


INTERFACES BETWEEN FINANCIAL EDUCATION AND CRITICAL MATHEMATICS EDUCATION, IN THE SPECTRUM OF THE SEMANTIC FIELDS MODEL: ANALYSIS OF INVESTMENT IN A RESIDENTIAL PHOTOVOLTAIC PLANT

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ABSTRACT

This article is the result of a research project that aims to analyze contributions of a proposal for teaching School Financial Education, based on Critical Mathematics Education, in the spectrum of the Semantic Fields Model, in teaching and learning processes in the training of mathematics educators. The epistemological foundation adopted seeks to understand what the participants think when they produce meanings in directions other than those expected by the teacher. From this perspective, we chose to make readings of processes of production of meanings, as a central aspect of learning. The proposed practice aims to foster a discussion about this framework, in the expectation of leveraging studies that use it as well as promoting reflections on teaching practice, in view of processes of production of meanings that are usually triggered in classrooms. The research is qualitative and descriptive, approaching a case study. The purpose of this practice, aimed at the possibility of implementing a residential photovoltaic system, as a generating theme, was to subsidize the participants in the feasibility analysis of implementing this system. Electricity bills of a residence were analyzed to verify the feasibility of implementing or not a residential photovoltaic plant, connected to the electricity grid, aiming at reducing electricity bills, through the compensation system. For the sake of space, for this article, we brought an analyzed case. To achieve the objective, we used the guiding principles of Investigative Educational Practices as an investigation strategy,

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observing knowledge produced from the graphic analysis involving consumption, cost and time.

Keywords: Critical Mathematics Education. School Financial Education. Energy Bill. Compensation System. Semantic Fields Model.

INTRODUCTION⁴

This article is the result of doctoral research projects, developed from the Study and Research Group on the Model of Semantic Fields and Mathematics Education (Gepemem), of a qualitative nature (BODGAN; BIKLEN, 2013), with a descriptive method, and the methodological procedure adopted is the *analysis of the production of meanings* (LINS, 2012; 1999; LINS; GIMÉNEZ, 1997; SILVA, 1997; 2022; 2003), based on ideas and notions related to the Semantic Fields Model (SCM), and the Sustainable Development Goals (SDGs), present in the 2030 Agenda of the United Nations (UN).

The application involved 6 (six) volunteers, undergraduate students in mathematics, participants of the Institutional Scholarship Program for Initiation to Teaching (Pibid), of the Federal Institute of Espírito Santo (Ifes), *Vitória campus*, members of Gepemem, from a formative action, from the perspective of Investigative Educational Practices (PEI), as endorsed in Chaves (2004), developed in the Laboratory of Integrated Teaching Practices of the Teaching Degree course in Mathematics (LPEI).

Although, in the *modus operandi* of today's societies, mathematics can be present in our daily lives, the teaching and learning processes in most basic education schools have occurred based on mnemonic methods, in the application of algorithms presented as central elements and not as a consequence (LINS; GIMÉNEZ, 1997; CHAVES, 2004), as a product (and not as a process) and by the formalization and fixation of ready-made and finished things (CHAVES, 2004). In this way, the teaching and learning processes minimize (and neglect) any *production of meanings* other than those *legitimized by the school* and academic communities, skills training, mechanization of processes and little connection and applicability to the daily lives of students and, as a consequence, many of these students do not perceive that there are utilities, as well as possibilities of applicability of what they supposedly "learn" or "should learn" in Mathematics classes .

However, studies in the field of Mathematics Education developed in recent decades (CHAVES, 2004; SKOVSMOSE, 2000; 2001) point out the importance of breaking with the exercise paradigm, which focuses exclusively on the transmission of ready-made and finished knowledge, with formulas and algorithms as a product, without prioritizing the

⁴ We emphasize that in addition to foreign words, we will write in italics ideas and terms related to MCS. We will also establish a differentiation in relation to mathematics: when we write Mathematics we will refer to the positivist, which its followers defend as unique, hegemonic and universal; However, we will graph mathematics, when we refer to the various mathematics, cultural products that meet different specificities in our daily lives: school mathematics, street mathematics, mathematics of the mathematics teacher, etc.

stimulation of reasoning, criticality, collaborative work, dialogue and its applicability, therefore, without focusing on the process.

In the traditional model, the teacher assumes a posture of transmitting knowledge and not guiding processes, privileging results and not ideas or reflections, reducing actions to the application of formulas, with little or no participation of students. Chaves (2004) and Skovsmose (2000) point out that traditional methodologies are based on the resolution of repetitive textbook tasks as a central part of the class, where such tasks are corrected without discussion and analysis of the students' ways of operating, not considering their respective logics of operations, nor the context.

Chaves (2004) also points out elements of reflection on the relations that maintain what he classifies as Traditional Mathematics Teaching (ETM) and its consequences; in this sense, it becomes essential to explore ways of relating classroom mathematics with the daily life of students, what Lins (1999) calls street mathematics. Thus, it is feasible and *legitimate* to develop approaches that foster the development of critical and reflective reasoning, which enable possible local interactions that can impact the quality of life of their surroundings (CHAVES, 2004).

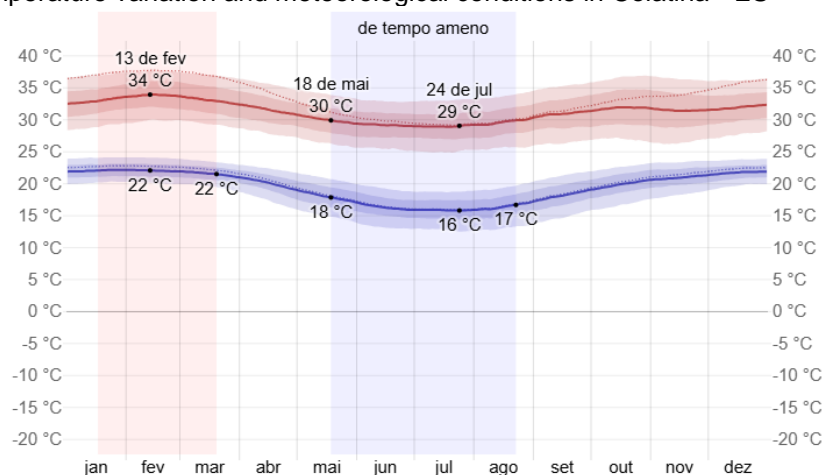
PROBLEMATIZATION

The expanding population growth and the demands for industrialization of a society immersed in capitalist molds are closely linked to the diversified consumption of energy and new ways of obtaining it. For example, the oil crisis in the 1970s and the consequences today, climate change and the silting of rivers have stimulated the search for renewable energy in the world, which is now on the agenda of the 2030 Agenda of the United Nations (UN), constituting one of the Sustainable Development Goals (SDGs) – SDG 7: Clean and Sustainable Energy. In this way, possible global efforts seek to articulate initiatives to promote the implementation of the 2030 Agenda and for institutions to undertake strategies engaging educators, researchers, students, public managers and civil society, in order to offer an opportunity to understand the 17 SDGs and their 169 goals. In this sense, possible changes that can mitigate and reduce the global impacts arising from exacerbated consumption, which leads to the large-scale production of garbage of various species, which impact the environment and, consequently, compromise the life of (and on) the planet, can be implemented (CABRAL; GEHRE, 2020).

