AERODYNAMIC ANALYSIS OF A DELTA KITE

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ABSTRACT

A kite despite being composed of a lightweight structure surrounded by a thin and light material, is designed to fly like a type of glider, however, fixed by a line. Mathematical and practical analyses can be done. The general objective of this work is to make a study of the aerodynamics behind kites, where the objective was to find the most effective aerodynamic factor on a kite in flight and propose changes in the design to optimize its performance. The study was done with a Delta Kite. This work is structured in two stages, first a literature review was made to study the history, types of kites and aerodynamic analysis. Subsequently, two analyses were performed, one mathematical and the other by the Kite Modeler software, and finally the values were compared. The main result of this project was the verification that the element that most effectively influences the flight of the kite is the location of its rein point. In addition, lift force was the most important aerodynamic factor in determining the resulting stress on the control line. Thus, improvements in the design of the kite were suggested.

Keywords: Kite Delta. Aerodynamics. Influences.

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INTRODUCTION

A kite is composed of a lightweight structure wrapped in a thin and light material, it can be plastic, cloth or paper, which is designed to fly through the air. Even though some references reinforce that kites were invented in China, they persist popularly throughout the world for years, not only among children, but also with adults. Diamond, Delta and Box kites are some of the most common types used for entertainment.

Kites were actually an important influence on the creation of aircraft. Thus, a kite is considered by many researchers to be a type of glider fixed by a line, where mathematical and practical analysis can be done.

According to the literature review, which can be seen below, some factors such as wind speed, angle of attack, position of rein point and wind window are observed, which have a significant influence on the performance of kites. For this project the question at hand is: what is the net force on the line of control through these factors? In addition, we estimate how the result of these effects on the kite can be modified and controlled.

The general objective of this work is to make a study of aerodynamics behind kites. Where the specific objective is to find the most effective aerodynamic factor on a kite in flight, to determine the tension in the control line of a Delta Kite, and also to propose how the design of this type of kite can be modified to optimize its performance.

To highlight the physical phenomenon behind a simple kite, a few years ago, studies of energy generation using kites were carried out in search of a new source of renewable energy. This research was applied and now the researchers are studying to improve this new method of energy generation, and everything is based only on the aerodynamics of the kites.

The methodology of this project will be divided into two parts so that the results can be compared. First, an analysis was made by the Kite Modeler software, which is specifically used for aerodynamic analysis of kites. Finally, the mathematical analysis was made by the equations presented in the literature review, to understand the main external effect on a kite in flight.

As the main result of this project, the element that effectively influences the flight of the kite is the location of its rein point. Lift force was the most important aerodynamic factor in determining the resulting stress on the control line. Thus, improvements in the design of the kite were suggested.

LITERATURE REVIEW

OVERVIEW

A brief literature review follows. Where this includes a short history of kites, types of kites, general kite structure and kite performance. The work also includes previous experimental and mathematical analyses made by using kites as a research instrument.

KITES

Many researchers consider the kite to be a type of aircraft/glider fixed by a line. Its lightweight structure covered by a thin and light material, it can be plastic, cloth or paper, which is designed to fly through the air.

History of Kites

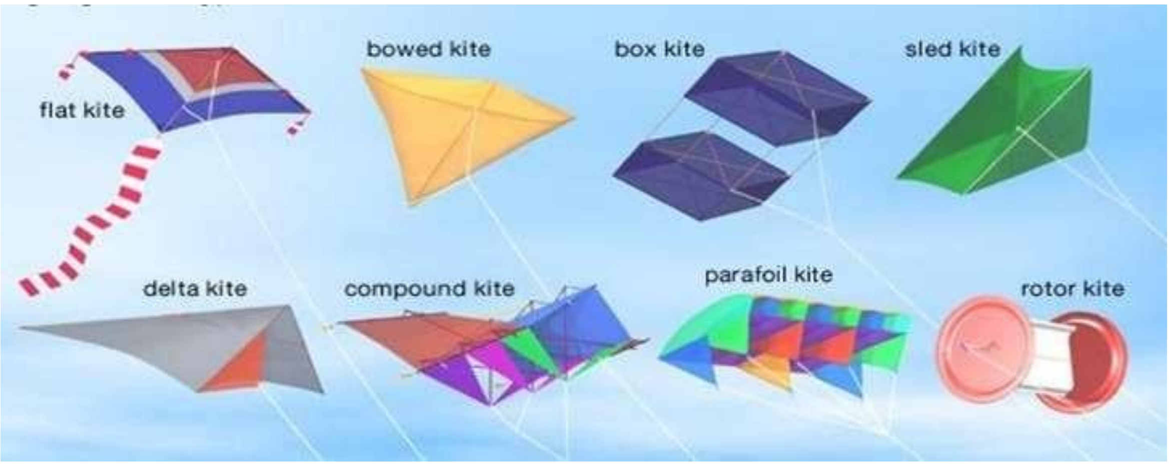
There is some documented evidence that shows that kites were invented in China. It is said that the Chinese began to think about kites after observing the effect of wind on the leaves of trees. According to researchers it is assumed that the first kite was made by a famous philosopher, Mo Di (468-376 BC). What's more, before being used for recreation, kites were used for military purposes. Consequently, their use extended throughout Asia, and even today, they persist as important in the culture and customs of peoples. Not only with children, but also with adults, and remains popular around the world. Above all, it can be said that the kite is officially united with the creation of aircraft, because it was one of the main inspirations for man. (RAY, 2004).

Types of Kites

There are different types of kites, some are used in clearer breezes, and on the other hand, other models need constant winds. Each may require a complex structure or it may be made with just two poles covered with a candle material. Thus, the shape and size of the structure are considerably varied (MAXWELL, 2013).

In the following illustration (Figure 1) there are some examples of kites. According to the National Aeronautics And Space Administration, nowadays, the Diamond, Delta, Winged Box, Sled and Box models are the most common types of kites, but the names of these kites are not regular. (NASA, [201-]).

Figure 1 - Types of kites



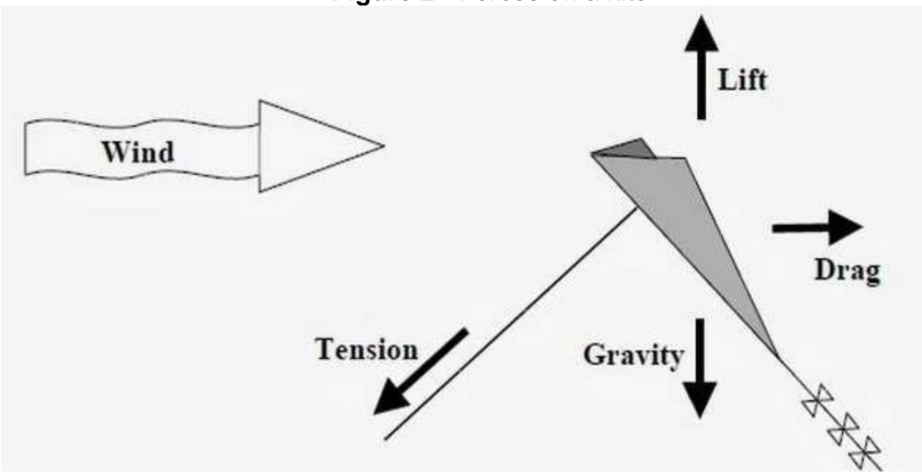
Source: RUSSEL, J., 2012.

Forces Acting on a Kite

Each type of kite can have different shapes, but the forces acting are exactly the same in every type of kite. Furthermore, forces acting on airline planes or even on a fighter plane are the same forces acting on kites, called aerodynamic forces. It is a fact that aircraft and kites are heavier than air, so they depend on these aerodynamic forces to fly. In addition, kites and aircraft have a rigid structure. The kite is covered by a thin material, which can be paper, plastic or cloth, to generate the necessary support to overcome its weight. Therefore, kites must be light and strong to perform well and withstand strong winds. (NASA, [201-])

Lift force, gravity and drag are the main aerodynamic forces on kites, responsible for controlling kite flight, which can be seen in Figure 2. The reason for the kite's flight is the overcoming of the force of gravity and air resistance by the force of wind support (MAXWELL, 2013).

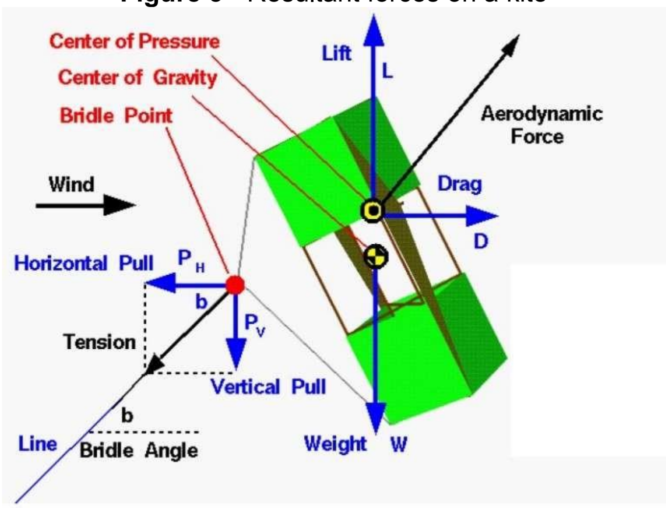
Figure 2 - Forces on a kite



Source: DAVISON, G., 2014

As mentioned earlier, aircraft and kites are heavier than air, so they depend on surfaces to generate the aerodynamic forces necessary for flight. In addition, they need strong structures to support surfaces and transmit forces. Therefore, a kite can fly due to the balance between the weight of the kite and aerodynamic forces. Where the aerodynamic forces of lift and drag are created by the defection of the wind from the surfaces of the kite. What's more, in flight, kite and driver are connected by a control line. Where the tension on the line, resulting from the aerodynamic forces on the kite, can be felt by the driver. By the way, kite and line are connected by a rein, which is usually called a *string bridle*. The point where the line connects with the bridle point is called *bridle point*, which can be seen in Figure 3. It is also said that the kite rotates around this point when it is in flight. Therefore, one way to change the flight characteristics of the kite would be to adjust the *bridle point* (NASA, [201-]).

