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AN APPROACH TO THE STUDY OF THE QUADRATIC FUNCTION USING THE ALGEBRAIC FORM OF A COMPLEX NUMBER

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ABSTRACT
This article aims to present a new method for calculating the roots of a quadratic function, also called polynomial function of the 2nd degree, without resorting to formulas known in the literature in Mathematics. In this sense, the complex number is used in the algebraic form x=m+ni. In the application session, some problems are addressed, including in the area of Physics, in which the calculation of roots is compared, considering the development from known expressions, and compared with the method presented in this work in order to validate what was proposed. It was found that the method, presented in the form of a theorem, does not have the need to apply the known formulas, because the mathematical development leads to solving only a linear system with two variables m and n, for example, in which m will represent the abscissa of the vertex of the parabola of the given function and n can be written as an expression that involves the discriminant Δ. In addition, a discussion of the nature of the roots of the quadratic function is made from the analysis of the parameter n, which will indicate whether they are real or complex. It was concluded that, considering the complex number in the form x=m+ni, the method presented is of simple applicability and great relevance.

Keywords: Mathematics Teaching, Quadratic function, Complex number, Root calculation.

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INTRODUCTION

The complex number arose from the solution of a polynomial equation of the 2nd degree (BERRIMAN, 1956), when a negative value was obtained for the delta discriminant (). In the beginning, there was no logical explanation capable of interpreting situations in which there was a square root calculus where the value under the radical was a negative sign, hence the great difficulty in finding such results. Historically, solving equations has always brought great fascination to a significant number of mathematicians (FRAGOSO, 1999), especially the ancient mathematicians of Babylon who in the beginning managed to solve some equations of the 2nd degree using the idea of completing squares. The Greeks, however, began to play an important role in the formalism of mathematics where they solved some types of equations of the 2nd degree with ruler and compass (ΔGARBI, 2010).

Hindu mathematicians were the ones who managed to advance in research in Algebra, and Bhaskara was the mathematician who developed a study for the calculus of roots of a quadratic function. It is due to him that the expression "Bhaskara's formula" was consecrated, however, it is worth mentioning that the discovery of this formula is attributed to the Hindu mathematician Sridhara (CARMO, MORGADO and WAGNER, 1992). Therefore, Bhaskara's formula leads to the roots being:

$$x_1 = \frac{-b + \sqrt{\Delta}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{\Delta}}{2a}. \tag{1}$$

If the number is real, it is possible that it will have a negative value, which caused enormous disturbance to many mathematicians at the time. The answer to this situation was the question of no solution. This idea of non-solution remained for a long time until in Italy, in the sixteenth century, from a dispute between Cardano and Tartaglia for the solution of an equation of the 3rd degree that it was thought that the real numbers were not enough to answer the problem presented. Thus, the need was felt to introduce in the area of mathematics a set that was "larger" than the set of real numbers, opening paths for a mathematical development in which a better algebraic interpretation could be used to solve this type of problem (CARMO $\Delta = b^2 - 4ac\mathbb{C}$*et al.*, 1991).

Of the many mathematicians who began to deal with the subject, taking into account the complex body, Leonhard Euler was the one who worked the most in production and publication. Of the various works developed, his effort was remarkable in improving the

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symbology. Many of the notations used today were introduced by this brilliant mathematician. Among the representations proposed by Euler, the i stands out, replacing it with . Euler went on to study numbers in the form where and are real numbers, such that . Elements represented in this way are called complex numbers.$\sqrt{-1} x = m + nimni^2 = -1$

Currently, complex numbers can be used in different scientific areas such as Physics, Mathematics, Engineering, etc. (ÁVILA, 1996).

In this article, we use the complex number in order to calculate the roots of a polynomial equation of degree 2, without resorting to the well-known Bhaskara formula. To better develop this research, some points were addressed as prerequisites for a better understanding of the subject and, then, to develop the method of solving a polynomial equation of the 2nd degree using the number as a way to obtain the roots of this equation. $x = m + ni$

To better interpret and analyze the relevance of the work, a theorem was demonstrated that leads to the values of the parameters and that represent the real part and the imaginary part, respectively, of the complex number in the algebraic form: . The theorem was applied to some mathematical and physical problems to consolidate and show the veracity and importance of the theorem, also using a comparison between the two formalisms. It is believed that the applications are important alternatives for a better understanding and consolidation of this theme.$mnx = m + ni$

The methodology proposed in the research focused on the fact that any quadratic equation, regardless of whether the delta value is positive or negative, can be solved not by the usual method (RIBEIRO, 2009), but based on two parameters, and . We can also consider the condition that the numerical value of n can be used to discuss the roots of the quadratic equation in the same way as it is done with the discriminant of the equation of the 2nd degree (REVISTA DO PROFESSOR DE MATEMÁTICA nº 39, 1999).mn

It was verified in the development of the problems that the solution of the equation happens with the solution of a system of two linear equations and two unknowns, arising from the operation of equality between complex numbers, in which the solution does not involve the use of Bhaskara's formula. Another issue to be observed is the reason for noting that the complex number, despite having been a great impasse that influenced and motivated numerous mathematicians to analyze and develop a solution for the case in which the discriminant was negative, can be used to calculate roots of any polynomial equation of the 2nd degree, as will be seen throughout this approach.