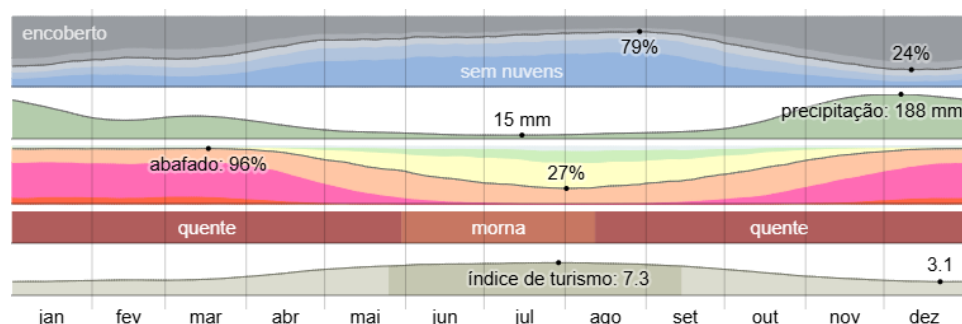
In Brazil, our energy matrix differs considerably from the global one. According to data from the National Electric Energy Agency (Aneel), about 47.4% of our energy matrix is composed of renewable sources – but not inexhaustible – However, in prolonged periods of rainfall scarcity, there is a substantial risk of electricity rationing due to the reduction of water reserves in dams, in addition to the socio-environmental impacts caused by the construction of these hydroelectric dams. An initial solution for these moments was the activation of thermoelectric plants that burn fossil fuels, such as coal, natural gas and oil, which in addition to polluting are costly. To alleviate the situation and reduce consumption, governments adopt the system of energy tariff flags, which makes energy more expensive when the reservoirs are at critical levels, requiring the activation of these thermoelectric plants.

In this sense, the installation of residential photovoltaic plants, with the compensation system, can, in some specific situations, contribute to a substantial reduction in the impact of the electricity bill on the family budget, which can have a favorable impact on people's quality of life, especially in the case of a residential photovoltaic energy project, installed in Colatina – ES, which has average maximum temperatures above 30 °C for most of the year⁵. During the year, the climate is warm and in general the temperature ranges from 16 °C to 34 °C. The hottest month is February, sometimes exceeding 38°C and a minimum of 22°C, on average. Only in winter, in a few hours of the day and in a very few days, the temperature orbits around 25 °C, with the coldest month of the year being in July, with a minimum of 16 °C and a maximum of 29 °C, on average (Figure 1).

Figure 1: Temperature variation and meteorological conditions in Colatina - ES



⁵ <<https://pt.weatherspark.com/y/30869/Clima-caracter%C3%ADstico-em-Colatina-Esp%C3%ADrito-Santo-Brasil-durante-o-ano>>. Accessed on 24 Feb. 2025.



Source: <<https://pt.weatherspark.com/y/30869/Clima-caracter%C3%ADstico-em-Colatina-Esp%C3%ADrito-Santo-Brasil-durante-o-ano>> Accessed on 24 Feb. 2025

With the installation of a residential photovoltaic system, the consumer pays only a minimum monthly tariff according to the type of residential connection. The main motivation of this study was to analyze whether the installation of photovoltaic solar energy systems, in the current case, in a residence would be feasible and whether it could contribute to changes in the quality of life of families without increasing indebtedness and preserving the environment.

The IEP and its principles, as well as the CME proposal, emerged as possible alternatives and a different way of working, starting from the real concerns of the students aiming at a participatory, reflective, critical practice, based on investigation and dialogicity and possible socio-environmental engagement, which encouraged learning, the involvement of students with the generating theme, the development of reasoning, collaborative work, in the face of socio-environmental problems. The mathematics contained in the process then began to serve as possible tools for reading the world (CHAVES, 2004) and, with that, ceasing to be hollow and banking (FREIRE, 2002), focused only on the paradigm of exercise (SKOVSMOSE, 2000).

In view of the exposed problem, in the present work we seek to answer the following research question: *how the development of investigative Educational Practices, from the perspective of the Semantic Fields Model (SCM) and Critical Mathematics Education (CME), can contribute to the teaching and learning processes of a School Financial Education, in order to promote a critical and reflective education?*

Thus, the general objective of this study is to analyze possible contributions of a proposal for Investigative Educational Practices (CHAVES, 2004) of School Financial Education (SILVA; POWELL, 2013), from the perspective of MCS and CME, in teaching and learning processes in high school.

THEORETICAL-EPISTEMOLOGICAL FRAMEWORK

In order to achieve the proposed actions, regarding the object of study of our research, this project was based on theoretical-epistemological conceptions that deal with School Financial Education (SILVA; POWELL, 2013), based on the PEI (CHAVES, 2004), from a perspective of the MCS (LINS, 1999; 2012; 2020; LINS; GIMÉNEZ, 1997; SILVA, 1997; 2003; 2012; 2022) and CME (SKOVSMOSE, 2000; 2001; 2013), as possible alternatives, applied to the teaching and learning processes.

INVESTIGATIVE EDUCATIONAL PRACTICES (PEI)

To break with this decontextualization of mathematics teaching, Chaves (2004) proposes that priority be given to work with *readings* and analysis of information that may interest students – as far as possible, information brought by them – so that, in a collaborative way, possible solutions to the proposed problems can be sought.

The practices defended in Chaves (2004) have an investigative character, allowing the student to assume a prominent role in confronting their own values and *meanings produced* by answering the questions proposed during the investigation, which is conducted collaboratively and in work groups. This research approach aims to break with the values of the ETM and proposes what it calls Investigative Educational Practices (PEI) and points out a set of guiding principles, based on basic notions of the Semantic Fields Model (SCM), developed by the mathematical educator Romulo Campos Lins (1955-2017), and of Critical Mathematics Education (CME), from the perspective presented in Skovsmose (2000; 2001).

Chaves (2004), based on the IEP, proposes a more than interdisciplinary perspective to break algorithmic education, which he called hollow and banking, as well as Freire (2002). Thus, it presents a list of principles, which it calls guidelines, namely:

(1) freedom of expression — with a view to fostering dialogue, the exchange of ideas and the *sharing of communicative spaces* as a means of *producing meanings and knowledge*;

(2) of the natural order – prioritizing investigative and dialogical approaches, developed in work groups – promoting *co-work* – with a view to identifying and solving problems, so that mathematical ideas and tools emerge from the need to respond to the research situation;

(3) collaborative — emphasizes and values group work, so that the teacher acts as a guide of the process and not as a centralizing element of "ready-made knowledge" that imputes ways of acting;

(4) integration — so that the resolution of proposed problems can be integrated, from generating themes, to the curricular units related (or not) to mathematics, which become means and not ends. The focus is no longer on Mathematics and becomes the solution of the problems investigated.

(5) interaction — formerly the principle of intervention, aimed at the implementation of investigative practices, aimed at solving local socio-environmental, sociocultural or socioeconomic problems, valuing group work and the ways of operating of each group;

(6) the tactical device – valuing the debate to minimize the possibility of initiating a process that leads students not to share new interlocutors in a situation of face-to-face interaction, different from those to which they were facing; of not proposing to produce meanings in another direction" (SILVA, 2012, p. 79); (ii) *epistemological limits* – "[...] the student's impossibility to produce meaning for a statement" (SILVA, 1997, p. 17-18); (iii) *epistemological obstacles* – "[...] a process in which a student operating within a semantic field could potentially produce meaning for a statement, but does not" (SILVA, 1997, p. 18). From the tactical device, the teacher seeks to identify the *students' ways of operating* and their respective *logics of operations*, with a view to analyzing the *production of meanings and knowledge*;

(7) freedom as an end — with a view to privileging the spontaneity and creativity of students, with a view to minimizing the process of reading through lack⁶, which ends up serving as a springboard for competition and meritocracy.

Based on the principles mentioned above, the teacher is challenged not only to develop new methodologies and procedures that can contribute to stimulate students, but mainly to learn to listen, observe and try to understand their *ways of operating*, seeking to subvert the order of traditional school systems and providing a new system where "[...] the mathematics education of our students must correspond to an 'education FOR

⁶ "In Piagetian theories this *lack of capacity* is interpreted in terms of developmental stages: *the child has not yet reached the stage that would allow him to learn this or that*. In both cases, the person is read by *the lack*: 'I, who have already developed (have already learned), and who know that you are just like me, can see what is missing in your development (knowledge), see what you *are not yet*'. I want to remind the reader, as mentioned earlier, that Piagetian theories and traditional teaching have this assumption in common, and also that it is only in other assumptions – center in the child or center in official Mathematics – that we will be able to understand how these two strands differ within Mathematics Education" (LINS, 1999, p. 78, highlights of the original).