Figure 3 - Resultant forces on a kite

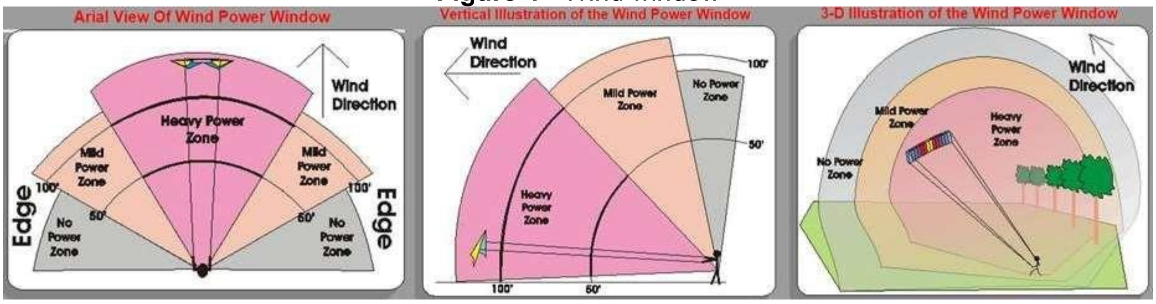


Source: NASA, [201-]

Kite Performance

The wind window represents the area within the set of possible positions of the kite along with the control line in space, where the kite performs best. The wind window is organized into three zones, the heavy *power zone*, the mild *power zone* and the low or *no power zone*. The high power zone will give the highest amount of drag and a better speed control. The medium power zone is where the kite will have reduced power and will slow down as it approaches the low power zone. In addition, the kite is launched and landed, as well as the turns or maneuvers are made in the low power zone. Finally, low or no power zone is where the kite regularly does not suffer air action, because there is the least amount of wind in the kite. (A WIND OF CHANGE, [201-]).

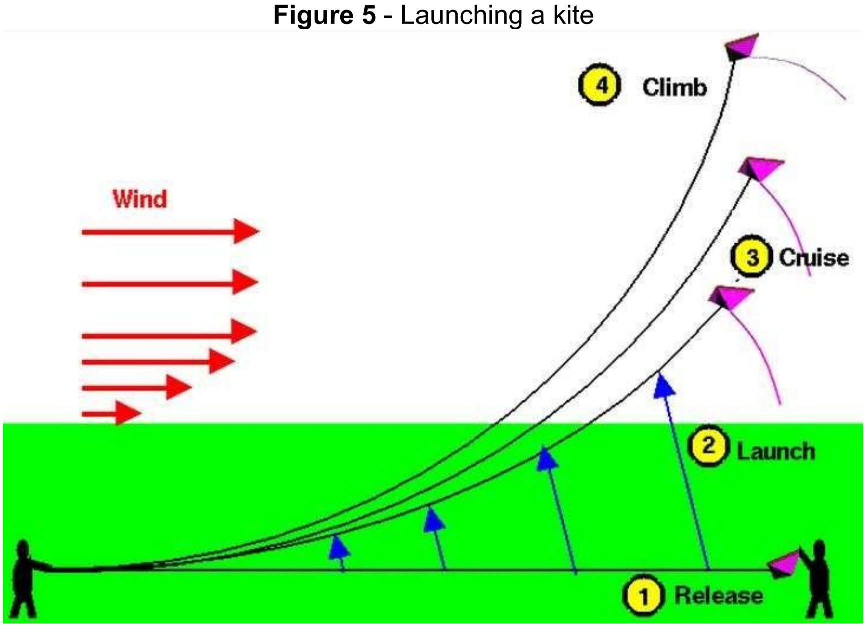
Figure 4 - Wind window



Source: A WIND OF CHANGE, [201-].

Basically, to launch a kite it is necessary to create a lift force greater than the weight of the kite, where many factors affect the dimension of the lift force, but the most important factor to generate lift is the relative speed between the air and the kite. During the launch, which can be seen in Figure 5, from position 1 (*release)* to 2 (*launch),* the driver can usually stand still and the kite will fly, due to the wind speed increasing the lift force. From position 2 (*launch)* to position 3 (*cruise),* the kite continues to rise because the lift force is already greater than its weight, this is due to the fact that commonly the wind speed increases with the increase in altitude. At a certain altitude, in position 3 (*cruise),* the speed and lift force become approximately constant, the kite will then fly to an altitude where the forces and torques will be balanced. On the other hand, by pulling the control line you can then increase the speed of the kite, thus increasing the lift that will make the kite rise reaching position 4 (*climb).* (NASA, [201-])

Figure 5 - Launching a kite



Source: NASA, [201-].

In addition, adjusting the kite's rein point will affect the kite's performance, as it can change the angle of attack (angle between the kite's surface and the wind direction) that the kite flies. For single and double line kites the rein settings are fixed to a specific angle of attack, so the angle of attack cannot be adjusted during flight. According to the wind condition, stunt double-line kites require adjustment of the rein on a regular basis (A WIND OF CHANGE, [201-]).

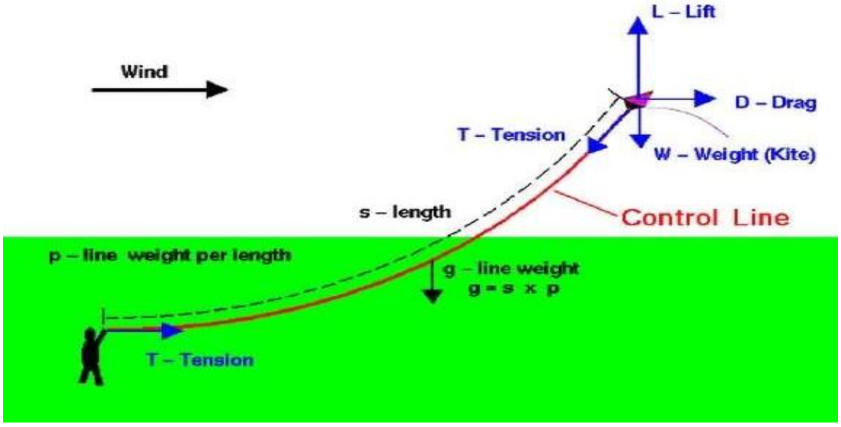
Control Line

A kite line or control line is the line that connects the kite and conductor. The driver can control the kite by pulling and releasing the line. There are kite lines with different lengths, diameters and weights, each one also has particular characteristics. Nowadays different types of kite lines are available. What's more, one of the factors that affect kite performance is the length of the line. Typically, drag is greater on longer lines than shorter lines. However, kites with longer lines travel through the power window in a longer arc, so the kite's response and speed will be slower. (A WIND OF CHANGE, [201-])

What's more, larger arcs in the power window result in higher horsepower for the kites, which will be achieved when using longer lines. Thus, kite drivers for propulsion commonly make use of longer lines than aerobatic kite drivers. Shorter lines will allow you to take the kite out of the power zone more quickly in stronger winds. Consequently short line is useful for controlling larger kites. On the other hand, in moderate wind, longer lines make it possible to keep the kite in the wind window for longer. Usually, stunt kite drivers make use of shorter lines because this can help improve the performance of the kite. (A WIND OF CHANGE, [201-])

It is also important to note that the voltage value that the kite control line can withstand without failure is called, in many references, the line weight (*line wheigth*), which is represented in Figure 6. Therefore, propulsion kites require lines with greater weight than acrobatic kites. (A WIND OF CHANGE, [201-])

Figure 6 - Kite Control Line



Source: NASA, [201-].