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THEORETICAL FOUNDATION

The following is a brief approach to the study of the quadratic function that can be found in several literatures in mathematics.

DEFINITION OF QUADRATIC FUNCTION AND CANONICAL FORM

According to Iezzi & Murakami (2013, p. 137) "An application of in is called a quadratic function or the 2nd degree when it associates to each , the element , in which , and are real numbers given and ."$f\mathbb{R}\mathbb{R}x \in \mathbb{R}(ax^2 + bx + c) \in \mathbb{R}abca \neq 0$

$$f(x) = ax^2 + bx + c, \qquad a \neq 0 \qquad (2)$$

From expression (2) it can be seen that:

$$f(x) = ax^2 + bx + c \quad \Longrightarrow \quad f(x) = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right) \qquad (3)$$

By introducing the term and its symmetrical, it is not changed. Like this:$\frac{b^2}{4a^2} f(x)$

$$f(x) = a\left(x^2 + \frac{b}{a}x + \frac{c}{a} + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right) \qquad (4)$$

$$f(x) = a\left[\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + \left(\frac{c}{a} - \frac{b^2}{4a^2}\right)\right] \qquad (5)$$

$$f(x) = a\left[\left(x + \frac{b}{2a}\right)^2 + \left(\frac{4ac - b^2}{4a^2}\right)\right] \qquad (6)$$

Inasmuch

$$\Delta = b^2 - 4ac \qquad (7)$$

So

$$f(x) = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right] \qquad (8)$$

The relation (7) is called the discriminant of the polynomial function of the 2nd

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degree and the expression (8) is known as the canonical form of the quadratic function, the latter being able to be used to determine the roots of this function and the coordinate of the vertex point.

ROOTS OF THE QUADRATIC FUNCTION AND THE STUDY OF THE DISCRIMINANT

Taking the expression given by (8) then transforms into the following equation:$f(x) = 0$

$$a\left[\left(x+\frac{b}{2a}\right)^2-\frac{\Delta}{4a^2}\right]=0. \qquad (9)$$

How, then:$a \neq 0$

$$\left(x+\frac{b}{2a}\right)^2-\frac{\Delta}{4a^2}=0 \iff \left(x+\frac{b}{2a}\right)^2=\frac{\Delta}{4a^2} \iff x=-\frac{b}{2a}\pm\frac{\sqrt{\Delta}}{2a} \qquad (10)$$

or, equivalently,

$$x=\frac{-b\pm\sqrt{\Delta}}{2a}. \qquad (11)$$

Therefore, the values of such that are given by:$xf(x) = 0$

$$x_1=\frac{-b+\sqrt{\Delta}}{2a} \quad \text{and} \quad x_2=\frac{-b-\sqrt{\Delta}}{2a}. \qquad (12)$$

In other words, the values of and make the quadratic function null, and so they are called roots or zeros of that function.$x_1 x_2$

According to Giovanni Junior & Castrucci (2018, p. 101) the expression (11) "is called the solving formula of the complete equation of the 2nd degree", which is also known in textbooks as Bhaskara's formula.

Cases for the study of the discriminant:

i. If (null discriminant), the equation will have two equal real roots, i.e. ;$\Delta = 0 f(x) = 0 x_1 = x_2 = -\frac{b}{2a}$

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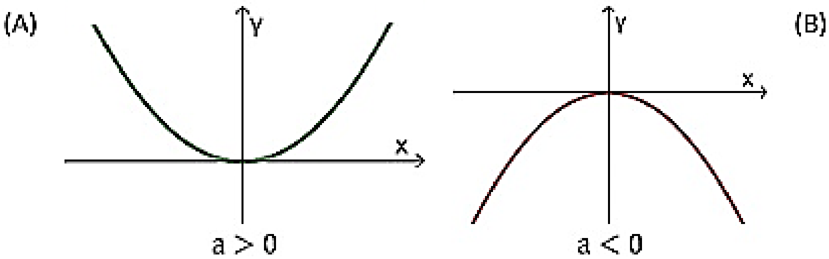
ii. If (positive discriminant), the equation will have two different real roots, i.e., ;$\Delta >$ $0 f(x) = 0 x_1 \neq x_2$

iii. If (negative discriminant), the equation will not have real roots, since .$\Delta < 0 f(x) =$ $0 \sqrt{\Delta} \notin \mathbb{R}$