Mathematics' and not an education FOR Mathematics" (LINS, 2020, p. 14, emphasis in the original), so that it is possible to:

- 1 to make explicit, in school, the modes of production of meanings of the street;
- 2 to produce legitimacy, within the school, for the modes of production of meaning in the street (political act, pedagogical act);
- 3 propose new modes of production of meanings, which join those of the street, instead of replacing them (LINS, 1999, p. 92).

From this perspective, the IEP constitute a pedagogical alternative in the development of teaching and learning processes, breaking the inertia of what Skovsmose (2000) classified as the exercise paradigm.

Skovsmose (2013), in the same direction as Chaves (2004), proposes a critical teaching of mathematics, in which students can debate economic, political and socio-environmental issues in society, producing possibilities of changes in the environment in which they live, without the exacerbated concern of formalization in the development of mathematics FOR Mathematics.

In this sense, because he understands that the proposal for the development of PEI can come to value and raise debates and interactions regarding socio-environmental aspects, Chaves (2004) suggests themes that are of interest to the group (such as the climate issue and alternative sources of energy production), whose work is collaborative, with approximations to the historical-cultural theory elaborated by Lev Semionovitch Vygotsky (1896-1934), epistemological ballast of the MCS.

SCHOOL FINANCIAL EDUCATION

According to the Organization for Economic Co-operation and Development (OECD), consumers around the world lack critical and reflective knowledge about product offerings and financial notions. This shortage can affect not only its financial stability but also the economic health of the country as a whole.

A study by the National Confederation of Trade in Goods, Services and Tourism (CNC) showed that, in 2018, 59.6% of Brazilian families had overdue bills. Abusive interest on credit cards is the largest source of debt, representing 76.4% of them. It is worth mentioning that, in January 2024, according to the Consumer Indebtedness and Delinquency Survey (Peic), 28.3% of Brazilian households had overdue debts and delinquency remains on the rise among Brazilians, with 29.4% of households reporting overdue debts – the highest level since October 2023. The number of consumers who say

they are unable to pay off their debts increased to 12.9% (in October, this percentage was 12.6% and, in November 2024, 12.5%).

The "ease" of credit and the lack of financial planning impact budgets – personal and family. We are constantly bombarded by media incentives to consume, and there are many subtle influences of consumption in society that are accepted as normal.

According to Bauman (2008), consumers become commodities in the midst of a sea of options and also highlights an important characteristic of this consumer society, which he calls "the transformation of consumers into commodities", that is, "their dissolution in the sea of commodities".

Bauman (2008) emphasizes that, in order to maintain this consumption system, it is necessary to perpetuate consumer dissatisfaction, constantly introducing new products into the market. This leads to a cycle of exchanges to follow fashion trends, based on the market principle of planned obsolescence, based on what Bauman (2008) calls the "economy of deception", which exploits the emotions of consumers to the detriment of reason. This culture of buying leads to the accumulation of waste (garbage), negatively impacting the environment. Consumerism, the result of exacerbated consumption, can also bring psychological and physical problems to individuals, since many work excessively to maintain their consumption patterns. Confronting this capitalist pattern requires a School Financial Education that is critical, reflective, concerned with the criticality of the student so that he does not become a compulsive consumer, passive to the media traps, as suggested by Bauman (2008) and Silva and Powell (2013).

Financial Education proposes approaches to economic and financial situations that are part of the daily lives of students and their families. School Financial Education (SILVA; POWELL, 2013), in addition, based on its guiding axes⁷, it proposes a broader spectrum, which leads us to bring the UN 2030 Agenda (CABRAL; GEHRE, 2020), with its SDGs at the heart of the problems to be discussed. However, the approaches in basic education and

⁷ *Basic notions of Finance and Economics* — from topics involving money and its function in society and the relationship between money and time, notions of interest, savings, inflation, profitability and liquidity of an investment, financial institutions, notion of assets and liabilities and financial investments. *Personal and family finance* — in which financial planning, management of personal and family finances, strategies for money management, savings and investment of finances, household budgeting, and taxes are discussed. *The opportunities, risks, and pitfalls of money management in a consumer society* — opening debates to discuss investment opportunities and risks, the consumption pitfalls behind marketing strategies, and how the media encourages people's consumption. *The social, economic, political, cultural and psychological dimensions that involve Financial Education* — aiming to broaden discussions on consumerism and consumption, relations between consumerism, waste production and socio-environmental impacts, wages, classes and social inequalities, need *versus* desire, ethics, etc. (SILVA; POWELL, 2013).

in the training of Mathematics teachers, related to Financial Education, are generally restricted to ideas and objects pertinent to Financial Mathematics.

However, we emphasize that there are profound differences between Financial Mathematics and Financial Education, especially with regard to the proposal of School Financial Education (Silva; Powell, 2013) that we follow, since we understand Financial Mathematics as a possible tool at the service of School Financial Education, as well as mathematics as possible tools for the implementation of IEP, but not as an area of inquiry, at least in education; this is because, according to our readings, School Financial Education, as well as the development of IEP, go beyond the algorithmic conception of a school mathematics committed to ETM, as they seek to develop the student's reflection and criticality – based on dialogicity and collaborative work – in decision-making in everyday situations, providing a possible transformation in their personal and family finances, in addition to worrying about the impacts on the environment.

The proposal for School Financial Education (SILVA; POWELL, 2013), is configured as a set of information in which students are introduced to the universe of money, finance and economics, so that they can analyze an economic-financial situation and make critical decisions about their personal finances, family or the society in which they are inserted. As well as in issues related to the production of garbage, the socio-environmental impact of consumption and the relations between consumption and consumerism, being aligned with the UN SDGs.

In view of these arguments, we emphasize the importance of addressing actions and practices that involve daily financial and commercial operations, in order to contextualize what is being taught in the classroom, so that it is possible to make changes, leading the student to constitute himself as the protagonist of the process in the *production of knowledge*, in a critical, reflective and autonomous way. Chaves (2004) based on the ideas of Patrick Geddes (1854-1923) in the defense that students, guided by their teachers, can interact with the "realities" of their environment and thus develop attitudes that are creative in relation to it, and then it is up to the teachers to act as interlocutors of an education that can incorporate an analysis of socio-environmental "realities" as opposed to the one in which the student is led NOT to reflect on the consequences of their actions (CHAVES, 2004).

MODEL OF SEMANTIC FIELDS

The epistemological model developed by the mathematical educator, Romulo Campos Lins (1955-2017), entitled the Semantic Fields Model (SCM), incorporates ideas from the thought of Lev Semionovitch Vygotsky (1896 - 1994), Alexis Nikolaevich Leontiev (1903 - 1979), Vasily Vasilovich Davydov (1930 - 1998) and Nelson Goodman (1906-1998), among others.

The CSM is not configured as a theory, but a theorization to be studied and mainly used, because the central aspect of all learning is the *production of meanings*, in which a *semantic field* is "[...] a process of production of meaning, in relation to a nucleus, within an activity" (LINS, 2012, p. 17). The Model was developed with the aim of understanding what students think when they "make mistakes", but without reinforcing the idea of "error" (LINS, 2012).

The MCS adopts as its foundation the processes of *production of knowledge* and meanings *regarding mathematical and non-mathematical objects*. From this perspective, a *text* is only a *residue of enunciation* and only comes into existence at the moment it becomes *meaningful* for the *reader*. In the meantime, in order to *produce knowledge* about an *object*, it is necessary to *produce meanings* about that *object*.

For Lins, the MCS is evidenced from the ideas of "[...] meaning, knowledge, interlocutors, nuclei, local stipulations, objects, in addition to the notions of activity, communicative space, text and legitimacy" (LINS, 1999, p. 88).

In the spectrum of the MCS, the individual *produces knowledge* when he is speaking (*enunciating*), and believing what he is saying (*belief-affirmation*) together with a *justification* (which authorizes the subject to say/do what he says); if the subject makes an *enunciation* (speaks/does) about a certain *object* (physical or not), based on a *justification* he becomes an *author* (LINS, 2012) in the process of *knowledge production*.