Pipa Delta

A hang gliding kite, also known as a hang glider, is a type of kite designed normally with a simple triangular shape, but it can have a diversity of models based on the triangular shape. An example of a delta kite with two control lines can be seen in Figure 7. Delta kites can fly in light winds and usually at high angles, which is their distinction from other kites. In most models the wing stringers of a delta can move freely, as they are not attached to the nose, so the kite can mount air currents by a self-steering action. In short, they are easy to assemble and use. However, although kites can have different shapes, they are aerodynamically similar. (WHAT IS A DELTA, [201-])

Figure 7 - Delta kite

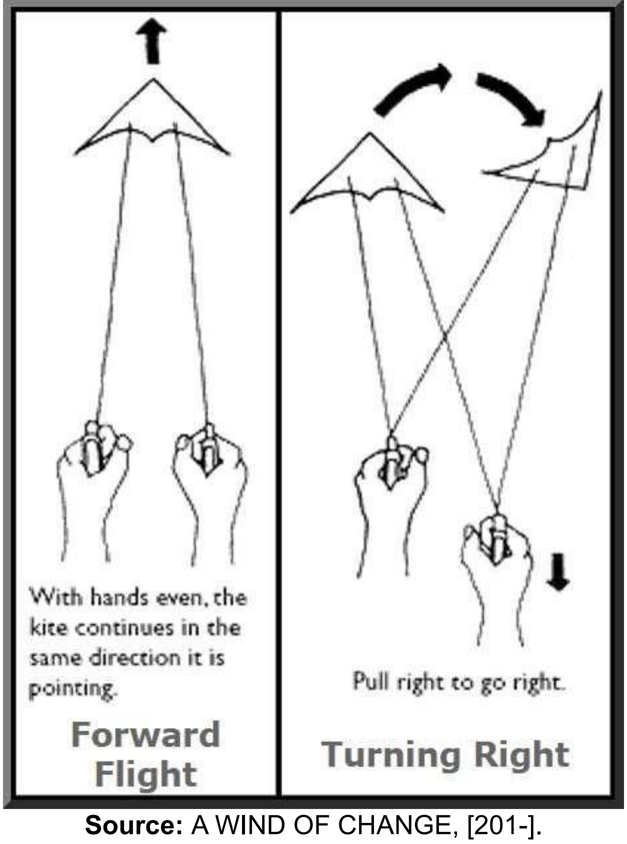
Source: MY BEST KITE, [201-].

The delta kite is commonly described by books as derived from the flexible kites of Rogallo, patented in 1948, who was also the one who developed the hang glider. Rogallo argued that the wings should not be rigid, as being flexible allows their shape to be influenced by the air current. *Fantastic Kites* , which became Gayla Kites in 1961, was the first industry to manufacture and market Delta Kites. Already in the 70's, delta kites were being tested and developed by the American Flier industry (WEBSTER, [ ]).

Two-Line Kite

Two-line kites are typically maneuverable stunt kites. Where it is possible to control the kite in any direction with just hand movements. Normally, to turn the kite to the right, the driver needs to pull the right hand back or push the left hand forward, similarly, to turn the kite to the left, the driver needs to pull the left hand or push the right hand (see Figure 8). In addition, you can combine hand movements to obtain curves of different styles, both accurate and fast. Thus, with these movements, the kite driver will have better control of it. In addition, it is possible to change the angle of attack by adjusting the kite's rein point. For two-line kites, the angle of attack cannot be regulated in flight either, the angle itself is a peculiar feature of the flight itself. That is, the rein point is regulated to obtain a required flight. (A WIND OF CHANGE, [201-]).

Figure 8 - Control of a kite with two control lines



Source: A WIND OF CHANGE, [201-].

MATHEMATICAL ANALYSIS

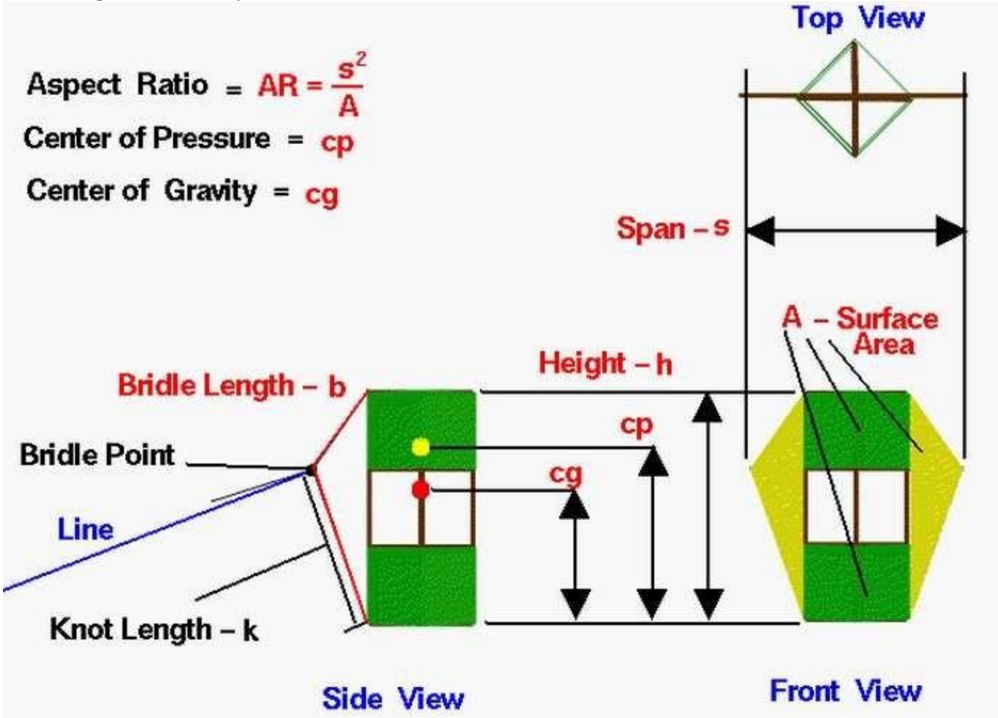
In this part, the mathematical analysis presented is only in the form of an equation. The calculations made for comparison purposes will be presented in the next chapter.

The analytical concepts that will be presented in this part were taken from the official website of NASA – *National Aeronautics and Space Administration* available at the link www.grc.nasa.gov. Where the concepts given below are considered true for most types of kites.

Geometry

Kites are significantly diverse in types and geometries, and change in size and shapes. However, the following definitions are typically applied to most types of kites. The surface area ***A*** (*surface area)* used to calculate the aerodynamic forces acting on a kite is the projected area of the front view, which is exemplified in Figure 9.

Figure 9 – Representation of the three views of a kite from the box model



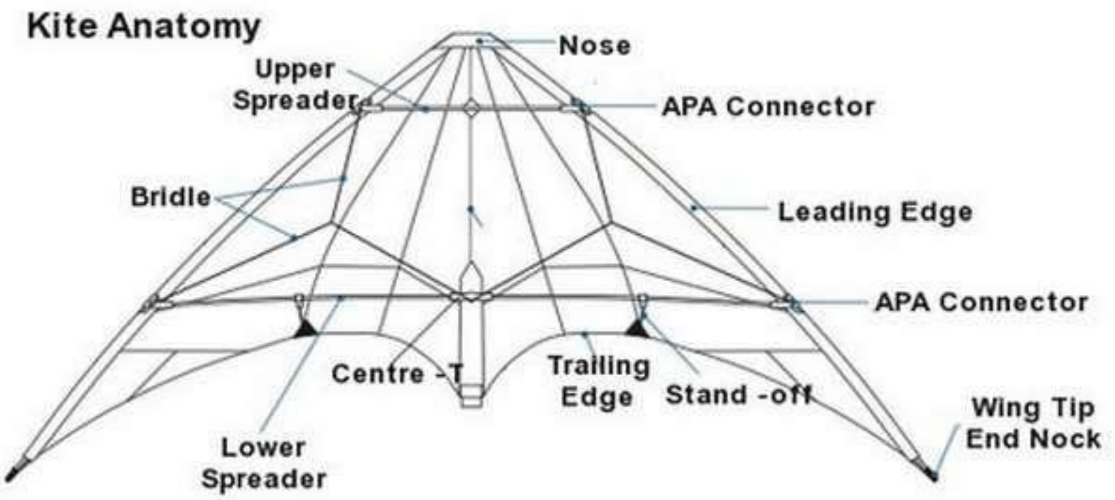
Source: NASA, [201-].

To check the location of the center of gravity and the center of pressure of the kite, the best view is the side *view*, since some kites are not like flat plates, such as the winged box kite, which can be seen in Figure 9. In addition, aerodynamic forces are located at the center of pressure ***cp*** (*center of pressure*) and the center of gravity ***cg*** (*center of gravity*), which is where the weight force of a kite is located. In the side view it is also possible to see the *bridle point*. In addition, the length of the wire from one end attached to the bottom to the other end attached to the top, is called *bridle length* b which is always greater than the height ***h*** (*height*) of the kite because there needs to be some slack in this wire. The *bridle point* is where the control line is attached to the bridle at a point. The distance from the bottom of the kite to the bridle point knot is the knot *length*. With node length ***k*** and rein length ***b,*** the location of the rein point can be determined. Also, the width of the kite is called palmo (*span*), which is used to calculate the aspect ratio ***AR*** (*aspect ratio*) of the kite which is the square of the palm divided by the surface area.

$$AR = \frac{s^2}{\text{The}} \tag{1}$$

The following figure shows the front view of a two-line delta kite with its main components.

Figure 10 - Front view of a two-line delta kite



Source: ROSE, 2016.