GRAPH, VERTEX, MAXIMUM AND MINIMUM VALUES, AND SYMMETRY AXIS

According to (LIMA, 2013) "The graph of a quadratic function is a parabola", which depending on whether it is or , the parabola has its concavity facing up or down, respectively. See Figure 1.$a > 0 a < 0$

Figure 1. Graph of a quadratic function: (A) concavity facing upwards; (B) Downward-facing concavity



Source: The authors (2023)

Now, let the expression be given in (8), we have that:

$$f(x) = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right] = a\left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a}. \qquad (13)$$

If

$$x = -\frac{b}{2a} \quad \Longrightarrow \quad f(x) = -\frac{\Delta}{4a}. \qquad (14)$$

The coordinate point$V$

$$V = \left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right) \qquad (15)$$

is called the vertex of the quadratic function and represents the extreme point of the graph of . Therefore, it is verified that the canonical form can be used to obtain the roots of

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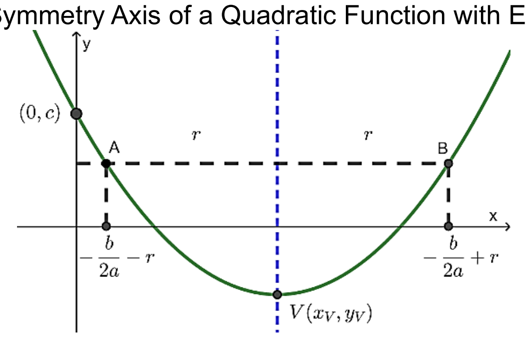
the function and to determine the coordinates of the vertex.f

It is also noted that, if , the quadratic function has a minimum value for and when , the quadratic function has a maximum value for .$a > 0 f(x) = ax^2 + bx + c y = -\frac{\Delta}{4a} x = -\frac{b}{2a} a < 0 f(x) = ax^2 + bx + c y = -\frac{\Delta}{4a} x = -\frac{b}{2a}$

The quadratic function graph admits an axis of symmetry perpendicular to the axis and that passes through the vertex of the parabola (IEZZI & MURAKAMI, 2013, p. 152) (Figure 2).x

Figure 2. Symmetry Axis of a Quadratic Function with E $a > 0 \Delta > 0$



Source: The authors (2023)

For , it follows that:$x_A = -\frac{b}{2a} - r$

$$f(x_A) = f\left(-\frac{b}{2a} - r\right) = a\left[\left(-\frac{b}{2a} - r + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right] = a\left[r^2 - \frac{\Delta}{4a^2}\right] \quad (16)$$

and to :$x_B = -\frac{b}{2a} + r$

$$f(x_B) = f\left(-\frac{b}{2a} + r\right) = a\left[\left(-\frac{b}{2a} + r + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right] = a\left[r^2 - \frac{\Delta}{4a^2}\right]. \quad (17)$$

Soon

$$f(x_A) = f(x_B), \quad (18)$$

if and only if, the parabola has an axis of symmetry, that is, and belong to the graph of the function.$f(x_A)$ $f(x_B)$

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CALCULATING ROOTS FROM THE COMPLEX NUMBER

Before presenting and demonstrating the theorem in a generalized way that leads to the determination of the roots of a quadratic function using the complex number in algebraic form, let us first consider the following examples:

Example 1: Let the quadratic function be we want to get the roots and without using the solving formula.$f(x) = x^2 - 6x + 8, x_1 x_2$

Solution: One way to find such roots is in the procedure you follow.

We denote that in where represents the root of the function. Substituting in the given function, we have to$x = m + nixx$

$$f(m+ni) = (m+ni)^2 - 6(m+ni) + 8 \quad \Longrightarrow \quad 0 = (m+ni)^2 - 6(m+ni) + 8.$$

Developing the last expression, it comes down to the following:

$$m^2 + 2mni + (ni)^2 - 6m - 6mi + 8 = 0 \Longrightarrow m^2 + 2mni - n^2 - 6m - 6ni + 8 = 0$$

Grouping the terms to obtain the new complex number obtains:

$$(m^2 - n^2 - 6m + 8) + (2mn - 6n)i = 0 + 0i \quad \Longrightarrow \quad \begin{cases} m^2 - n^2 - 6m + 8 = 0 & (I) \\ 2mn - 6n = 0 & (II) \end{cases}$$

In this case, we observe that the equality of complex numbers leads to a linear system of two equations and two variables. Therefore, from , we have:(II)

$$2mn - 6n = 0 \quad \Longrightarrow \quad n(2m - 6) = 0$$

Immediately, we will consider and later we will discuss the reason for this condition. Like this$n \neq 0$

$$2m - 6 = 0 \quad \Longrightarrow \quad m = \frac{6}{2} \quad \Longrightarrow \quad m = 3$$

Substituting for in , it comes that:$m(I)$

$$m^2 - n^2 - 6m + 8 = 0 \;\Longrightarrow\; 3^2 - n^2 - 6.3 + 8 = 0 \;\Longrightarrow\; -n^2 - 1 = 0 \;\Longrightarrow\; n = \pm i$$