When two people are talking, they are taking on the roles of *author* and *reader*. For effective communication between subjects (*cognitive beings*) it is necessary to establish *communicative spaces*, a process of sharing between *cognitive beings* who speak in the direction of the same *interlocutor* and thus, the *production of meanings* occurs from the *internalization of interlocutors* and *legitimacy* (LINS, 2012).

In the SCM, when *meaning is produced* for a given *object*, it is important that there is a reference to the context of which it is spoken/made, because the *meaning produced* by

the *subject* is always local and punctual, relative to that activity. These *productions of local meanings* are constituted in a *nucleus*, formed by *objects*.

For Lins (2012, p. 26) "the core of a semantic field is constituted by local stipulations, which are, locally, absolute truths, which do not require justification locally". The true term "[...] it is not an attribute of what is affirmed (when there is production of knowledge), but an attribute of the knowledge produced. Legitimacy, on the other hand, applies (or not) to modes of production of meaning" (LINS, 2012, p. 21).

It is in the conception of the MCS that we conceive the production of knowledge of the students, offering us relevant elements to the analysis of possible contributions of an IEP proposal in the context of School Financial Education, thus configuring our epistemological conception.

3.4 CRITICAL MATHEMATICS EDUCATION (EMC)

Research supported by Alves and Matos (2008) indicates that a critical view of Mathematics Education has emerged as an alternative to the absolutist conceptions of Mathematics, while Chaves (2004) advocates that it is an opportunity to confront the panoptic conception of what he calls ETM, which works as an instrument of control to the positivist character imputed to school mathematics.

EMC seeks the development of teaching projects that can go beyond just transmitting information, or applying algorithms, as it aims to prepare individuals engaged in cultural, socio-environmental and political issues of their context, using mathematics as possible tools for reading the world (CHAVES, 2004).

According to Skovsmose (2001), reflective knowledge is a possible key to understanding and debating mathematical models that can be present in society, influencing our choices, behaviors and decision-making. These models arise from the combination of mathematical and technological knowledge, which, by themselves, are limited; that is, they do not anticipate socio-environmental and political consequences of their implementation. Therefore, for a critical analysis of the possible applicability of a model and its socio-environmental repercussions, and to develop the criticality of evaluating it, it is essential to foster the *production of* critical and reflective knowledge.

The proposal presented by Freire (2002) is configured as problematizing, focused on the student's daily life, in which it places the teacher as the advisor of an educational process. However, it is explicit that the proposals of Paulo Reglus Neves Freire (1921 –

1997), patron of Brazilian education, serve as an epistemological ballast for the CME proposal, advocated by the mathematical educator Ole Skovsmose (1944 – 2025).

Freire (2002) points out that the liberation of the oppressed is only possible through awareness, a process in which they become aware of their situation and fight to transform it. This process involves overcoming what he calls naïve consciousness and thus begins to develop a critical consciousness, capable of questioning and acting on reality.

According to Skovsmose (2000), the idea is to stimulate students to formulate problematic situations in their daily lives and to carry out a critical analysis of the results found in the studied scenario, as well as Chaves (2004), when proposing that a student can be put in contact with the "reality" of his environment, envisioning that he can broaden his spectrum in relation to learning, as well as being able to develop attitudes that are creative towards the world around them, not just being trained to ignore the consequences of their actions. In this context, from Freire's (2018) perspective, students are encouraged to discuss the sociopolitical roles of what they produce, strengthening the relations between school and society, causing a possible change in reality and not adapting to it.

With regard to the classroom, Skovsmose (2001) uses the expression "hidden curriculum" of mathematics to describe the ideologies that remain obscure behind exercises that provide detailed instructions on how to proceed, contributing to repress students' creativity and initiative, a limitation that can extend outside the school environment and that Chaves (2004) calls the tactical and strategic device of the ETM panopticon, in the maintenance of Mathematics that is assumed to be universalist and hegemonic, to promote meritocracy, exclusion and to widen the abyssal socioeconomic rift existing in Brazilian society.

In contrast to this hidden curriculum, we suppose that a critical understanding can offer a more solid foundation for understanding and analyzing events and processes and help in the construction of an individual identity. Therefore, the importance of *producing reflective knowledge* arises, since they represent ways to recover the critical aspect of mathematics, breaking with its positivist and uncritical conception, in the perspective of Chaves (2004).

Based on the central principles of CME, proposed in Skovsmose (2001), the importance of incorporating critical education in basic education, as well as in teacher training, should be highlighted. This is because, through it, it is possible to train individuals prepared to manage situations related to the family budget, consumption, and sustainability,

for example. So that they can develop criticality in making decisions regarding when, how much, where and if an investment is possibly necessary, enabling them to reflect on these issues, plan for the short, medium and long term and, thus, be able to avoid future debt.

In CME, educators seek to go beyond ETM, which is often based on mnemonic methods of fixing algorithms, techniques, and procedures, which privilege an education FOR Mathematics to the detriment of an education FOR Mathematics, as proposed by Lins (2020), in which it is possible to make explicit, at school, the *modes of production of meanings* on the street, as well as being able to *produce legitimacy*, within the school, for the *modes of production of meaning* of the street, as a political act and pedagogical act, and also that new *modes of production of meanings can be proposed, which join those of the street, instead of simply denying or replacing them (Lins, 1999)*, as Romulo Campos Lins proposes when he glimpses a profile of Mathematics Education that he defended and practiced, seeking to involve students in critical questions about the role of school mathematics in society, its possible applications and implications.

Ole Skovsmose, when developing his proposal for CME, sees it as an extension of this idea to the specific context of school mathematics and argues that Mathematics is not neutral and, therefore, can be used to reproduce (and why not denounce, as Chaves (2004) proposes) social inequalities, so that students can critically analyze these issues.

Freire, on the other hand, emphasizes the dialogue between the educator and the students, encouraging a horizontal approach so that both can learn together. He promotes awareness through action-reflection cycles, while Skovsmose proposes practices that can lead students to question and challenge mathematics (ETM), examining how school mathematics can be used to perpetuate dominant ideologies, in a way that can include the relationship between school mathematics and street mathematics, as pointed out in Lins (1999) and already presented previously.

In short, while Paulo Freire provides a broader theoretical basis for critical and socially engaged education, Ole Skovsmose brings these principles into the specific context of Mathematics Education, focusing on the critical analysis of school mathematics and how it relates to broader social issues. However, we emphasize that both Freire's proposals, as well as Skovsmose's CME or even Chaves' (2004) PEI, are strongly close to what is proposed in Lins (1999) as a possible skeleton of Mathematics Education, by defending that the modes of production of meaning of the street, together with those of the school, be legitimized, explicit and expanded instead of ignoring and replacing them.

In our research, CME assumes the key principle of: (1) contextualization of mathematics: mathematics is taught in order to relate them to real situations and problems of students' daily lives, enabling learning processes, enabling students to *produce meanings* for mathematics as possible useful tools to understand and solve everyday issues; (2) dialogue and participation: valuing dialogue as an essential tool for the *production of knowledge*, in which students are encouraged to actively participate in IEP, sharing their experiences, doubts and perspectives and the teacher acts as a facilitator of dialogue, promoting critical and collaborative discussions; (3) critical awareness: students are encouraged to reflect critically on mathematics, questioning and analyzing patterns and applications and being encouraged to consider how mathematics can be used to perpetuate or challenge social inequalities and power structures; (4) empowerment and transformation: the ultimate goal is to provide subsidies to students so that they can use mathematics as tools for possible social transformations and for the promotion of justice and equity, involving not only the development of mathematical ideas, but also a broader awareness of the world around and the potential to act on it; (5) interdisciplinarity: recognizing that mathematics is interconnected with other areas of knowledge and aspects of life, therefore, the approach of a CME, in the adopted perspective, may involve the integration of mathematical ideas and themes with other areas, such as social sciences, history, art, among others.

For the development of this research, we used an approach in which class themes could be brought by the teacher, but ideally this choice would be made in a collaborative way, considering the interests, experiences and contexts of the students. Therefore, while the teacher can bring suggestions for themes or topics to be discussed, it is important that there is space for students to express their own ideas, doubts and concerns. The objective is to establish a learning environment in which *knowledge* is produced from collaborative work, based on the experiences and perspectives of all those involved.