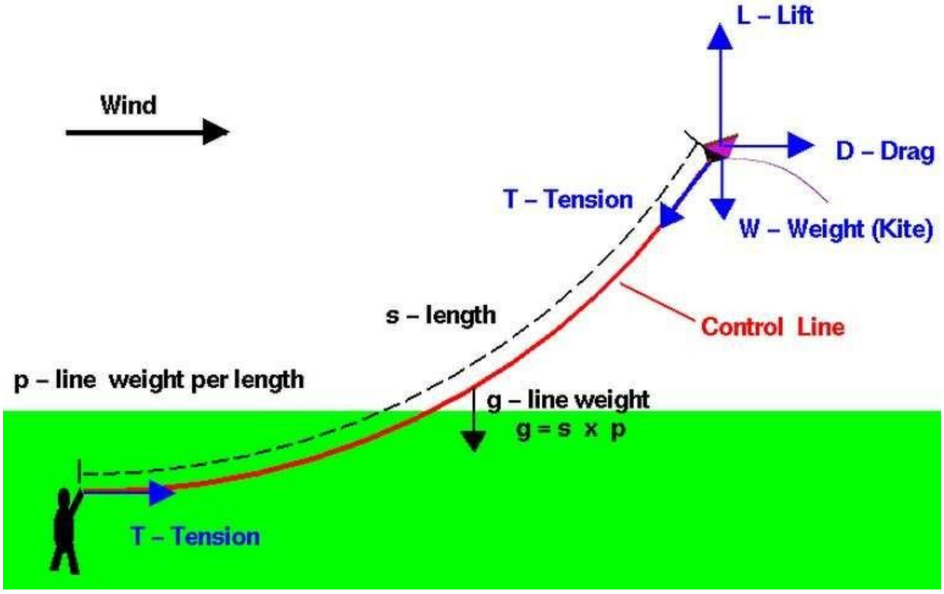
Control Line

The kite rotates over the rein point when it is flying due to a torque created by aerodynamic forces. Thus, the tension in the line is a result of the action of the forces due to the control of the line. In addition, the total weight of the control line ***g*** (*line weight*) that can be seen in Figure 11 is designated by:

$$g = s * p \tag{2}$$

Where ***s*** is the kite line length and ***p*** is the line *weight per length*. Figure 11 shows the side view with the person or controller with a kite in flight and the four representations of the main forces acting on the kite, the lift ***L****(lift)*, the drag ***D****(drag)*, the weight ***W****(weight)* and the tension ***T****(tension)* on the line.

Figure 11 – Representations of the forces acting on a kite in flight



Source: NASA, [201-].

Center of Gravity and Pressure

The center of gravity ***cg*** is the position where the weight force of the kite acts. Knowing the location of the kite, relative to a reference, of each of the parts of the kite, including the structure of surface rods, it can then be said that the weight of the kite times the position of its ***cg*** is equal to the sum of the weight of each part multiplied by the distance of that part to a reference.

The center of pressure ***cp*** is the average position of pressure, where aerodynamic forces act. However, the pressure varies across a surface, so the center of pressure varies with the angle of attack. Therefore, the center of pressure can be calculated by:

$$cp = \frac{\int x\, p(x)\, dx}{\int p(x)\, dx} \tag{3}$$

As aerodynamic forces are more important, to calculate the center of pressure we can disregard the effect of the structure composed of the rods and focus only on the flat area of the kite, the surfaces. These surfaces behave like a thin plate with low velocity. When determining the *cp* of each surface, the *cp* of the kite can be calculated as the weighted average area of each surface. Thus, *cp* will be the sum of the surface area of the kite multiplied by its respective distance from the reference line for the force, divided by the total surface area. Version used in the work.

$$\text{cp} = \frac{\sum \text{Aidi}}{\sum \text{there}} \tag{4}$$

In particular for thin aerodynamic profiles, aerodynamic forces act on a point called the aerodynamic center where the aerodynamic moment remains constant with the angle of attack.

Reining Point

The rein point is the place where the rein and the control line are joined with a knot, and where the kite rotates when flying. The trim *angle* at which the kite flies and the torque balance on the kite are determined by the location of the rein point relative to the center of pressure (***cp***) and the center of gravity (***cg***). The length of the rein ***b*** or ***B*** is measured along the rein wire, from the top to the bottom of the kite, as mentioned earlier. By placing an X-Y coordinate system fixed to the bottom of the kite, which can be seen in Figure 12, the ***Yb*** and ***Xb coordinates*** of the rein point are:

$$\mathrm{Yb} = \mathrm{K} * \cos(\mathrm{The}) \tag{5}$$

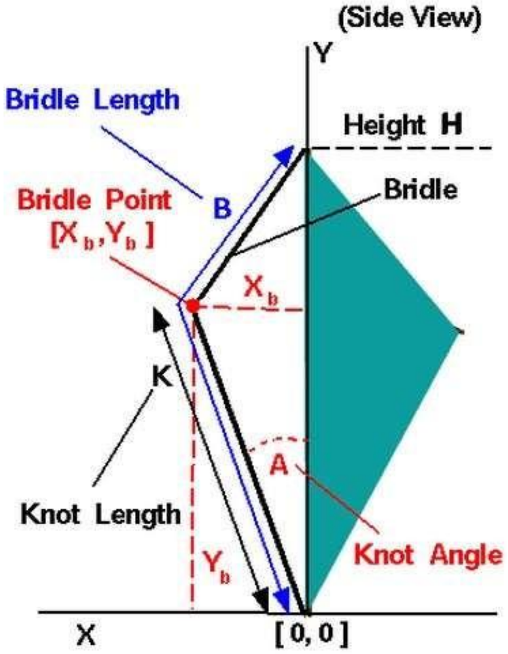
$$\mathrm{Xb} = \mathrm{K} * \sin(\mathrm{The}), \tag{6}$$

where ***A*** is the knot *angle* with the Y-axis and ***K*** *(knot lengh)* the length of the knot. And the equation for determining the knot angle is:

$$\cos(A) = \frac{K^2+H^2-(B-K)^2}{2*K*H} \tag{7}$$

where ***H*** is the height of the kite, as shown in Figure 12 below.

Figure 12 - X-Y coordinates fixed to the bottom of the kite



Source: NASA, [201-].

2.3.5 Control Line and Kite Weight

The weight of the W kite is related to the weight of the Ws surface and the weight of the structure

$$W = Ws + W_f \quad (8)$$

$$Ws = d_s * As \quad (9)$$

$$Wf = df + Lf \quad (10)$$

The weight of the surface and the weight of the structure is related to the shape, size and material that the kite is made of. The surface size and shape can be characterized as the surface area ***and*** the surface material with its density ***ds***. Also, the size and shape of the frame can be characterized as the length of all the rods used in the Lf frame and the frame material with the density ***df.***

2.3.6 Forces at Work in Pipa

There are numerous types of kites with different shapes and sizes, but the forces acting on the kite are the same for all types. The four main forces that act on the kite, as mentioned earlier, are the L lift, the D drag, the W weight and the T tension on the line. Lift and drag are aerodynamic forces that act along the center of pressure, where lift acts perpendicular to the wind and drag acts parallel to the wind. For simplicity, the wind is considered constant and parallel to the earth's surface when away from the ground. The weight force acts on the center of gravity and the tension on the line acts along the reining point. The forces remain constant as long as the kite is in stable flight, so there are no external forces resulting. By dividing the tension in the line into two components, ***PV*** and ***PH,*** see Figure 13, vertical and horizontal respectively, and using Newton's first law of equilibrium, it is possible to obtain equilibrium equations in the vertical and horizontal direction:

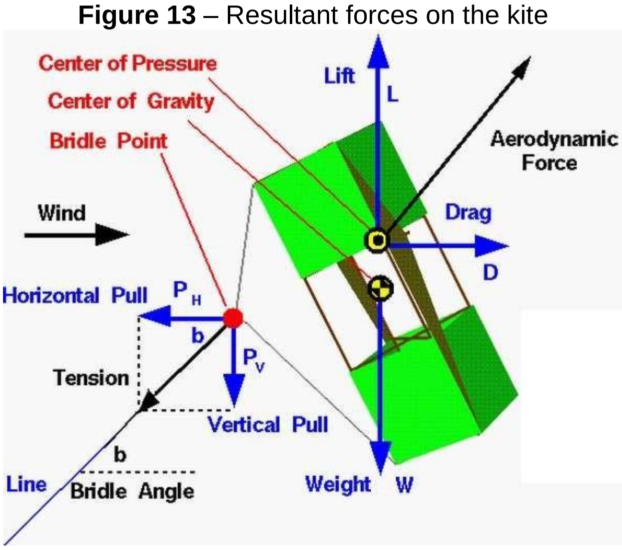
$$PV + W - L = 0 \tag{11}$$

$$PH - D = 0 \tag{12}$$

And the rein angle ***b*** is related to the PV and ***PH voltage components:***

$$\tan(b) = \frac{PV}{PH} \tag{13}$$

Figure 13 – Resultant forces on the kite



Source: NASA, [201-].

