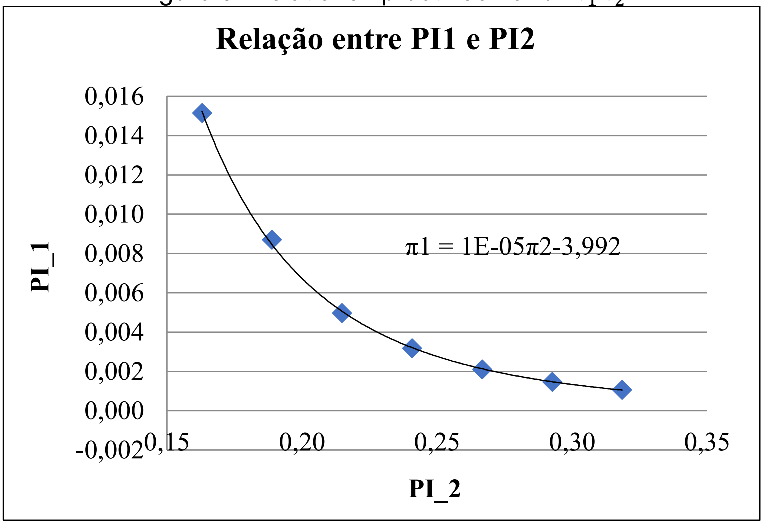
Constant Variables	Dimensions
Bar Length (C)	0.054 (m)
Applied Force (F)	800.0 (N)
Modulus of elasticity (E)	7.0 x 1010 (N/m2)

Varying the diameter (Ø) of the cylindrical bar to obtain the behavior of the dependent variable displacement by bending of the member (δ):

Table 6. Simulation Data.

"Simulation Data"							
Ø (m)	**0,0088**	**0,0102**	**0,0116**	**0,0130**	**0,0144**	**0,0158**	**0,0172**
PI_1	0,01515	0,00870	0,00498	0,00319	0,00211	0,00147	0,00107
PI_2	0,16296	0,18889	0,21481	0,24074	0,26667	0,29259	0,31852

Figure 3. Relationship between and .$\pi_1 \pi_2$



Towards:

$$F(\overline{\pi_2}, \pi_3) = (\pi_1)_{\overline{\pi_2}} \ = H\pi_3{}^n \tag{22}$$

Π 2_constante = Ø ÷ C = 0.0172 ÷ 0.054 = 0.31851852 (23)

Table 7. Constant Variables of $\overline{\pi_2}$.

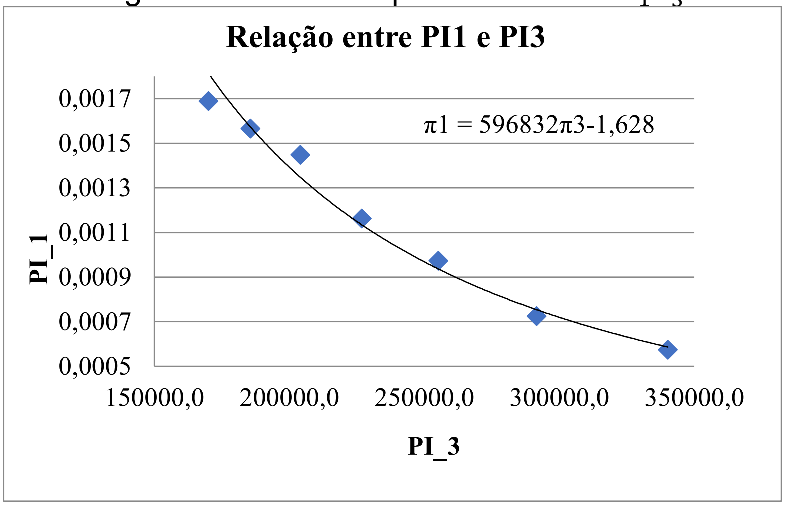
Constant Variables	Dimensions
Bar Length (C)	0.054 (m)
Bar diameter (Ø)	0.0172 (m)
Modulus of elasticity (E)	7.0 x 1010 (N/m2)

Varying the applied force (F), loading, on the cylindrical bar to obtain the behavior of the dependent variable displacement by bending of the bar (δ):

Table 8. Simulation Data.