Since we assumed a non-zero value, we find two values for : and , therefore:$nnn_1 = in_2 = -i$

$$x_1 = m + n_1 i \quad \Longrightarrow \quad x_1 = 3 + i.i \quad \Longrightarrow \quad x_1 = 3 - 1 \quad \Longrightarrow \quad x_1 = 2$$

$$x_2 = m + n_2 i \quad \Longrightarrow \quad x_2 = 3 + (-i).i \quad \Longrightarrow \quad x_2 = 3 + 1 \quad \Longrightarrow \quad x_2 = 4$$

To verify whether the numbers e are really roots of this function, the check is made

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by substituting the value found in the unknown. Therefore, 24

i) For : $x = 2$

$$f(x) = x^2 - 6x + 8 \quad \Longrightarrow \quad f(2) = 2^2 - 6.2 + 8 = 4 - 12 + 8 = 0.$$

ii) To : $x = 4$

$$f(x) = x^2 - 6x + 8 \quad \Longrightarrow \quad f(4) = 4^2 - 6.4 + 8 = 16 - 24 + 8 = 0.$$

Thus, since each of the sentences is true, it is concluded that the given function admits roots and . 24

Example 2: Given the function and is a complex number as being the roots of the given function, get the values of those roots. $y = 9x^2 + 6x + 10$

Solution: Let and substituting in the function the value given to , that is, substituting the complex number given in the polynomial function of the 2nd degree, it comes that: $x = m + nix$

$$y = 9(m + ni)^2 + 6(m + ni) + 10 \Longrightarrow y = 9m^2 - 9n^2 + 18mni + 6m + 6ni + 10.$$

Being the root of the function, then: x

$$y = 0 \quad \Longrightarrow \quad \begin{cases} 9m^2 - 9n^2 + 6m + 10 = 0 & (I) \\ 18mn + 6n = 0 & (II) \end{cases}$$

From , it comes that: (II)

$$n(18m + 6) = 0 \quad \Leftrightarrow \quad n \neq 0 \text{ e } m = -\frac{1}{3}.$$

Thus, from , we have: (I)

$$9m^2 - 9n^2 + 6m + 10 = 0 \quad \Longrightarrow \quad 9\left(-\frac{1}{3}\right)^2 - 9n^2 + 6\left(-\frac{1}{3}\right) + 10 = 0$$

$$\Longrightarrow \quad -9n^2 + 9 = 0 \quad \Longrightarrow \quad n = \pm 1$$

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Therefore, for ,$n_1 = 1$

$$x_1 = m + n_1 i \quad \Longrightarrow \quad x_1 = -\frac{1}{3} + i$$

and, for ,$n_2 = -1$

$$x_2 = m + n_2 i \quad \Longrightarrow \quad x_2 = -\frac{1}{3} - i$$

Now, verifying, we have:

i) For :$x = -\frac{1}{3} + i$

$$\begin{aligned} y = 9x^2 + 6x + 10 \quad \Longrightarrow \quad y &= 9\left(-\frac{1}{3} + i\right)^2 + 6\left(-\frac{1}{3} + i\right) + 10 \\ &= 9\left(\frac{1}{9} - \frac{2}{3}i - 1\right) - 2 + 6i + 10 \\ &= 1 - 6i - 9 + 6i + 8 = 0 \end{aligned}$$

ii) To :$x = -\frac{1}{3} - i$

$$\begin{aligned} y = 9x^2 + 6x + 10 \quad \Longrightarrow \quad y &= 9\left(-\frac{1}{3} - i\right)^2 + 6\left(-\frac{1}{3} - i\right) + 10 \\ &= 9\left(\frac{1}{9} + \frac{2}{3}i - 1\right) - 2 - 6i + 10 \\ &= 1 + 6i - 9 - 6i + 8 = 0 \end{aligned}$$

In view of the above, complex numbers and are roots of this function.$-\frac{1}{3} + i\ -\frac{1}{3} - i$

The two examples given above show that it is possible to obtain the roots of a complete quadratic function without the need to use the solving formula. In view of these results, the following theorem is introduced.

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ROOT THEOREM OF THE QUADRATIC FUNCTION

"Given the quadratic function , the roots and of are obtained by considering the algebraic form of the complex number , such that$f(x) = ax^2 + bx + cx_1 x_2 f x = m + ni$

$$m = -\frac{b}{2a} \text{ e } n = \pm\frac{\sqrt{-\Delta}}{2a}, \text{com } \Delta = b^2 - 4ac.\text{"} \quad (19)$$

Demonstration:

Let the function and be the complex number so that it is root of , then,$f(x) = ax^2 + bx + cx = m + nixf$

$$f(m + ni) = a(m + ni)^2 + b(m + ni) + c \quad (20)$$

$$a(m + ni)^2 + b(m + ni) + c = 0 \quad (21)$$

Developing the expression (21) and making the groupings, it follows that:

$$(am^2 - an^2 + bm + c) + (2amn + bn)i = 0 \quad (22)$$

It is observed that the equality of complex numbers leads to a linear system of two equations and two variables

$$(am^2 - an^2 + bm + c) + (2amn + bn)i = 0 \quad \begin{cases} am^2 - an^2 + bm + c = 0 & (23) \\ 2amn + bn = 0 & (24) \end{cases}$$

From , it comes that:(II)

$$2amn + bn = 0 \implies n(2am + b) = 0 \quad (25)$$

Considering, then:$n \neq 0$

$$2am + b = 0 \quad (26)$$

$$m = -\frac{b}{2a} \quad (27)$$

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The relation (27) shows that the real part of the complex number represents the abscissa of the vertex of the graph of .f

Substituting the result (27) in (23), we have:

$$a\left(-\frac{b}{2a}\right)^2 - an^2 + b\left(-\frac{b}{2a}\right) + c = 0 \Rightarrow \quad n^2 = \frac{-(b^2-4ac)}{4a^2} \qquad (28)$$

By doing so you get:$\Delta = b^2 - 4ac,$

$$n^2 = -\frac{\Delta}{4a^2} \qquad (29)$$

Or

$$n = \pm\frac{\sqrt{-\Delta}}{2a} \qquad (30)$$

The expression (30) shows that o represents a relationship with the discriminant of the function, which would assume the null value only when o is also null. Thus, the roots of the functions will be equal real numbers, which ensures that the equation will have to be a perfect square trinomial. That is why it is considered in examples 1 and 2.$n\Delta n \neq 0$

In this way, you can substitute the values obtained from and in , resulting in,mnx

$$x = -\frac{b}{2a} \pm \frac{\sqrt{-\Delta}}{2a} i \quad \Rightarrow \quad x = -\frac{b}{2a} \pm \frac{\sqrt{\Delta}}{2a} i.i \quad \Rightarrow \begin{cases} x_1 = \dfrac{-b-\sqrt{\Delta}}{2a} \\ x_2 = \dfrac{-b+\sqrt{\Delta}}{2a} \end{cases} \qquad (31)$$

Therefore, a way to obtain the roots of the polynomial function of the 2nd degree using the algebraic form of the complex number is demonstrated.

It is worth mentioning that although the parameters and are associated with an expression involving the coefficients , and the quadratic function, for the calculation of these it is sufficient to make use of resources of basic mathematics, such as solving a linear system of variables with two variables, development of remarkable products and calculation

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of numerical value.$mnabc$

DISCUSSION OF ROOTS, CANONICAL FORM AND VERTEX

It should be noted that:

- If it is pure imaginary, then it will be real roots;$nx_1 \neq x_2$
- If it is real non-zero, then it will be imaginary roots (complex, but not real);$nx_1 \neq x_2$
- If it is null, then it will be real roots, and so the equation will be a perfect square trinomial;$nx_1 = x_2$

It is important at this point to consider that in terms of a complex number, the imaginary part carries the condition that discusses the nature of the roots and can compare the value of with the discriminant of the function that also analyzes the nature of the roots, as shown in nChart 1.

Table 1. Comparison between the discriminant and for discussion of the roots of the quadratic equation$n$

DISCRIMINATING	IMAGINARY PART n	ROOTS OF THE QUADRATIC EQUATION
$\Delta > 0$	n It's pure imaginary	$x_1 \neq x_2$ will be distinct royal roots
$\Delta < 0$	n it's real, not null	$x_1 \neq x_2$ will be imaginary roots
$\Delta = 0 \quad \Rightarrow \quad x_1 = x_2 = -\dfrac{b}{2a}$	n It's null	$x_1 = x_2$ will be real roots the same

Source: The authors (2023)

Using (8), we can write the canonical form of the quadratic function by making changes to variables. Like this

$$f(x) = a\left[\left(x + \frac{b}{2a}\right)^2 - \frac{\Delta}{4a^2}\right] = a[(x-m)^2 + n^2] \qquad (32)$$

Let the expression be given in (8), we have that:

$$f(x) = a[(x-m)^2 + n^2] = a(x-m)^2 + an^2. \qquad (33)$$

is called the vertex of the quadratic function and represents the extreme point of the graph of . Therefore, it is verified that the canonical form can be used to obtain the roots of the function and to determine the coordinates of the vertex.f

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Therefore, the parameter represents the abscissa of the vertex of the parabola of the function, while it configures the ordinate of the vertex of this parabola. Like this:$m(x_V)an^2(y_V)$

$$f(x) = a(x-m)^2 + an^2 \quad \Longrightarrow \quad f(x) = a(x-x_V)^2 + y_V \tag{34}$$

So

$$V = (x_V, y_V) = (m, an^2) \tag{35}$$

and called the vertex of the quadratic function.f

Examples involving the mathematical foundation constructed in sections 3.1 and 3.2 are discussed below.