According to NASA, the forces on an aircraft are equal to the forces on a kite. (NASA, [201-]). Because of this, equations developed to predict the performance of an aircraft can be used to predict the aerodynamic performance of a kite, so the equations below to determine lift and drag can be used for kites:

$$L = \frac{C_l * A * r * V^2}{2} \quad (14)$$

$$D = \frac{C_d * A * r * V^2}{2}, \quad (15)$$

where ***Cl*** is the lift coefficient, ***Cd*** is the drag coefficient, ***A*** is the projected area, ***r*** is the density of the air, and ***V*** is the relative speed between the wind and the kite. In general, the density of the air depends on where one is on the land, since the density of the air decreases with increasing altitude. This explains why aircraft have a *Flight Ceiling* or a maximum altitude that cannot be exceeded. The density of air at sea level is given by *1,229kg/m3.*

The lift and drag forces depend on the lift and drag coefficients respectively for any object. These coefficients are usually determined experimentally. However, experimental values of these coefficients for flat plates (***Cdo*** and ***Clo***) can be used for kites, since most kites can be considered as thin and simple flat plates. When considering that ***a*** is the angle of attack of the kite's inclination with the wind, the values for ***Clo*** and ***Cdo*** are:

$$Clo = 2\pi a \quad (16)$$

$$Cdo = 1.28 * \sin(the) \quad (17)$$

Most kites have a low AR aspect ratio since the length of the kite width is very small compared to the value of its surface area. It is important to know how *downwash* effect affects the kite in vain, this effect, in short is the airflow moving back and down which depends on the nature of the wing profile, because of the pressure difference near the edges of the wing, the airflow flows from the bottom to the top, which creates the *Downwash* which consequently changes the effective angle of attack of the fluid on a part of the wing. Where for small AR values the portion of the affected area is larger than for large AR values**.** Thus, this effect must be included in the drag coefficient. Where this effect for the drag coefficient is called induced wing drag.

$$C_l = \frac{C_{lo}}{1 + \frac{C_{lo}}{\pi * AR}} \tag{18}$$

$$C_d = C_{do} + \frac{{C_l}^2}{0.7 * \pi * AR} \tag{19}$$

Cd is named "drag due to lift" or induced drag due to the *downwash effect* over the drag coefficient.

Torques na Pipa

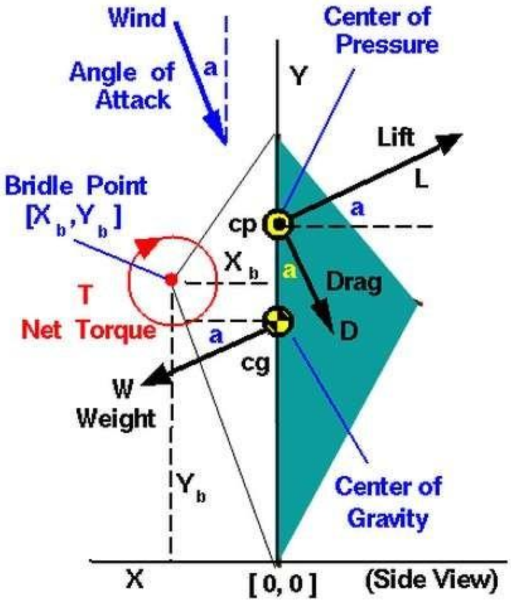
As mentioned earlier, the kite rotates on its rein point because of the aerodynamic forces of lift and attack and the weight force. Point over which torque can be calculated by knowing the position of ***cg*** and ***cp*** (center of gravity and pressure), the angle of inclination of the wind and the position of the rein point [Xb, Yb]. When using X-Y coordinates, see Figure 14, the torque around the rein angle is:

$$\begin{aligned} T = &-[L * \cos(a) * (Y_b - cp)] \\ &- [L * \sin(a) * X_b] + [D * \cos(a) * X_b] \\ &- [D * \sin(a) * (Y_b - Cp)] \\ &+ [W * \cos(a) * (Y_b - cg)] + [W * \sin(a) * X_b] \end{aligned} \tag{20}$$

Where ***a*** is the angle of attack of the kite or the angle of inclination of the wind direction with the surface of the kite. According to Newton's First Law if the external forces are balanced, the body will then remain in static equilibrium (*v = 0)* or dynamic equilibrium (*v = constant).* The same goes for momentum, when the resulting torque is zero the body will have zero or constant angular velocity.

Equation 20 above is used to determine the angle ***a*** for when the resulting torque is zero. In these conditions ***the*** is called the *kite trim angle*. However, as the kite rotates around the rein point, the value of ***the*** seedling. A project that does not reach equilibrium is said to be unbalanced. For a balanced design with lift and drag forces, there is then a trim *angle* where the resulting torque is zero. Furthermore, if after a disturbance the kite naturally returns to its equilibrium condition, it is said to be stable.

Figure 14 – Torque on the kite

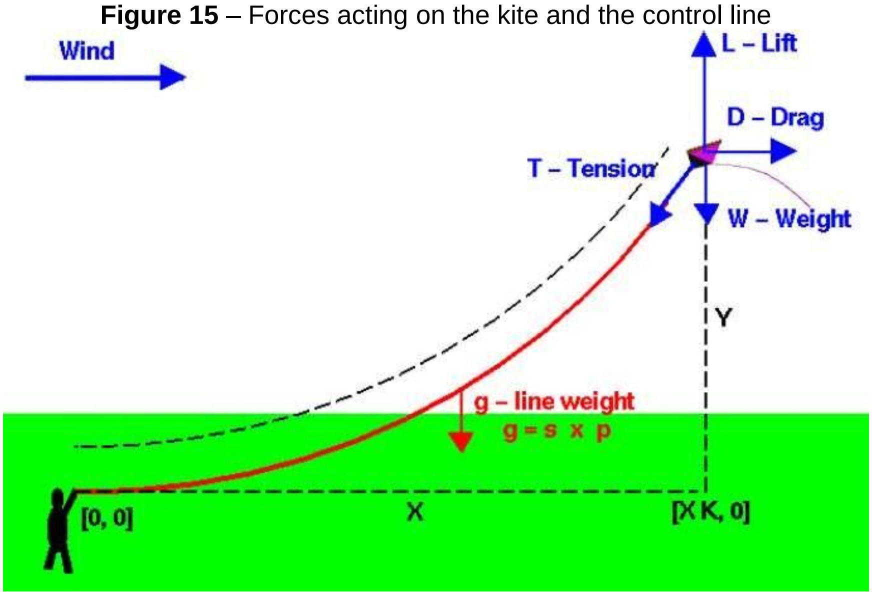


Source: NASA, [201-].

Kite Height

The forces acting on the kite are governed according to Newton's laws. With the kite in flight it can be observed that the control line produces a delicate curve between the driver's hand and the kite, so the line makes a curve under its own weight. Through the equations that describe the curve of the control line, the altitude of the kite can be determined.

Figure 15 – Forces acting on the kite and the control line



Source: NASA, [201-].

In addition, the weight of the control line ***g*** is evenly distributed along its length, which causes it to sag. Thus, a coordinate system is established, where the X axis is positioned along the ground, the Y axis perpendicular to it, and the origin fixed in the hand of the kite driver. As seen in the references, there is a differential equation that describes the transmission of forces along a string, where when solving this equation, it is possible to relate the shape of the control line (Y(X)) to the distribution of the tension present in this control line. The resulting equation is shown below.

$$Y = C_2 + \frac{D}{p}\cosh\left(\frac{P}{D}X + C_1\right) \tag{21}$$

where ***C1*** and ***C2*** are constants of the integrations. The above equation is commonly known as the Catenary Equation. This aims to describe the curve of cables that flex under their own weight. Examples include telephone cables, cables on a suspension bridge, and even a skipping rope.

To calculate C1 and C2, the boundary condition *X=0* and *Y=0 is used. By* substituting the values of *X* and *Y* in the Catenary equation, a relationship between the two constants is given:

$$C_2 = -\frac{D}{p}\cosh C_1 \tag{22}$$

Thus, by determining *C1*, one can then obtain *C2*. To determine *C1*, first the derivative of the Catenary equation, Equation 21, is made, then X=0 is taken as the point of origin, where the hand of the kite controller is located, as we have:

$$\frac{dy}{dx} = \sinh\left(\frac{P}{D}X + C_1\right) \tag{23}$$

$$\frac{dy}{dx} = \sinh C_1 \tag{24}$$

Furthermore, one can then infinitesimally decompose the line tension into the *X* and *Y axes*, and applying Newton's law of equilibrium we have that:

$$\sum Fx = 0 = D - dX$$

$$dX = D \tag{25}$$

$$\sum Fy = 0 = L - W - g - dY$$

$$dY = L - W - g. \tag{26}$$

Like *this dY/dX:*

$$\frac{dY}{dX} = \frac{L - W - g}{D} \tag{27}$$

By equating the previous equation 27 with 24 and rearranging the terms, we have:

$$C_1 = \sinh^{-1}\left(\frac{L - W - g}{D}\right). \tag{28}$$

With the values of *C1* and *C2* it is then possible to know the function that describes the curve that the control line makes along its entire length.

The kite flies at a distance *k* (on the X axis) from the kite driver. In *X=k* the weight of the line is not considered, so equations 23 and 28 can be rewritten.

$$\frac{dy}{dx} = \sinh\left(\frac{P}{D}k + C_1\right) \tag{29}$$

$$\frac{dy}{dx} = \frac{L - W}{D} \tag{30}$$

By equalizing the previous equations and rearranging, one can then predict the distance *k*:

$$k = \frac{D}{P}\left[\sinh^{-1}\left(\frac{L - W}{D}\right) - C_1\right] \tag{31}$$

Using the value of *X=k* in the Catenary equation, it is then possible to know at what height the kite flies, such value is:

$$Y(k) = -\frac{D}{p}\cosh(C_1) + \frac{D}{p}\cosh\left(\frac{p}{D}k + \sinh^{-1}\left(\frac{L - W}{D}\right)\right) \tag{32}$$

PREVIOUS MATHEMATICAL ANALYSES FOR KITES

Next, research with mathematical analysis previously done by researchers who had kites as an object of analysis will be presented.