"Simulation Data"							
F (N)	**600,0**	**700,0**	**800,0**	**900,0**	**1000,0**	**1100,0**	**1200,0**
PI_1	0,000575	0,000725	0,000973	0,001162	0,00144852	0,00156593	0,00168944
PI_3	340200,0	291600,0	255150,0	226800,0	204120,0	185563,6	170100,0

Figure 4. Relationship between and .$\pi_1\pi_3$



7 VALIDITY TEST

The validity test is obtained by an equation in which one of the PI-terms is held constant, for a value different from those of the first data series.

$$\frac{F(\pi_3,\overline{\pi_2})}{F(\overline{\pi_2},\pi_3)} = \frac{F(\pi_3,\overline{\overline{\pi_2}})}{F(\overline{\pi_3},\overline{\overline{\pi_2}})} \tag{24}$$

New simulation for a new constant:$\overline{\overline{\pi_2}}$

So for:

$$F(\pi_3,\overline{\overline{\pi_2}}) = (\pi_1)_{\overline{\overline{\pi_2}}} = L{\pi_3}^{v} \tag{25}$$

Table 9. New Constant Variables of $\overline{\overline{\pi_2}}$.

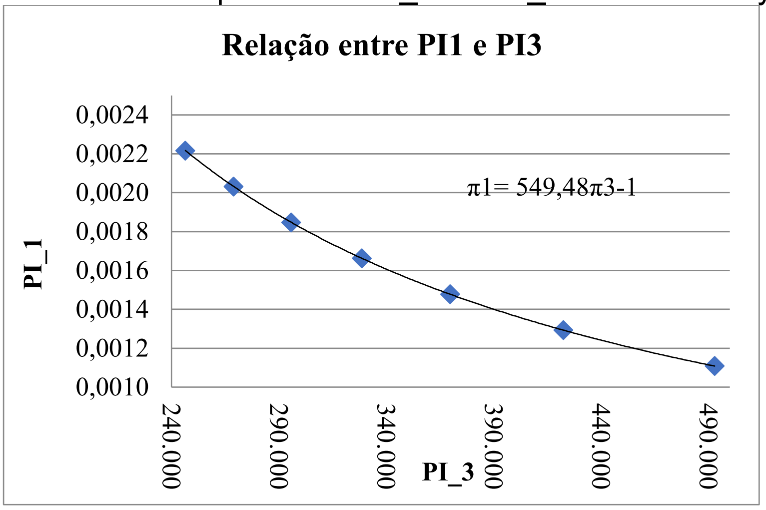
Constant Variables	Dimensions
Bar Length (C)	0.065 (m)
Bar diameter (Ø)	0.0172 (m)
Modulus of elasticity (E)	7.0 x 1010 (N/m2)

Varying the applied force (F), loading, on the cylindrical bar to obtain the behavior of the dependent variable displacement by bending of the bar (δ):

Table 10. Simulation Data.

"Simulation Data"							
F (N)	**600,0**	**700,0**	**800,0**	**900,0**	**1000,0**	**1100,0**	**1200,0**
PI_1	0,00110831	0,00129292	0,00147754	0,00166308	0,00184769	0,00203231	0,00221692
PI_3	492916,67	422500,00	369687,50	328611,11	295750,00	268863,64	246458,33

Figure 5. Relationship Between π_1 and π_3 for the Validity Test



Thus, the equation for the validity test will be as follows:

$$F(\pi_3, \overline{\overline{\pi_2}}) = (\pi_1)_{\overline{\overline{\pi_2}}} = L\pi_3{}^{v} = 549{,}48\pi_3{}^{-1} \quad (26)$$

$$F(\overline{\pi_2}, \pi_3) = (\pi_1)_{\overline{\pi_2}} = H\pi_3{}^{n} = 596832\pi_3{}^{-1{,}628} \quad (27)$$