Example 3: Write the canonical form of the following quadratic functions, and then find the vertex of the parabola of these same functions.

a) ;$f(x) = x^2 - 4x + 5$

b) $y = -5x^2 + 2x + 3$

Solution: In view of the theorem developed earlier, the value of represents the solution of the equation. Soon$x = m + ni$

a) To have that:$f(x) = x^2 - 4x + 5$

$(m+ni)^2 - 4(m+ni) + 5 = 0 \quad \Longrightarrow \quad m^2 + 2nmi - n^2 - 4m - 4ni + 5 = 0.$

Thus, and .$m^2 - n^2 - 4m + 5 = 0(2m-4)n = 0$

Be it, then, or .$n \neq 02m - 4 = 0m = 2$

Substituting in , results in and therefore$m = 2m^2 - n^2 - 4m + 5 = 04 - n^2 - 8 + 5 = 0$

$$n^2 = 1$$

Using the values of and in the expression (33), it is concluded that the canonical form of the function is:mn^2

$$f(x) = 1[(x-2)^2 + 1]$$

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From the last result it is possible to directly extract the coordinates of the vertex of the parabola, being:

$$V = (2{,}1).$$

b) Similarly, for we have that:$y = -5x^2 + 2x + 3$

$-5(m + ni)^2 + 2(m + ni) + 3 = 0 \implies -5m^2 - 10nmi + 5n^2 + 2m + 2ni + 3 = 0$

Thus, and .$-5m^2 + 5n^2 + 2m + 3 = 0$ $(-10m + 2)n = 0$

Be it, then, or$n \neq 0 - 10m + 2 = 0$

$$m = \frac{1}{5}$$

Substituting the value of in , results in$m$$-5m^2 + 5n^2 + 2m + 3 = 0$

$$-5\left(\frac{1}{5}\right)^2 + 5n^2 + 2\left(\frac{1}{5}\right) + 3 = 0 \implies -\frac{1}{5} + 5n^2 + \frac{2}{5} + 3 = 0$$

and therefore

$$n^2 = -\frac{16}{25}$$

Using the values of and and substituting in the expression (8), it is concluded that the canonical form of the function is:mn^2

$$f(x) = -5\left[\left(x - \frac{1}{5}\right)^2 - \frac{16}{25}\right].$$

The apex of the parable is given by

$$V = \left(\frac{1}{5}, \frac{16}{5}\right).$$

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OTHER APPLICATIONS OF THE THEOREM

This subsection contemplates applications in Mathematics and Physics using what was proposed in this work, reaffirming the simple applicability and relevance of the theorem presented and demonstrated previously.

Application 1:

Find the points of intersection of the function parabola with the axis of the abscissas using:$f(x) = 3x^2 - 5x - 2$

(a) the resolute formula;

b) the algebraic form of complex numbers.

Solution: At the instant when the parabola intersects the axis of the abscissas, the value of is equal to zero, that is, one wants to find the roots of the function. Like this:$f(x)$

a) Being so, it has to be so,$f(x) = 3x^2 - 5x - 2, a = 3; b = -5$ e $c = -2,$

$$\Delta = b^2 - 4ac = (-5)^2 - 4.3.(-2) = 25 + 24 = 49$$

soon

$$x = \frac{-b \mp \sqrt{\Delta}}{2a} = \frac{-(-5) \pm \sqrt{49}}{2.3} = \frac{5 \pm 7}{6} \implies \begin{cases} x_1 = \dfrac{5+7}{6} = \dfrac{12}{6} = 2 \\ x_2 = \dfrac{5-7}{6} = \dfrac{-2}{6} = -\dfrac{1}{3} \end{cases}$$

b) According to the theorem in 3.1, let be root of , then:$x = m + ni\ f(x)$

$$f(m + ni) = 3(m + ni)^2 - 5(m + ni) - 2$$

$$3m^2 - 3n^2 + 6mni - 5m - 5ni - 2 = 0$$

Like this

$$(3m^2 - 3n^2 - 5m - 2) + (6mn - 5n)i = 0 + 0i \implies \begin{cases} 3m^2 - 3n^2 - 3m - 2 = 0 \\ 6mn - 5n = 0 \end{cases}$$

Using the second equation of the system, it follows that:

$$n(6m - 5) = 0 \iff n \neq 0 \text{ e } m = \frac{5}{6}.$$

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Considering the first equation of the system and substituting the value of :m

$$3m^2 - 3n^2 - 3m - 2 = 0 \implies 3\left(\frac{5}{6}\right)^2 - 3n^2 - 3\left(\frac{5}{6}\right) - 2 = 0 \implies -\frac{147}{36} = 3n^2$$

$$\implies n^2 = -\frac{49}{36} \implies n = \pm\frac{7}{6}i \implies \begin{cases} n_1 = \frac{7}{6}i \\ n_2 = -\frac{7}{6}i \end{cases}$$

So

$$x_1 = m + n_1 i \implies x_1 = \frac{5}{6} + \frac{7}{6}i.i \implies x_1 = \frac{5}{6} - \frac{7}{6} \implies x_1 = -\frac{1}{3}$$

$$x_2 = m + n_2 i \implies x_1 = \frac{5}{6} - \frac{7}{6}i.i \implies x_1 = \frac{5}{6} + \frac{7}{6} \implies x_2 = 2$$

which shows that the same values are obtained.