A study conducted by Argatov, Rautakorpi and Silvennoinen (2009) is one among many analytical studies on energy production through the force of winds at high altitudes by means of kites. Where the fundamental purpose of this article is to estimate the mechanical energy output of a wind power generator using kites. In this study, an analytical modeling of a generator called "*Pumping Kite Generetor*" was made, which makes use of kites fixed by lines to mechanically drive an electric generator with a fixed base. As a result of this study, a simple approximate formula was obtained to calculate the average mechanical power produced by kites. To conclude, they presented an example that estimated the numerical value for the generation of energy by a kite with specified parameters. (ARGATOV; RAUTAKORPI; SILVENNOINEN, 2009)

What's more, there is an equation in this work that is a formula for calculating the aerodynamic force F_r^{aer} acting on the control line.

$$F_r^{aer} = \frac{1}{2}\rho_a A C_L G_e \sqrt{1 + G_e^2}(V_\parallel - V_L)^2 \tag{33}$$

$$G_e = \frac{L}{D + F_f} \tag{34}$$

Where ρa is the density of the air, ***A*** is the characteristic area of the kite, ***CL*** is the lift coefficient, ***Ge*** is the effective glide ratio, V‖ is the wind speed component along the control line, ***VL*** is the longitudinal speed of the kite, and ***Ff*** is the net force on the line due to thelift and drag, concepts that will be analyzed later. With this work they concluded that the aerodynamic force F_r^{aer} is the greatest force, of all the other forces acting on the kite line.

In 2011, Argatov, Rautakorpi and Silvennoinen conducted another study, where once again the generation of energy through kites was the subject in question. However, the purpose of this work was to analyze the effect of the curve made by the kite control line on the efficiency of the propulsion kite, which had not been studied in the previous work. The main objective of this work was to make an analytical modeling of the apparent effects of wind load on the line that connects the kite to the power generator. The method used was to introduce a correction in the angle of attack of the kite due to the curve made by the line by a simple analytical formula, which made use of a model of kite orientation and the mathematical model of a well-stretched string. As the main result, the formulas for the refined crosswind motion law, the angle of attack of the kite and the angle of fall of the line were deduced. As a conclusion, they found that the angle of fall of the line did not depend considerably on the wind speed. Some simplifications were employed in the analytical modeling of the line curve effect. Also, some simplified hypotheses were assumed in the modeling of the effect of apparent wind load. (ARGATOV; RAUTAKORPI; SILVENNOINEN, 2011)

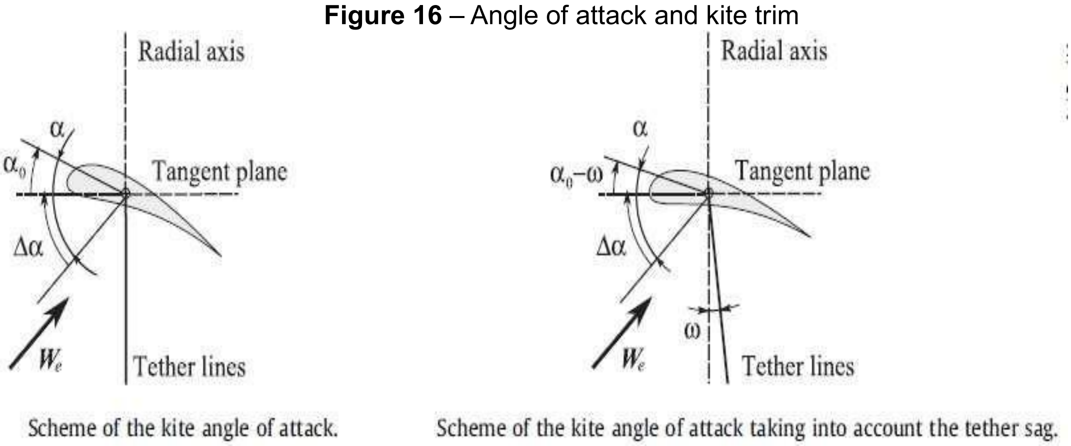
Therefore, from this second work there are two important equations:

$$\propto=\propto 0-\omega+\Delta\propto \qquad (35)$$

$$\omega=\arctan\left(\frac{rdC\perp}{4ACL\cos(\text{Ⱶ})}\right) \qquad (36)$$

The first formula is to calculate the angle of attack of the α kite, where ***αo*** is the base angle of attack, ***ω*** is the angle of fall of the line, and ***Δα*** is the angle between the effective vector of the wind and the tangent plane (see Figure 16 below). In addition, the second formula is to calculate the value of the line's fall angle, where *r* is a spherical coordinate, *d* is the diameter of the line, Ⱶis the control angle of the kite, and $C\perp$ is the normal drag coefficient to the line (see Figure 16).

Figure 16 – Angle of attack and kite trim



Source: ARGATOV; RAUTAKORPI; SILVENNOINEN, 2011

PREVIOUS PRACTICAL ANALYSIS WITH KITES

Next, a research with practical analyses previously made by researchers will be presented.

The following study was developed by Alexander and Stevenson in 2005. The objective of the study was to measure aerodynamic properties of a kite and compare it with the performance of the kite. There is fast operation test and comparative results that can be immediately available, also without the need for wind tunnels. In this work the ratio between lift and drag force can be determined by flying a controllable kite on a windless day, walking in small circles and flying the kite in a larger horizontal circle. This test improves the accuracy of the results while reducing the time taken for each test. (STEVENSON; ALEXANDER; LYNN, 2005)

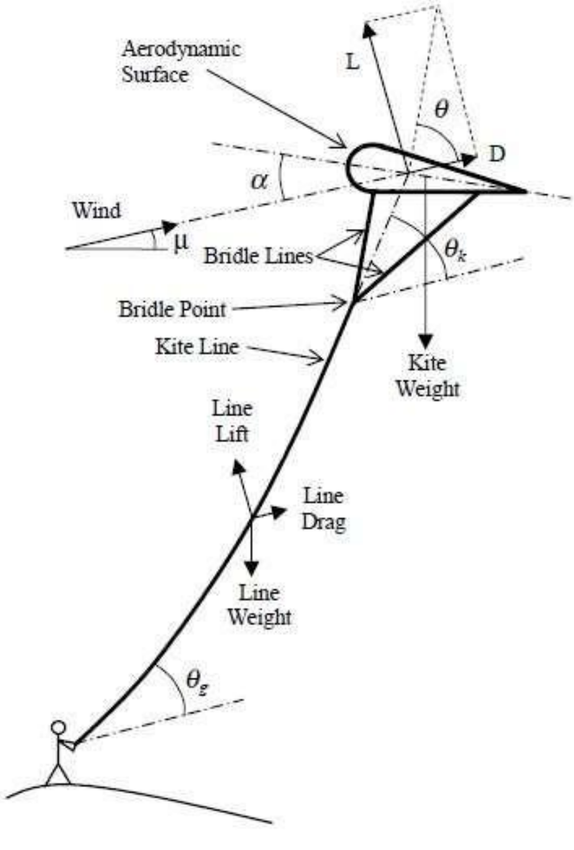
The equation of the ratio between the lift and drag forces, ***LTDk***, and for the angle of the kite line, ***θk***, which consider the weight of the kite and the μ angle of the wind with the horizontal, presented in this article, can be seen below.

$$LTD_k = \frac{L - m_k g Cos(\mu)}{D + m_k g Sin(\mu)} \quad (37)$$

$$\theta_k = Tan^{-1}(LTD_k) \quad (38)$$

Where ***mk*** is the mass of the kite and ***g*** is gravity. And equation 38 is for calculating the angle of the kite line ***θk*** (see Figure 17).

Figure 17 - Representation of forces in the kite



System diagram for a flying kite.

Source: STEVENSON; ALEXANDER; LYNN, 2005.

CRITICAL EVALUATION

All the experiments mentioned above made use of kites in their work, where they were able to obtain important equations for an aerodynamic analysis of kites, these studies also managed to highlight the factors that most effectively influence the kite in flight. Some of these that are basic equations will be used. In addition, it could be noted that most of the works studied above used the potential of the kite for sustainable energy generation.

Recently, research into energy generation through kites has increased remarkably, since it can be used as a renewable energy.