By involving students in the choice of the class topic, the teacher can promote greater engagement and relevance in the learning process, in addition to stimulating critical reflection and the application of mathematics in contexts relevant to students. This approach can also help to promote greater autonomy and empowerment of students, as they are encouraged to actively participate in the definition of the curriculum and learning objectives.

4 METHODOLOGY

The methodological process we adopted was of a qualitative nature, in the sense proposed by Bodgan and Bilken (1994), as we consider that it addressed subjective aspects of social phenomena and human behavior and has nuances of a case study, as it constitutes an investigation with a qualitative approach that benefited from several sources of evidence, as highlighted by Yin (2001). To apply the research in the classroom, we used principles from a case study experienced by the authors, from the perspective proposed in Yin (2001).

Because we understand that the elementary notions of the SCM, in a *process of production of meanings and knowledge*, are in line with what Bodgan and Bilken (1994) and Yin (2001) propose, we proposed to carry out an analysis in order to list the *meanings produced*, the *constituted objects* and the *ways of operating* of those involved in the process.

The field action described and analyzed here was carried out at the Federal Institute of Espírito Santo (Ifes) – *Vitória campus*, in the Laboratory of Integrated Teaching Practices (LPEI),⁸ with the target audience of 6 (six) participants, volunteers, mathematics undergraduates, members of Gepemem and participants of Pibid, Mathematics subproject. To this end, we have set up 2 (two) working groups. As an action to teaching, we present a set of Investigative Educational Practices (IEP), as conceived in Chaves (2004), which takes the MCS as its epistemological foundation (Lins, 1999; 2012; Lins; Giménez, 1997; Silva, 1997; 2003; 2022) and CME, as presented in Skovsmose (2000; 2001; 2013) and, thus, developed in investigative environments, striving for collaborative work and dialogue as constitutive elements for the sharing of communicative spaces and the production of meanings and knowledge.

In the development of IEP, we were guided by the levels of functioning of human activity (action itself, actions and operations), according to the ideas of Alexis Nikolaevich Leontiev (1903-1979) and the action we brought to this article refers to the analysis of graphs aimed at comparing costs and consumption in relation to time.

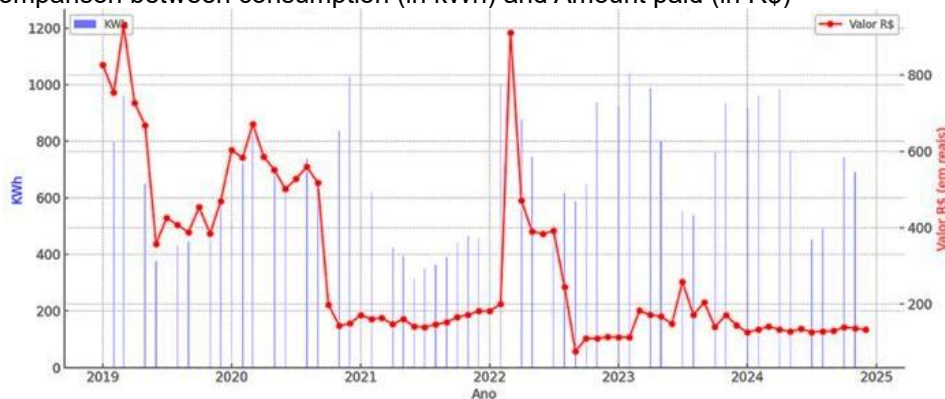
⁸ Funded by the Coordination for the Improvement of Higher Education Personnel (Capes), of the Ministry of Education (MEC).

SOME DISCUSSIONS ARISING FROM ENERGY BILL READINGS

The analysis was based on the *reading* of data regarding energy consumption (in kWh) and the amounts paid (in R\$) before and after the installation of the photovoltaic system in the residence of one of those involved in the PEI and, for this, we considered monthly records of electricity bills and information about consumption and generation of energy credits.

The residence analyzed, proposed by the students based on the comparison of electricity bills, had the photovoltaic system installed at the end of 2020. From that date on, the consumption of the electricity grid began to be complemented by the energy generated by the system, with the surplus being injected into the grid for later compensation. In 2022, there was an interruption in generation due to the circuit breaker being turned off, temporarily compromising the efficiency of the system (Figure 2).

Figure 2: Comparison between consumption (in kWh) and Amount paid (in R\$)



Source: Produced by the authors

The data were organized into time series, allowing the comparison between the pre- and post-installation periods of the system. In addition, seasonal variations in energy consumption and generation, associated with climatic and operational factors, were analyzed. When we analyzed Figure 2, those involved in the process *produced meaning* for the(s):

(i) Energy consumption in kWh, consisting of the trend line (in red) as the change in consumption (in kWh) for the indicated period. From the perspective of the MCS,

[...] *Meaning* is the set of things that is said about an object. Not the set of what *could* be said, but *what is actually said* within an activity. To produce meaning is, then, to talk about an object (LINS; GIMÉNEZ, 1997, p. 145-146, emphasis in the original).

As an object "[...] it is what meaning is produced for" (LINS, 2012, p. 28); However, with regard to the amount paid, these students found themselves in an *epistemological obstacle*.

In order for this *epistemological obstacle* not to evolve into an *epistemological limit*, we invited those involved to examine the column of values on the right of the graph (Figure 2) and they constituted objects 200, 400, 600 and 800 as values in reais paid for energy consumption.

(ii) Years 2019 and 2022 as the periods in which the highest consumption occurred, constituting as an object the around 1200 kWh as referring to this consumption.

(iii) "Last months" (*sic.*) of 2020 and "more or less the middle of the year" (*sic.*) of 2022 as the periods of least consumption, constituting mid-2022 as the period in which the minimum consumption occurred, staying below 200 kWh.

When we asked whether, from the reading of the graph (Figure 2), it was possible to estimate the value of the kWh, we observed, from their reactions (facial and body), that a *strangeness* began to take shape. In the MCS we consider *estrangement* as a process that "[...] can be indicated by imagining a situation in which there is, on the one hand, 'the one for whom a thing is natural – even if strange – and on the other hand, the one for whom what [is said by the former] cannot be said'" (LINS, 2004, p. 116, emphasis in the original).

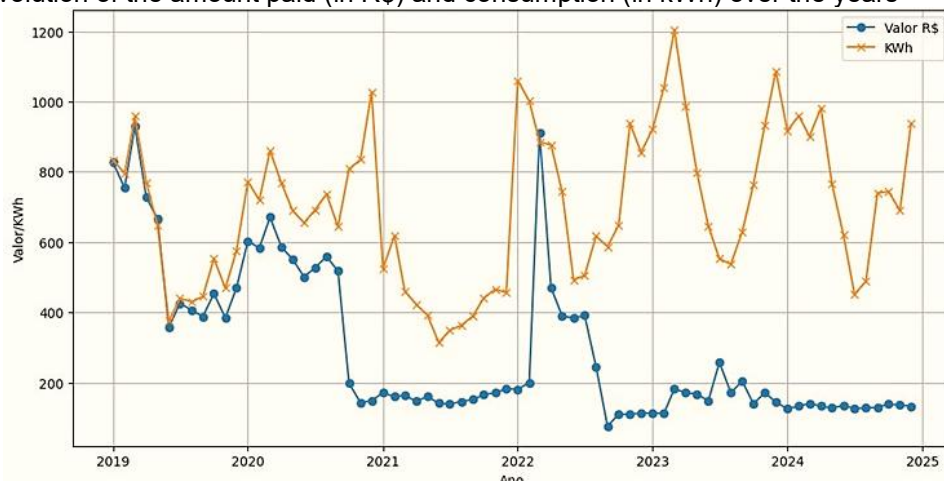
To prevent this process from becoming a *waterproofing*, we tried to exercise a *decentralization*, which is the process by which one tries to change one's place in the world, to change one's interlocutor; that is, "[...] to change the center, is to leave yourself as the center and try to go to the place where the other is as the center" (VIOLA DOS SANTOS; LINS, 2016, p. 337), that is, the teacher puts himself in the student's shoes.