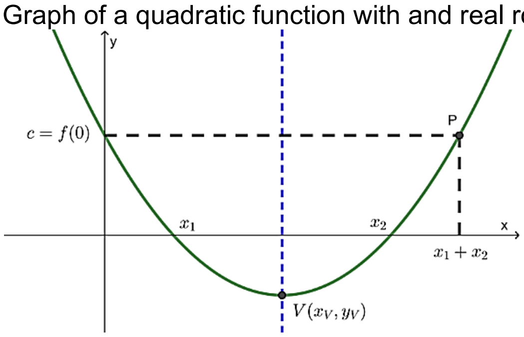
Application 2:

Using the algebraic form of complex numbers, prove the reciprocal of Etiene's theorem.

Solution: Etiene's Theorem stated in (MUNIZ, 2019), says that: "Consider the quadratic function , . Then the symmetric point, with respect to the axis of symmetry of the parabola, to the point is such that the number is equal to the sums of the roots of the function or, equivalently, ."$f(x) = ax^2 + bx + ca \neq 0P = (x_e, c)(0, c)x_e y = f(x)x_e = -\frac{b}{a}$

Figure *3* represents a possible graphical representation for the theorem exposed.

Figure 3. Graph of a quadratic function with and real roots$a > 0$



Source: The authors (2023)

Let be and the roots of the quadratic function, we have that:$x_1 = m + nix_2 = m -$

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$nif(x) = ax^2 + bx + c$

$x_e = x_1 + x_2 \implies x_e = (m + ni) + (m - ni) \implies x_e = 2m.$

Like this

$$f(x_e) = ax_e^2 + bx_e + c \implies f(2m) = a(2m)^2 + b(2m) + c$$
$$\implies f(2m) = 4am^2 + 2bm + c.$$

As , it turns out that:$m = -\frac{b}{2a}$

$$f(2m) = 4a\left(-\frac{b}{2a}\right)^2 + 2b\left(-\frac{b}{2a}\right) + c \implies f(2m) = \frac{b^2}{a} - \frac{b^2}{a} + c \implies f(x_e) = c$$

which proves the reciprocal of Etiene's theorem.

Application 3:

Given the functions below, determine its roots.

a) $f(x) = \frac{1}{2}[(x - 4)^2 + 100]$

b) $f(x) = (-2)[(x + 3)^2 - 25]$

Solution: Comparing the given functions with the expression (8), we realize that they are presented in canonical form. Like this:

a) $m = 4 \text{ e } n^2 = 100 \implies n = \pm 10$

Thus, as it is real, then the roots of are complex (see $nf(x)$*Box 1*). Indeed

- towards;$n = 10x_1 = 4 + 10i$
- towards.$n = -10x_2 = 4 - 10i$

b) $m = -4 \text{ e } n^2 = -25 \implies n = \pm 5i$

So as it is complex, then the roots of are real. Indeed$nf(x)$

- towards;$n = 5ix_1 = -3 + 5i.i = -3 - 5 = -8$
- towards.$n = -5ix_2 = -3 - 5i.i = -3 + 5 = 2$

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Application 4: A particle moves from a position of and under an angle of with the horizontal. It is known that at the beginning of the oblique throw, the particle's velocity was . Let , get the time the object stayed in the air. $10\ m30^0 20\ m/s g\ =\ 10\ m/s^2$

Solution: Data , and . In addition, as the movement is parabolic, it is stated that: $g\ =\ 10\ m/s^2 H_0 = 10\ m v = 20\ m/s$

$$H = H_0 + v_{0y}t + \frac{1}{2}gt^2 \quad \Longrightarrow \quad H = 10 + v_0 . sen30^0 . t + \frac{1}{2}(-10)t^2$$

$$\Longrightarrow \quad H = 10 + 20 . \frac{1}{2} . t - 5t^2$$

$$\Longrightarrow \quad H = 10 + 10t - 5t^2$$

Let be , the time the particle remains in the air, with , then, when the particle hits the ground it has . Like this $t = m + ni t > 0 H = 0$

$$H = 10 + 10t - 5t^2 \quad \Longrightarrow \quad 10 + 10(m + ni) - 5(m + ni)^2 = 0$$

$$\Longrightarrow \quad 10 + 10m + 10ni - 5m^2 + 5n^2 - 10mni = 0$$

Soon

$$(10 + 10m - 5m^2 + 5n^2) + (10n - 10mn)i = 0 \Longrightarrow \begin{cases} 10 + 10m - 5m^2 + 5n^2 = 0\ (I) \\ 10n - 10mn = 0\ (II) \end{cases}$$