It is known from the first simulation that:

$$\overline{\pi_3} = \frac{E(L)^2}{F} = 0{,}319 \quad (28)$$

Like this:

$$F(\overline{\pi_2}, \overline{\pi_3}) = 596832(0{,}319)^{-1{,}628} = 380369{,}7 \quad (29)$$

$$F(\overline{\pi_2}, \overline{\overline{\pi_3}}) = 549{,}48(0{,}319)^{-1} = 1722{,}5 \quad (30)$$

Therefore:

$$\frac{F(\pi_2, \overline{\pi_3})}{F(\overline{\pi_2}, \overline{\pi_3})} = \frac{F(\pi_2, \overline{\overline{\pi_3}})}{F(\overline{\pi_2}, \overline{\overline{\pi_3}})} \quad (31)$$

$$\frac{596832\pi_3{}^{-1{,}628}}{380369{,}7} = \frac{549{,}48\pi_3{}^{-1}}{1722{,}5} \quad (32)$$

$$1{,}57\, \pi_3{}^{-1{,}628} = 0{,}319\, \pi_3{}^{-1} \quad (33)$$

$0{,}096 \approx 0{,}068$ *(reducing decimal places)* → 0.1 ≈ 0.1

It is denoted that the values are very close and the validity test is accepted.

8 GENERAL PREDICTIVE EQUATION

Once the product function has passed the validity test, we have that the predictive equation can be determined as follows:

$$F(\pi_2, \overline{\pi_3}) = (\pi_1)_{\overline{\pi_3}} = G\pi_2{}^m = 0{,}00005\pi_2{}^{-3{,}992} \quad (34)$$

$$F(\overline{\pi_2}, \pi_3) = (\pi_1)_{\overline{\pi_2}} = H\pi_3{}^n = 596832\pi_3{}^{-1{,}628} \quad (35)$$

$$\overline{\pi_2} = \frac{\emptyset}{L} = 0{,}319 \quad (36)$$

$$F(\overline{\pi_2}, \overline{\pi_3}) = 596832(0{,}319)^{-1{,}628} = 380369{,}7 \quad (37)$$

From the product function, we have that:

$$\pi_1 = \frac{F(\pi_2, \overline{\pi_3})F(\overline{\pi_2}, \pi_3)}{F(\overline{\pi_2}, \overline{\pi_3})} \quad (38)$$

$$\pi_1 = \frac{(0{,}00005\pi_2{}^{-3{,}992})(596832\pi_3{}^{-1{,}628})}{596832(0{,}319)^{-1{,}628}} \quad (39)$$

Evidently, the final predictive equation is as follows:

$$\pi_1 = 7{,}84x10^{-4}(\pi_2{}^{-3{,}992})(\pi_3{}^{-1{,}628}) \quad (40)$$

Substituting the values of the π-terms:

$$\pi_1 = \frac{\delta}{L} \; ; \; \pi_2 = \frac{\emptyset}{L} \; ; \; \pi_3 = \frac{EL^2}{F} \quad (41)$$

Finally:

$$\delta = 7{,}84x10^{-4}\,(L)\left(\frac{\emptyset}{L}\right)^{-3{,}992}\left(\frac{EL^2}{F}\right)^{-1{,}628} \quad (42)$$

For more accurate validation, the final equation must be validated in some way by data from experimental models. However, with the intention of meeting the needs of adaptation and adaptation, this work proposed the analysis of the behavior of the axis of an ergonomic structure, which includes an ergometer for people with physical disabilities dependent on wheelchairs. Such analysis will contribute to the habilitation and physical rehabilitation of people with disabilities, based on the personal anthropometric dimensions of each person.

NOMENCLATURE

F	Applied strength.
C	Bar length.
And	Modulus of elasticity.
L	Basic Dimension Length.
K	Polynomial constant.
G	Polynomial constant.

Greek Letters

δ	Axis displacement.
π	Dimensionless terms.
Ø	Diameter of the circular bar.

Overwritten.

m	Polynomial exponent
n	Polynomial exponent

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