Thus, as a *mode of production of meanings* – which "[...] they are 'idealized semantic fields' that exist in the form of repertoires according to which we prepare ourselves to try to anticipate what others are talking about or whether what they say is legitimate or not" (LINS, 2012, p. 29, emphasis in the original) – we estimate that a single trend line could constitute an *epistemological limit*, since we usually work – whether in Mathematics or Physics – with functions involving a single independent variable and, in this case, we were dealing with a function involving 4 (four) quantities: consumption, energy production and cost, varying as a function of time.

Hence, in the case of an IEP developed with students and future teachers who teach mathematics, participants of Pibid (starting their journeys in the core schools), as a second

option, with the purpose of expanding the possibilities of *producing meanings* and *knowledge* and *sharing communicative spaces*⁹ regarding possible *readings* of the quantities involved, we presented, discussed and compared a graph with two trend lines: in blue for values in reais and in orange for kWh (Figure 3).

Figure 3: Evolution of the amount paid (in R\$) and consumption (in kWh) over the years



Source: Produced by the authors

The color differentiation we proposed was not based on a simple didactic resource. We are guided by the ideas of Alexander Romanovich Luria (1902-1977), with regard to the development of operations (in the Leontievian sense) based on tasks that aim to enable the triggering of processes of perception (naming and grouping of colors, naming and grouping of geometric figures, responses to visual illusions), abstraction and generalization (comparison, discrimination and grouping of objects, definition of concepts), deduction and inference (establishment of logical conclusions from given information), mathematical problem solving (solving problems based on hypothetical situations presented orally), imagination (elaboration of questions to the experimenter) and self-analysis (evaluation of their own characteristics).

The strategy we adopted, based on a *process of decentralization*, as we predicted, enabled the participants to carry out new *readings*, expand the spectrum of *meanings produced* and, consequently, constitute other *objects*, such as stating that in 2019 the cost of the kWh was R\$ 1.00.

If such an assertion were not justified, we could consider it as a *local stipulation*, that is, as an absolute truth that does not require a *justification locally* (Lins, 2012). However, in

⁹ "sharing a communicative space is sharing interlocutors and this, together with the elaboration I made of the production of meanings in the direction of interlocutors, ensures that all production of meanings is dialogical in the cognitive sense" (LINS, 1999, p. 88).

order to analyze whether or not there was (or not) *production of knowledge*, from the perspective of the MCS, it is essential that the *subject of knowledge enunciates* something and justifies it, because, "Knowledge consists of a belief-affirmation (the subject enunciates something in which he believes) together with a justification (what the subject understands as authorized to say what he says)" (LINS, 2012, p. 12, emphasis in the original). Thus, based on this understanding, we ask:

"What do you mean?" What led you to say that the cost of kWh in 2019 was R\$ 1.00? (Professor).
 "It's because in 2019 both the yellow line, for kWh, and the blue line, for value, start from the same point! (Mermaid Man – pseudonym of one of the participants).
 - Okay, but how did you get to this value of R\$ 1.00? (Professor).
 "It's because if he consumed 800 and a little bit of energy and paid the same 800 and a little bit of bill, it's because the cost of the kWh in 2019 was R\$ 1.00 (Little Mussel – pseudonym of a participant in the Mermaid Man working group).
 "And what operation did you do to get to this result?" (Professor).
 "We divide 800 and a little bit of the amount paid by 800 and a little bit for consumption! (Little Mussel).

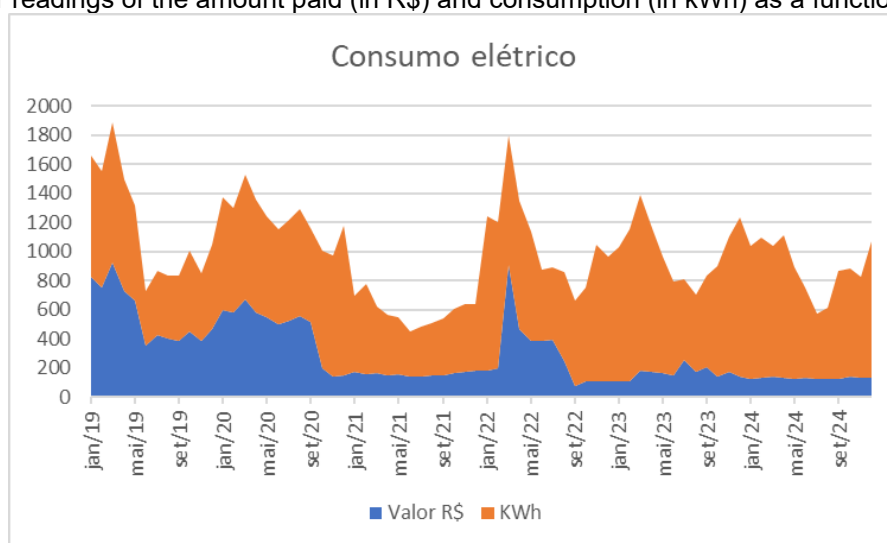
If we observe, the *subject of knowledge* Mermaid Man constituted the points of values and consumption as *objects* and, because these points coincide, *produced meaning* for the fact that values are equal. In the same way, it *produced meaning* for the trend lines, differentiating them not only by the colors, but also by the quantities involved: "Mermaid Man — [...] *yellow line, of kWh, and the blue line, of the value [...]*" (*sic.*). This led us to the understanding that, through this *residue of enunciation*, Mermaid Man operated in a *geometric semantic field*.

The *subject of knowledge* Mexilhãozinho, on the other hand, constituted the values "800 and a little bit" as *objects*. We pluralize values because, like Mermaid Man, he constituted cost (R\$) and consumption (kWh) as *objects* and, in order to *produce the knowledge* that the cost of kWh was R\$ 1.00, he performed the operation of division; this led us to the understanding that, through his *residues of enunciation*, Mexilhãozinho operated, in an *arithmetic semantic field*, which Lins and Giménez (1997) characterize as *arithmetic*, one of the characteristics of the development of *algebraic thinking*, which refers to the *production of meanings* only in relation to numbers and arithmetic operations.

In a new *process of decentralization*, we inferred that if we changed the *design* of the graphic (Figure 3) we could start working from a new *mode of meaning production* in order to make new readings and, therefore, observe other *meanings* that would be produced. To this end, we were also guided by the Lulean principles regarding the development of

operations based on dialogues that could enable the triggering of perception processes from the naming and grouping of colors with responses to visual illusions, as well as abstraction and generalization from the comparison, discrimination and grouping of *objects*, aiming that they could deduce and infer (produce new meanings) from logical conclusions, in view of the information given. This is because we observed that we could explore more the time variable that was underlying and, for this, we presented a new format for the graph involving consumption and cost varying as a function of time (Figure 4).

Figure 4: Other readings of the amount paid (in R\$) and consumption (in kWh) as a function of time



Source: Santa Maria Light and Power Company – electricity bills

From the *enunciative actions* established with the participants Mermaid Man and Little Mussel and the presentation of this graph (Figure 4), new dialogues were developed, aiming at the *sharing of communicative spaces*. See:

"Guys, is this graph the same as the first one?" (referring to graph 3). See that the first one we obtained from the information on the energy bills and we were the ones who produced it. The second was supplied by the energy company. (Professor).
 "Wait a minute!" ... Yes, it is. (exclaimed Mexilhãozinho).
 "No, it's not!" ... (said Mermaid Man).
 "Why?" (exclaimed Mexilhãozinho).
 "Because in the other the consumption and the cost were the same, 800 and a little bit and now the cost was 800 reais and consumption doubled, it went to 1600. (said Mermaid Man).
 - And how much was the value of the kWh at the beginning? (for a few moments they stared at the slide of the graphs). You can discuss it in the working groups. (Professor).

If we observe, the Mermaid Man participant constituted the two graphs (Figures 3 and 4) as distinct *objects* and *produced knowledge* for the difference between these graphs, as he presented the *belief-affirmation* that they are not equal and declares the *justification* that such difference occurs because in the first (graph 3) consumption and cost

are numerically equal, but in the second (Figure 4) they are numerically different, alleging that "[...] *consumption doubled, it went to 1600*" (*sic.*).