From , it comes that: (II)

$$n(10 - 10m) = 0 \quad \Leftrightarrow \quad n \neq 0 \text{ e } m = 1.$$

Now, from and by doing, we get: $(I) m = 1$

$$10 + 10m - 5m^2 + 5n^2 = 0 \quad \Longrightarrow \quad 10 + 10.1 - 5.1^2 + 5n^2 = 0 \quad \Longrightarrow \quad 15 + 5n^2 = 0$$

$$\Longrightarrow \quad 5n^2 = -15 \quad \Longrightarrow \quad n^2 = -3 \quad \Longrightarrow \quad n = \pm\sqrt{3}i$$

As , it has to $t = m + ni$

$$t = 1 \pm \sqrt{3}i . i \quad \Longrightarrow \quad t = 1 \mp \sqrt{3}$$

Possessing that, then, , that is, the particle takes a time of approximately seconds

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$t > 0t = \left(1 + \sqrt{3}\right)s2{,}73$of remaining in the air.

CONCLUSION

Throughout the development of this article, a study with the quadratic function and calculus roots of a quadratic function was presented by applying the algebraic form of complex numbers with the resolution of a system of two linear equations and two unknowns, and . Based on this study, it became possible to elaborate a solving technique to find roots of a polynomial function of degree 2 without the need to introduce formulas present in the literature in mathematics. In this new solution method, the student develops the calculation without the need to memorize expressions, just using the mathematical formalism, as presented in examples 1 and 2 of section 3.1.mn

This methodology of finding the roots of a polynomial function of the 2nd degree, as it was exposed in this article, has great relevance and special attention, because the theory presented has the advantage of showing that although the algebraic form of a complex number is not included in the set of real numbers, it can be used and applied in problems that can lead to real or even imaginary values as observed in the examples and applications given.

Another fact of interest in this learning is that the discussion for the study of the roots of the equation is no longer restricted to the discriminant, now, the discussion of the roots focuses on the parameter (Δn*Chart 1*), which actually represents a new approach to the discussion of the roots of the polynomial equation of the 2nd degree, since it was found that for complex the roots are real, for real the roots are complex and for n null the roots are equal.nn

Therefore, the developed approach can be used as an alternative way to calculate roots of a quadratic function by not resorting to the help of formulas present in the mathematical literature and by making use of basic tools such as numerical value of an algebraic expression, the development of remarkable products, initial knowledge of the set of complex numbers and solving a linear system of two equations and two unknowns.

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REFERENCES

1. Ávila, G. (1996). Variáveis complexas e aplicações. Rio de Janeiro: LTC.

2. Berriman, A. E. (1956). The Babylonian quadratic equation. The Mathematical Gazette, 40(333), 185–192. https://doi.org/10.2307/3608807

3. Carmo, M. P., Sampaio, A. P., & Wagner, E. (1991). Trigonometria e números complexos. Coleção do Professor de Matemática. Rio de Janeiro: SBM.

4. Carmo, M. P., Morgado, A. C., & Wagner, E. (1992). Trigonometria - Números complexos. Rio de Janeiro: Editora IMPAVITAE.

5. Fragoso, W. C. (1999). Equação do 2º grau: uma abordagem histórica (2ª ed.). Ijuí: Unijuí.

6. Garbi, G. G. (2010). O romance das equações algébricas (4. ed.). São Paulo: Livraria da Física.

7. Giovanni Junior, J. R., & Castrucci, B. (2018). A conquista da matemática: 9º ano: ensino fundamental: anos finais (4. ed.). São Paulo: FTD.

8. Iezzi, G., & Murakami, C. (2013). Fundamentos de matemática elementar: 1: conjuntos, funções (9. ed.). São Paulo: Atual.

9. Lima, E. L. (2013). Números e funções reais. Rio de Janeiro: SBM.

10. Muniz, L. de O. (2019). O teorema de Etiene. RPM, 99, 32–33. Disponível em: https://portal1.iff.edu.br/nossos-campi/bom-jesus-do-itabapoana/noticias/teorema-de-etiene-estudante-do-curso-tecnico-em-quimica-cria-teorema-matematico

11. Revista do Professor de Matemática. (1999). A fórmula é de Bhaskara? RPM, 1º Quadrimestre, 39, 54. Disponível em: http://www.rpm.org.br/cdrpm/39/12.htm

12. Ribeiro, A. J. (2009). A noção de equação e suas diferentes concepções: uma investigação baseada em aspectos históricos e epistemológicos. Revista Brasileira de Ensino de Ciência e Tecnologia, 2(1), 70–86. https://doi.org/10.3895/S1982-873X2009000100005