METHODOLOGY

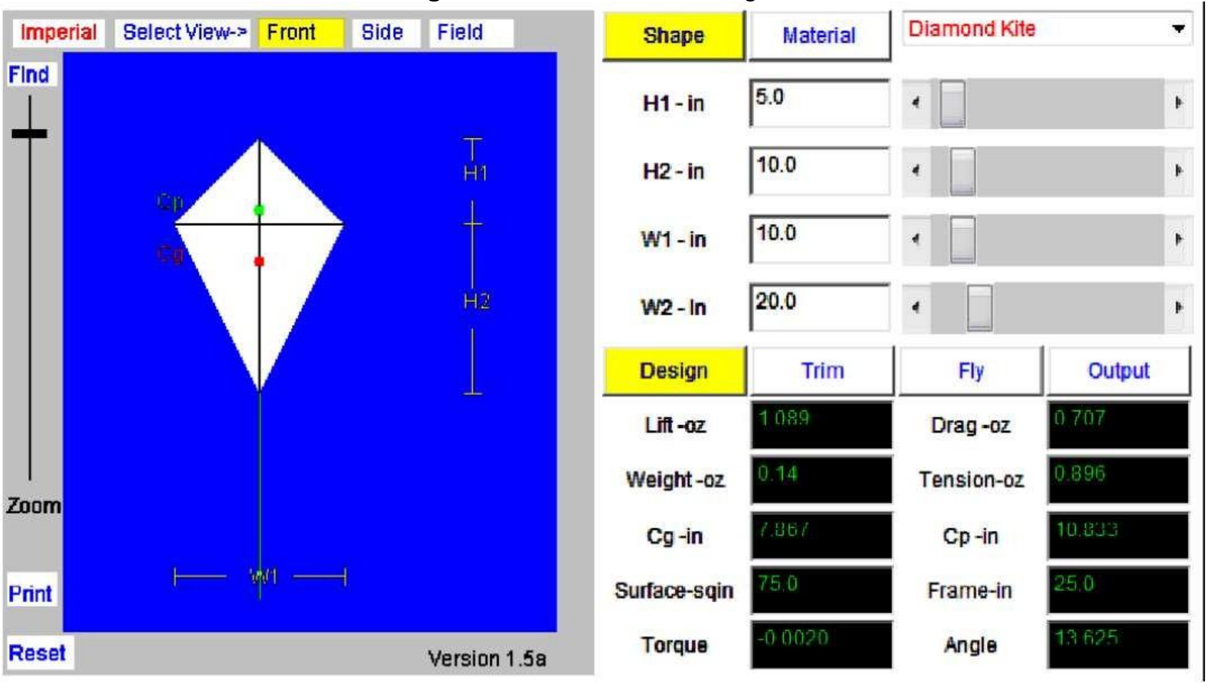
The research method used in this study has been divided into three main sections. Where the first part, already seen before, was to make a survey of important concepts about kites and search for research studies that had the kite as an object of study. In addition, the second part made an analytical study on kite aerodynamics, where general formulas and deductions from studies cited in the literature review were highlighted. Finally, in the third and last part, an analysis of the model was made by the Kite Modeler Program software to analyze the design and flight of the kite. Finally, after finishing the mathematical and model analyses, they will be compared.

KITE MODELER PROGRAM

This is free software made available for download by NASA Glenn Research Center. With it, you can obtain physical and mathematical analyses that describe the flight of a kite. Inside it, you can choose the type of kite and change its shape, size and material. In addition to being able to change the value of some variables that affect the project and immediately see the new characteristics of the kite flight. What's more, this program indicates whether the kite design is stable or not, and it also calculates how high the kite will fly.

The layout of the software screen is simple to understand and can be seen in Figure 18 below. On the left side of the screen there is a graphic window to see the drawing of the kite. On the right side at the bottom is the control panel and outputs, so you can select the *"Design", "Trim", "Fly"* or *"Output" mode.* Finally, on the right side at the top is the input panel where you choose the dimensions and material of the kite. Where in the place of the entrances there is a white background and in the exits the black background.

Figure 18 - Kite Modeler Program



Source: NASA, [201-].

GEOMETRIC AND MATERIAL DATA OF THE KITE

In this first approach, a simple delta kite with only one control line was used, where the front view can be seen in Figure 19 below.

Figure 19 - Front view of a Delta kite

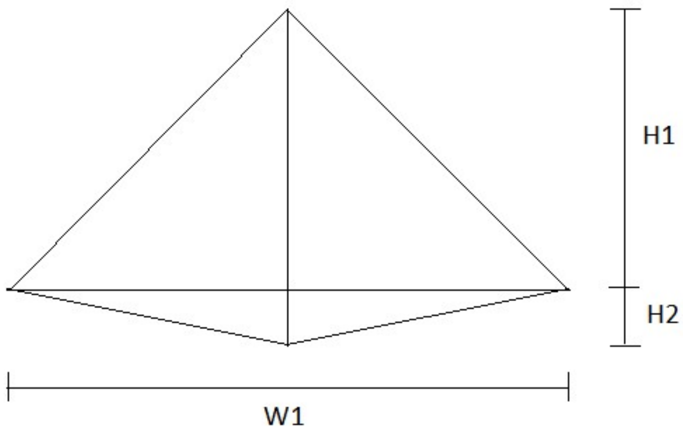


Table 1 below presents the dimensions of the kite according to the previous figure.

Table 1 – Kite Shape

Kite dimensions		
H1	50	Cm
H2	10	Cm
W1	100	Cm
B	70	Cm
Line Length	20	M

The materials used for kite analysis were chosen according to the materials available to be used in the Kite Modeler Program, for a better comparison later. Table 2 below summarizes these data.

Table 2 - Materials Used

Component	**Materials**	**Density**	**Unit**
Surface	Plastic	0,0021	g/cm2
Structure	1/4 ferry	0,0441	g/cm
Line	Nylon	0,3720	g/cm

With the geometric and material data it is then possible to calculate the aspect ratio, Equation 1, the center of pressure, Equation 4, the center of gravity, weight of the control line and weight of the kite. The values are summarized in tables 3, 4, 5, 6 and 7 below.

Table 3 - Aspect Ratio

Aspect Ratio		
s	100,00	Cm
The	3000,00	cm2
AIR	3,33	

Table 4 - Line of Control

Control Line Weight		
s	2000,00	Cm
p	0,37	g/cm
g	7299,57	g
p'	0,36	N/m
g'	71,61	N

Table 5 - Weight of the Kite

Kite weight		
The	3000,0000	cm2
Ds	0,0021	g/cm2
Df	0,0441	g/cm2
Lf	141,4210	Cm
Ws	0,0615	N
Wf	0,0612	N
W	0,1226	N

Table 6 - Center of Pressure

Pressure Center		
A1	2500,00	cm2
A2	500,00	cm2
d1	26,67	Cm
d2	6,67	Cm
Cp	23,33	Cm

Table 7 - Center of Gravity

Center of gravity		
Pe	0,06	N
from	35,00	Cm
Ps1	0,05	N
ds1	26,67	Cm
Ps2	0,01	N
ds2	6,67	Cm
W	0,12	N
Cg	29,15	Cm

From this data, the analyses can then be made by the Kite Modeler. To perform the analyses, five different values of the knot length and four values for wind speed ***V*** will be used.Their respective values can be seen in table 8 below.

Table 8 - K and V values

K	52,5	43,75	35	26,25	17,5	Cm
V	5	10	13	15	m/s	

Also for analysis, it will be assumed that the place where the kite flies is at sea level with a temperature of 15 $^{oC.}$ Therefore, it will have zero altitude and an air density of 1,225 kg/m3.

RESULT AND DISCUSSIONS

FIRST APPROACH: *KITE MODELER*

Tables with values obtained through the Kite Modeler program will be presented below. These results are the output parameters of the program, which in order to provide these values it is necessary to provide the input parameters, which are the geometric parameters previously fixed in Table 1, environmental conditions such as altitude and wind speed, and finally select the materials of the surface, structure and control line, which are summarized in Table 2.

Since the values of the center of pressure, center of gravity, surface area and total length of the rods of the structure do not vary with the change in wind speed, these values will be presented in table 9 below with their respective values for each value of the knot length.

Table 9 - Values of *cg, cp, As* and *L*

K (cm)		**52,5**	**43,75**	**35**	**26,25**	**17,5**
Cg	Cm	29,154	29,154	29,154	29,154	29,154
Cp	Cm	33,333	33,333	33,333	33,333	33,333
The	cm2	3000	3000	3000	3000	3000
L (rods)	Cm	141,421	141,421	141,421	141,421	141,421

Finally, the following tables bring twenty values of lift force, drag force, weight force, tension on the control line, *trim* angle, torque, distance and height of the kite. Each value refers to the combination of a wind speed and a knot length. For a better presentation, four tables are as follows, each referring to a wind speed value. However, within each table there are five columns for each node length value.

Tabela 10 - Parâmetros de Saída para V=5m/s

Velocidade do vento					**5**	**m/s**
Comprimento K (cm)		52,5	43,75	35	26,25	17,5 **Propriedade**
Unidade				Valor		
L	N	0,1881	0,4366	15,9728	0,0629	0,0595
W	N	0,1226	0,1226	0,1226	0,1226	0,1226
D	N	0,0395	0,0954	11,9131	0,0129	0,0129
T	N	0,0395	0,2592	19,7697	0,0129	0,0129
Ângulo Trim	graus	0,3750	0,8750	48,0000	0,1250	0,1250
Torque	N.m	-0.0005	-0.0013	-0.0052	-0.0010	-0.0099
Distancia X	m	13,0000	6,0000	12,0000	0,0000	0,0000
Altura Y	m	10,0000	18,0000	16,0000	0,0000	0,0000
Estabilidade		Estável	Estável	Estável	Instável	Instável

From Table 10 above, it can be seen that there is an increase in the values of the forces as the length of the knot (*K)* decreases. However, for the length of the node half of the rein length (*K=35)* there was an increase (peak) in the values of the calculated forces, noting a very large discrepancy.

Table 11 - Output Parameters for V=10m/s

		Wind speed	10	m/s		
Length K (cm)		52,5	43,75	35	26,25	17,5
Property	Unit			Value		
L	N	0,2514	0,5022	63,6662	66,0983	66,0983
W	N	0,1226	0,1226	0,1226	0,1226	0,1226
D	N	0,0518	0,1044	47,3699	50,4597	0,1226
T	N	0,0761	0,3239	79,1986	83,0020	83,0020

Angle Trim	Degrees	0,1250	0,2500	47,2500	47,2500	47,2500
Torque	N.cm	-0,0130	-0,0045	-0,0201	-0,0201	-0,0201
Height X	M	10,0000	5,0000	12,0000	12,0000	12,0000
Y Distance	M	17,0000	19,0000	16,0000	16,0000	16,0000
Stability		Stable	Stable	Stable	Unstable	Unstable

For the knot length greater than half of the rein length (K>35) plausible values were observed for the aerodynamic forces, but for the other K values the forces were very large and the program indicated an instability of the kite.