After a while for the groups to debate, we return to the plenary:

- Professor, we found 2 reais. (says pseudonymous participant Little Chico).
" And how did you get to that amount?"
" Oh, we shared." $1600 \div 800 = 2$ (Little Chico) [...]

Let us see that, **at this moment**, the *subject of the* pseudonymous knowledge Little Chico *produced the knowledge* that the value of the kWh is R\$ 2.00. In a brief reading of the lack, based on common sense, the reader may object by stating that he did not produce knowledge, because "it is wrong!". However, we remember that, from the perspective of the MCS,

[...] "true" is not an attribute of what is affirmed (when there is production of knowledge), but an attribute of the knowledge produced [...] As a consequence of being enunciated in the direction of an interlocutor, and of having even been produced, all knowledge is true. This does not mean that what is stated is "true" (LINS, 2012, p. 21, emphasis in the original).

Continuing the dialogue:

— [...] Oh, we share. $1600 \div 800 = 2$ (Little Chico) [...]
" That's not it, ani%\$#@! (Birobiro).
" Calm down!" Not like that, Birobiro. (Professor, speaking in a lower tone, but looking directly into Birobiro's eyes).
" Little Chico, please, 1600 what?" (Professor).
— kWh. (Little Chico).
" All right. And 800 what? (Professor).
" Real. (Little Chico).
" Okay! Now tell me something: is it consumption that varies as a function of cost or is it cost that varies as a function of consumption? (Professor).
— The cost that varies according to consumption. (Little Chico).
" Yes! So, the more you consume, the more you pay the bill at the end of the month. Isn't that it? (Professor).
" yes!" (Birobiro).
" Great!" So cost and consumption are two quantities directly or inversely proportional?
— Directly proportional. (Birobiro).
" Why?" (Professor).
" Because they both increase. (Birobiro).
" Yes! Because the two increase, **a m a e m f u n t i o n d a o u t r a**. (Teacher, slightly increasing the tone of voice and spelling the highlighted sentence) [...]

First, it is worth noting that the reprimand of the participant Professor to the participant Birobiro, according to our *reading*, occurs with the purpose of minimizing the possibility of establishing two possible *epistemological obstacles* that could be transformed

into *waterproofing* processes: (1) of the participant Little Chico due to the way his colleague (Birobiro) addressed him; (2) Birobiro by the possible understanding that the teacher called his attention, hence the *enunciative action* of speaking (in a lower tone of voice) and doing (looking directly into Birobiro's eyes).

The second issue that we highlight refers to the *Professor's way of operating*, trying to avoid a reading by lack, starting from what was said by Little Chico so that he could abstract, generalize, deduce and infer (as stated in Luria (1990)) from his own enunciation, taking not only Little Chico, but the others involved, to conclude that the cost varies according to consumption and that these two quantities are directly proportional.

Thirdly, it is worth noting that, according to our *reading*, Professor sought *to share a communicative space*, bringing Birobiro into the context, asking him to explain to Little Chico his *way of operating*.

— [...] So, with serenity, Birobiro, explain now to comrade Little Chico how you did it! (Professor).
 "Oh! We take and divide the value by the consumption, like this, oh: $.800 \div 1600 = 0,5$ (Birobiro).
 " So, guys, what does that mean?" What does this mean that Birobiro's group found 0,5? (Professor).
 " That in January 2019 he was paying 50 cents per kWh. (Little Mussel).
 " All right, so you're dividing reais by kWh, aren't you?" (Professor).
 " yes!" (Little Chico).
 " So the amount of the bill payable you get how?" (Professor).
 - Looking at the amount on the bill, lol! (Little Mussel).
 The participants laugh effusively.
 " Okay! Obviously, the amount to be paid is expressed in the bill, but how do you arrive at this amount?
 " It's consumption times 50 cents. (Mermaid Man).
 " Guys, do you understand, what did Mermaid Man say?" (There was a collective affirmation).
 " Are we going to write this mathematically?" Who qualifies to go to the blackboard? (Professor).
 " I'm going!" (Mermaid Man).
 Consumption of the bill amount $\times 0,5 =$
 " That' 0,5 s what?" (Professor).
 — The value of the kWh. (Little Chico).
 — And looking at the graph (referring to graph 4) what was the consumption in January 2019?
 (Professor).
 — 1600 kWh. (Little Chico).
 " So how do you get to the value of R\$ 800? (Professor).
 — 1600 kWh times 50 cents. (Little Chico).
 " Shall we write this then?" (Professor addressing the blackboard).

$$1600 \text{ kWh} \times \text{R\$ } 0,50 / \text{kWh}$$

" Shall we solve this expression then?" (Professor addressing the blackboard).

$$1600 \text{ kWh} \times \text{R\$ } 0,50 / \text{kWh} =$$

$$= \frac{800 \text{ kWh} \cdot \text{R\$}}{\text{kWh}}$$

" Notice that the cost is 50 cents PER (speaking slowly with emphasis, with a different tone of voice) kWh. That's why we write it like this (Professor pointing to the preceding expression. Highlighting each term).

" And now, simplifying, let's see how it will be." (Professor).

$$\begin{aligned} 1600 \text{ kWh} \times \text{R\$ } 0,50 / \text{kWh} &= \\ &= \frac{800 \text{ kWh} \cdot \text{R\$}}{\text{kWh}} \\ &= \text{R\$ } 800 \end{aligned}$$

" So, that's why the final value is R\$ 800. (Professor) [...]"

In relation to the *enunciative actions* of the preceding framework, we understand that there was, on the part of the *subject of knowledge*, Teacher, a greater concern with the didactic-pedagogical procedure, without necessarily having ceased to operate in a *semantic field* that we call consumption-cost. However, it is possible to observe that there was a process that Silva (2003) calls *nucleation*, that is, the process of constitution and *transformation of local stipulations*, operations and logics that occurs when there is the incorporation of other *local stipulations* in the triggering of an *activity*; this occurred in the course of the *production of meaning* of the Teacher, which began to operate in a *semantic field* that we call didactic-pedagogical, even though it does not abandon the *semantic field* of consumption-cost, which has become underlying, implicit.

It is important to keep in mind that nucleus, in the sense proposed in the MCS, does not refer to something static, a set of things, but to a process that is constituted within an activity. In another activity, a new nucleus is constituted and this is the process (SILVA, 2022, p. 101).

But the *enunciative actions* did not end. Let's look at the continuation of the episode, when Birobiro asks the Professor about the change of focus in his *way of operating*.

— [...] Teacher, in a high school class is it really necessary to act like this? Don't you think it's too many details that can confuse or tire the student? (Birobiro).
" I wanted to reproduce the dynamics that I would use with basic education students, but I also wanted to draw attention to a fact that we usually neglect when we abandon the magnitudes involved in the process. Romulo and Giménez, there in *Perspectives in arithmetic and algebra for the twenty-first century* [referring to Lins and Giménez (1997)] call attention to the fact that, in school, numbers are not numbers of anything, except in problems with stories and even so it is usual to abandon stories and fixate on numerical value, completely abandoning the context, the story and, in this case, the greatness involved. Another thing is that, when the MCS deals with the 3 major categories in a process of producing meanings, Romulo and Giménez also call attention to the fact that, usually, when we solve a problem, we talk about the things we are trying to understand, but we end up silencing the things we take for granted [referring once again to Lins and Giménez (1997)] And that's why we are normalizing the abandonment of magnitudes, units of measurement, context and so on. And that's why numbers become numbers of nothing. I understand that drawing attention to the context is always important. (Professor) [...]"

Initially, we emphasize that the *enunciative actions* of the preceding table corroborate the analysis we carried out regarding the *nucleation process*. Secondly, we call attention to

the existence of non-hegemonic ways of mathematizing in school, as well as mathematical dynamics not legitimized by academia – especially when there are still so many training centers immersed in the paradigm of 3 plus 1, punctually when it comes to the training of mathematics teachers in basic schools, especially in relation to those who are guided exclusively by the paradigm of exercise.

These non-hegemonic modes constitute, and are constituted, by school knowledge that is part of the daily life of the mathematics teacher and that goes beyond the hegemonic ways of mathematizing, especially by changing the focus, bringing to the center of the process the student, criticality, dialogicity and collaborative work, which systematically break the *modus operandi* of the ETM, where numbers are not numbers of anything (LINS; GIMÉNEZ, 1997).

It is worth noting that the analysis we present in the previous paragraph is in line with what Skovsmose (2001) calls the "hidden curriculum" of mathematics to describe the ideologies that remain obscure behind tasks that end up repressing students' creativity and initiative. Is there then an ideology in school behind the practice of perpetuating numbers as numbers of nothing?

In his proposal for CME, Ole Skovsmose argues that mathematics is not neutral and, therefore, can also be used to reproduce or contest social inequalities, so that students can critically analyze these issues. But how to carry out any critical analysis based on results in a universe where numbers are not numbers of anything?

We understand that the *enunciative action* of the Teacher seeks to establish the *sharing of a communicative space* that brings to the heart of the matter things that are not usual to be discussed in the formative processes, in addition to seeking to emphasize the dialogue between *the subjects of knowledge*, encouraging a horizontal approach so that both can learn together, as we present when we talk about Paulo Freire and his proposal of education aimed at the liberation of the oppressed, especially when seeking to promote awareness through cycles of action-reflection.

Another aspect that we highlight refers to the possibility presented in the context of the discussion of a CME, as defended by Ole Skovsmose, by proposing and discussing practices that can lead students to question and challenge conventional Mathematics (MTE), examining how school mathematics can be used to perpetuate ideologies, including a critical analysis of problems that are part of the daily life of school mathematics.

With this analysis, we infer that there is no thematic M, in the positivist conception of universal and hegemonic being, but there are mathematics and, with the episode presented, we highlight the existence of a school mathematics and a mathematics of the mathematics teacher educator, which breaks the ETM paradigm.

But we were not yet satisfied, because we did not arrive at a *mode of production of meaning* that contemplated in the graphic analysis the question of dealing with temporal functions until the *subject of knowledge* with the pseudonym Ruivo brings his contribution.

— [...] Professor, I wanted to go back to the two graphs. Come back there please [referring to the slide with the comparison of both (Figures 3 and 4)]. There. Taking advantage of what you said in relation to the numbers of nothing, I would like to highlight the following. Did you notice that in the first graph [referring to Figure 3] the standard unit of time is year and in the second [referring to Figure 4] the standard unit of time is month? Another thing, did you see that between September 2020 and January 2022, despite consumption varying a lot, the cost remained practically constant? (Redhead).

As much as the *subject of knowledge* Professor tried to make a "landscape face" – as the elaborator of the MCS, Romulo Campos Lins, used to say – his satisfaction with Ruivo's observation was undoubted. His body showed signs of this by his facial expression and the synchronized movements of shaking his head. It is worth noting that we understand that "the body speaks" and this speaking of the body is what in the MCS we highlight as being a NON-differentiation between speaking and doing, because,

Either saying is understood as doing something, or doing is understood as an enunciative act. And doing includes, for example, gestures, arrangements or manipulations of physical objects, drawings and diagrams of all kinds (LINS, 2004, p. 5 *apud* SILVA, 2022, p. 89).

That is why we understand that the Teacher's gestures were constituted as an *enunciative action* and, therefore, it was up to us to analyze it as well. However, it should be noted that we do not know how to answer whether such satisfaction was restricted exclusively to *Ruivo's enunciative action* or to the process as a whole, due to the interaction and participation of the 6 involved.

According to our analysis, Ruivo began to operate in another *semantic field*, performing a *transformation of the nucleus* that had been constituted from the *consumption-cost semantic field*, making explicit the existence of a *consumption-cost-time* semantic field.

When they started to operate in the *semantic field* of consumption-cost-time, based on a *local reading method* (Silva, 2003), we found that the *knowledge produced*, presented in the form of topics, was:

1 – The monthly consumption of the residence – before the installation of the photovoltaic system – presented seasonal variations, with peaks in the summer months (January to March) and lower values during the winter (June to August);

2 – In 2019, monthly costs exceeded R\$ 900.00 in some periods, evidencing the high dependence on the conventional electricity grid.

3 – As of October 2020, with the activation of the photovoltaic system, there was a significant reduction in costs, remaining at values of approximately R\$ 200.00.

4 – Between January and May 2022, possibly due to a problem with a technical problem in the equipment, it resulted in the interruption of photovoltaic generation for approximately four months, substantially increasing consumption and, consequently, cost.

5 – After a brief explanation about the energy credit compensation system, the participants inferred that there are indications that the accumulated credits were consumed, increasing the monthly costs of the energy bill, as well as consumption, between January and May 2022.

6 – After asking what could be done to avoid problems like this, the participants suggested frequent monitoring of the system as a preventive action, since operational failures can compromise the efficiency of generation and the desired savings.

7 – After September 2022, there was probably a resumption of the system's operation, as costs stabilized again, even with the high variations in consumption, which shows a recovery in energy generation and the compensation of credits in the electricity grid.

8 – Between 2023 and 2024 produced significance for the existence of a standard of stability in terms of cost, similar to that recorded in 2021, however, energy consumption continued to suffer large variations.

In a *global reading process* in relation to the proposal for the implementation of a residential photovoltaic plant, the *subjects of the process knowledge produced the knowledge* that, in addition to the financial benefit, the monthly cost was reduced, ensuring financial savings to the user.

At the same time, when it comes to the location of high temperatures, the *subjects of knowledge* inferred that the expansion of energy capacity and consumption, without

increasing cost, can positively interfere in the quality of life of the residents and inferred that there was possibly a probable increase in the number of appliances connected and also in the time of use of them, mitigating the effects of the intense heat in the region, in addition to dealing with the consumption of clean and sustainable energy, meeting the following Sustainable Development Goals (SDGs): SDG 3 – Health and Well-being; SDG 7 – Clean and Sustainable Energy; SDG 12 – Responsible Consumption and Production; SDG 13 – Action Against Global Climate Change.

SOME CONSIDERATIONS

By adopting the guiding principles of the IEP in the teaching process, we observed that the participants assumed the leading role in the course of the practices, working collaboratively and being guided by dialogue and *co-collaboration*, in a critical and reflective way, thus directing themselves to what is proposed in CME, in the perspective conceived by Ole Skovsmose.

The search for *sharing communicative spaces*, trying to understand the *ways of operating* of colleagues in the work groups, performing *decentralization* and being guided by the dialogical relationship of speaking/doing, allowed us to observe the MCS in action, as recommended by its elaborator, Romulo Campos Lins.

Throughout the process, based on the *participants' statements* (speaking/doing), we observed possible biases between the guiding principles of the IEP, the basic principles of the CME and the ideas of the MCS.

As the purpose of the practice under analysis was focused on the possibility (or not) of acquiring a residential photovoltaic system, as a generating theme, the participants, when carrying out their analyses regarding the feasibility of implementing this system, in this specific case, concluded that, even adopting financing for the acquisition of the system, the recovery period of the invested capital will occur before the payment of the financing.

The proposed practice aimed to foster a discussion about our framework, in the expectation of leveraging studies that use it as well as promoting reflections on teaching practice. We understand that this objective was achieved, but it also allowed us to observe both MCS and EMC in action.

The *meanings* and *knowledge produced* by the participants, as well as examining their respective *ways of operating*, allowed us to observe that our guideline question (*How can the development of IEP, from the perspective of the MCS and CME, contribute to the*

*teaching and learning processes of a School Financial Education, in order to promote a critical and reflective education?) was achieved, allowing us to deepen our epistemological bases in order to achieve what was – already exposed – advocated by Romulo Campos Lins, of making explicit, in the school, the *modes of production of meanings* of the street, in order to make possible the *production of legitimacy*, within the school, for the *modes of production of meaning* of the street, as a political act and a pedagogical act.*

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