Table 12 - Output Parameters for V=13m/s

		Wind speed	13	m/s		
Length K (cm)		52,5	43,75	35	26,25	17,5
Property	Unit			Value		
L	N	0,4249	0,4249	107,5959	111,7061	111,7061
W	N	0,1226	0,1226	0,1226	0,1226	0,1226
D	N	0,0875	0,0875	80,0552	85,2769	85,2769
T	N	0,2454	0,2454	133,9539	140,3806	140,3806
Angle Trim	Degrees	0,1250	0,1250	47,2500	47,2500	47,2500
Torque	N.cm	-0,0470	-0,0010	-0,0502	-0,0502	-0,0502
Height X	M	6,0000	6,0000	12,0000	12,0000	12,0000
Y Distance	M	18,0000	18,0000	16,0000	16,0000	16,0000
Stability		Stable	Stable	Stable	Unstable	Unstable

For *V=13m/s* the observations are the same as the results for *V=10m/s.*

Table 13 - Output Parameters for V=15m/s

		Wind speed			**15**	**m/s**
Length K (cm)		52,5	43,75	35	26,25	17,5
Property	Unit	Value				
L	N	0,5657	0,5657	142,9953	148,7211	148,7211
W	N	0,1226	0,1226	0,1226	0,1226	0,1226
D	N	0,1165	0,1165	106,2641	113,5344	113,5344
T	N	0,3880	0,3880	177,9994	186,9488	186,9488
Angle Trim	Degrees	0,1250	0,1250	47,1250	47,1250	47,1250
Torque	N.cm	-0,0746	-0,0075	-0,0020	-0,0020	-0,0020
Height X	m	5,0000	5,0000	12,0000	12,0000	12,0000
Y Distance	m	19,0000	19,0000	16,0000	16,0000	16,0000
Stability		Stable	Stable	Stable	Unstable	Unstable

For *V=15m/s* the observations are the same as the results for *V=10m/s.*

SECOND APPROACH: MATHEMATICAL ANALYSIS

Next, the analytical results obtained through the calculations using the equations presented in the literature review will be shown. The calculations were made by Excel and only the result is presented.

Similar to what was exposed in the previous analysis, as the values of the center of pressure, center of gravity, knot angle, rein point coordinates, coefficients for plane plates and lift and drag coefficient do not vary with the change in wind speed, these values will be presented in the following table with their respective values for each value of the knot length **K**.

Table 14 - Values of *K, A, Xb, Yb, Clo, Cdo, Cl* and *Cd*

Knot Angle						
K	52,5	43,75	35	26,25	17,5	Cm
cos(A)	0,960317	0,919048	0,857143	0,753968	0,547619	
The	0,282658	0,405139	0,5411	0,716714	0,99128	Rad
Coordinates of the bridle point						
Xb	14,64274	17,24391	18,02776	17,24391	14,64274	Cm
Yb	50,41667	40,20833	30	19,79167	9,583333	Cm
Coefficient for flat plates						
Clo	0,041123	0,095954	5,263789	0,013708	0,013708	
Cdo	0,008378	0,019547	0,951225	0,002793	0,002793	
Coefficient of lift and drag						
Cl	0,039404	0,087088	0,799354	0,013511	0,013511	
CD	0,008589	0,020582	1,038392	0,002817	0,002817	

The angle of attack used for this mathematical analysis will be equal to the angle of attack calculated by the Kite Modeler program in order to compare the results. The following table shows these values for their respective speed and rein length.

Table 15 - Angle of Attack

Length K		Cm	52,5000	43,7500	35,0000	26,2500	17,5000
V=5m/s	**the**	Degrees	0,3750	0,8750	48,0000	0,1250	0,1250
V=10m/s	**the**	Degrees	0,125	0,125	47,25	47,25	47,25
V=13m/s	**the**	Degrees	0,125	0,125	47,25	47,25	47,25
V=15m/s	**the**	Degrees	0,125	0,125	47,25	47,25	47,25

Finally, also similar to the previous approach, the following tables bring twenty values of lift force, drag force, tension in the control line and its Pv and Ph coordinates (vertical and horizontal) and the rein angle. Each value refers to the combination of a wind speed and a knot length. For a better presentation, four tables are as follows, each referring to a wind speed value. However, within each table there are five columns for each node length value.

Table 16 - Results for V=5m/s

		Speed	From the wind		5	m/s
K	Cm	52,5	43,75	35	26,25	17,5
L	N	0,1810	0,4001	3,6720	0,0621	0,0621
D	N	0,0395	0,0945	4,7701	0,0129	0,0129
Pv	N	0,0584	0,2774	3,5494	-0,0606	-0,0606
Ph	N	0,0395	0,0945	4,7701	0,0129	0,0129
B	Rad	0,9763	1,2423	0,6397	-1,3603	-1,3603
T	N	0,0395	0,0945	4,7701	0,0129	0,0129

The results in Table 16 show that for moderate wind speed the calculated values are similar to the values generated by the Kite Modeler, except for the values generated for the node length half of the rein length (*K=35*), where there was a large discrepancy.

Table 17 - Results for V=10m/s

			V(m/s)		10	m/s
K	cm	52,5	43,75	35	26,25	17,5
L (N)	N	0,2483	0,4895	14,6528	14,6528	14,6528
D (N)	N	0,0518	0,1044	18,8653	18,8653	18,8653
Pv	N	0,1256	0,3669	14,6528	14,6528	14,6528
Ph	N	0,0518	0,1044	18,8653	18,8653	18,8653
b	rad	1,1799	1,2935	0,6604	0,6604	0,6604
T	N	0,1359	0,3814	23,8873	23,8873	23,8873

Increasing the wind speed, it was noticed that only for node length values greater than half the rein length the values calculated and generated by the program were very close. For the other K values, the results were quite different. The same discrepancy previously mentioned for *K=35 is also noted.*

Table 18 - Results for V=13m/s

V(m/s)				**13**		**m/s**
K	Cm	52,5	43,75	35	26,25	17,5
L (N)	N	0,4196	0,4196	24,7632	24,7632	24,7632
D(N)	N	0,0875	0,0875	31,8823	31,8823	31,8823
Pv	N	0,2969	0,2969	24,6406	24,6406	24,6406
Ph	N	0,0875	0,0875	31,8823	31,8823	31,8823
b	Rad	1,2842	1,2842	0,6580	0,6580	0,6580
T	N	0,3095	0,3095	40,2944	40,2944	40,2944

For *V=13m/s* the results behaved like the results for *V=10m/s*.

Table 19 - Results for V=15m/s

			V(m/s)	15		m/s
K	Cm	52,5	43,75	35	26,25	17,5
L (N)	N	0,5586	0,5586	32,9688	32,9688	32,9688
D(N)	N	0,1165	0,1165	42,4468	42,4468	42,4468
Pv	N	1,5065	3,4779	32,9256	0,4360	0,4360
Ph	N	0,3551	0,8509	42,9310	0,1165	0,1165
b	Rad	1,3393	1,3308	0,6543	1,3097	1,3097
T	N	1,5477	3,5805	54,1033	0,4513	0,4513

For *V=15m/s*, the results also behaved as the results for *V=10m/s*.

ANALYSIS OF THE RESULTS

As expected, it can be observed that for the main forces acting on the kite, being the lift force, drag and weight, there was an increase in its value when decreasing the value of the knot length and/or increasing the wind speed.

The value of the center of gravity was the same for both analyses, however the value of the center of pressure gave a difference of 10 cm, being higher than the one calculated by the Kite Modeler program.

Regarding the lift, drag and weight forces, for V=5m/s there were very similar values for the respective K values, except for K=35cm between the analyses. In which Kite Modeler generated a value much higher than calculated. For the other speeds, the values of the forces gave similar values for K=52.5cm and K=43.75cm. For the other K values, there was a very large discrepancy between the value generated by the Kite Modeler and the calculated value. A systematic review would be needed to determine the reason for such a discrepancy.

Evidently, based on the results, regarding the rein point, its best configuration occurred with values greater than 35cm, that is, values greater than half the length of the rein. The choice of this location was based on the purpose of the use of the kite.

CONCLUSION

It is a fact that aircraft and kites are heavier than air, so they depend on the aerodynamic forces of lift and drag to fly. This work showed that factors such as environmental condition and kite geometry have a significant influence on kite performance.

Through Newton's law of motion, it was possible to obtain that the factors that affect the tension in the control line are the lift force, the drag force and the weight force. However, taking into account the order of magnitude of these forces, it is clear that the lifting force is the one that most influences the resulting tension in the line. Consequently, the most important vector of the aerodynamic analysis of kites is the location of the rein point, which directly affects the angle of attack, where it is decisive in the calculation of the aerodynamic coefficients. Therefore, the characteristics of the environment, such as air density and wind speed, cannot be disregarded, since these factors also directly affect the calculation of lift and drag forces, and indirectly through the downwash effect on the aerodynamic coefficients.

However, this work can be considerably complemented through deeper analyses with systematic reviews, numerical analysis, as well as experimental analyses.

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