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QUADRILATERAL EDUCATION IN MOZAMBIQUE: ITS RELATIONSHIP WITH THE DIFFICULTIES PRESENTED BY PRIMARY SCHOOL STUDENTS

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ABSTRACT

The objective of this article is to analyze the relationship between the methodological strategies used by the teacher in the teaching of quadrilaterals and the difficulties that students have in including the square as a rectangle. We opted for a qualitative research based on interpretivism, outlined the Freirean action research, carried out in the 5th grade classes, at the Complete Primary School of Maragra, in Mozambique - Africa, with class observation, interview and questionnaire as instruments for field data collection. The students showed great difficulties, almost simulating those of their teachers. The use of the geoplane proved to be a useful strategy in the teaching and learning of quadrilaterals. It was concluded that the difficulties that students present in the inclusion of the square as a rectangle are related to the methodological strategies that teachers use in the teaching of quadrilaterals.

Keywords: Methodological strategies. Teaching quadrilaterals. Teaching and learning difficulties.

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INTRODUCTION

The emergence of geometry dates back many centuries, and it is present in our lives. As Lorenzato (1995, p. 5) states:

> Geometry has been around since before Christ, but you have to be able to see it... Even if we don't want to, we deal in our daily lives with the ideas of parallelism, perpendicularism, congruence, similarity, proportionality, measurement (length, area, volume) Symmetry: whether by visual (shapes), or by use in leisure, in the profession, in oral communication, we are involved with geometry daily

Geometry tends to be the part of mathematics that is least privileged in Basic Education (Lorenzato, 1995). Teachers do not emphasize the importance of geometric knowledge, leaving the contents of geometry to the last minute. It is notorious in textbooks that geometry content has always been placed in the last place in mathematics teaching programs. As Lorenzato (1995) states, most of the time, geometry is the last subject taught and without making the connection with arithmetic.

The article discussed the relationship between the methodological strategies used by teachers in the teaching of geometry, particularly of quadrilaterals, and the learning difficulties that primary school students present in the inclusion of the square as a rectangle. In this regard, it is necessary to understand that initial training also contributes a lot to the teaching work since many teachers use the same methodologies acquired in their initial training without improvements (Mozambique, 2019).

Regarding the difficulties in understanding geometry, during a professional internship carried out at the Chibututuine Primary Teacher Training Institute in Manhiça, Mozambique, the graduates of class 7 denied that the square is a rectangle, when, in fact, such knowledge should already be mastered by students at lower school levels.

In the class attendances in the 4th and 5th grades, it was observed that the teachers used the traditional teaching methods, relying on Paulo Freire's Pedagogy of the Oppressed, transferring their difficulties to their students. In a 5th-grade class, for example, the teacher drew squares, triangles, rectangles, non-rectangles, parallelograms, and non-square rhombuses. Then he asked the students to mark "X" on the rectangles, and they did not identify the square as a rectangle. Unfortunately, the teacher also did not identify the squares as rectangles. Therefore, he did not work on the properties of these figures but limited himself to the concepts defined with gaps.

Considering what has been experienced and reported above, in this work we will dedicate ourselves to the identification and classification of quadrilaterals, particularly the

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inclusion of the square as a rectangle, its treatment in the classroom and the relationship between teaching and the difficulties existing in the students of the 5th grade of the Complete Primary School of Maragra, a public institution that is located in the province of Maputo, Manhiça district in Mozambique.

The research aims to analyze the relationship between the methodological strategies used by primary school teachers in the teaching of quadrilaterals and the difficulties that 5th-grade students present in the inclusion of the square as a rectangle. As well as proposing the correct use of the geoplane as a methodological strategy for teaching quadrilaterals, with a view to improving the quality of learning, thus reducing difficulties in students.

RELEVANCE OF THE STUDY OF GEOMETRY

The concepts of geometry and spatiality are part of the experiences of all children when they try to understand how the universe that surrounds them works and when they distinguish objects and the proximity between them. Therefore, geometry provides the essential tools for the correlation between mathematics and reality, prioritizing the studies of shapes and space (Pereira, 2012). One can add the idea that geometry provides an understanding of the entire space in which the child lives.

It is pertinent to the learning of geometry in school because it favors the development of logical thinking, as Lorenzato (1995, p. 5) states:

> To justify the need to have geometry in school, it would be enough to argue that without studying geometry people do not develop geometric thinking or visual reasoning and, without this ability, they will hardly achieve the life situations that are geometrized; Nor can they use geometry as a highly facilitating factor for the understanding and resolution of questions from other areas of human knowledge. Without knowing geometry, the interpretative reading of the world becomes incomplete, the communication of ideas is reduced, and the vision of mathematics becomes distorted.

The author states that geometry is fundamental for the complete formation of a person because it has the ability to develop a broader communication and interpretation of the world. Corroborating Lorenzato (1995), Jones (2002) states that geometry provided students with a broad view of the universe that surrounds them, such as developing critical thinking, improving problem solving, increasing the capacity for intuition and visualization, and improving deduction, proofing, and argumentation reasoning.

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In the first phase, the student only identifies the geometric figures in an indivisible way, distinguishes them according to their characteristics, and then makes the correlation between the figures and their properties. After these stages, he begins to organize these ideas and create a concept of abstraction of objects starting from a concrete thought of objects to develop their connection in space and, little by little, they become more abstract (Pavanello, 2004).

About this content, the National Curriculum Parameters of Mathematics (PCNM) of Brazil, a Portuguese-speaking country with a primary education curriculum similar to that of Mozambique, highlight that:

> The importance of geometry and how this discipline is essential in the curriculum - even identifying its complete abandonment - because it enables the student to develop critical and organized thinking, acting mainly in the understanding, description, and representation of the world. Geometric shapes tend to attract the interest of young people naturally, including providing the ability to argue and demonstrate how to solve problems (Brasil, 1998, p. 122).

Geometry is also an essential subject in the school curriculum. according to Santos (2017, p. 23) "the student develops a specific way of thinking, providing the interest of students to expand their knowledge". The importance of this content in the school education process is thus evident.

TEACHING AND LEARNING GEOMETRY

Regarding the teaching of geometry, Pereira, (2012, p. 9), states that "geometry presents questionable methods, purposes and contents in the presentation of the discipline in the classroom, thus allowing it to be the most controversial unit in the field of mathematics". Based on the works of Lorenzato, (1995) and Santos, (2017, p. 22) "the teaching of geometry is not very present in classrooms, even without due appreciation of the teaching of this discipline, and when present, geometry is disconnected from the reality of the students". These authors Lorenzato (1995) and Santos (2017), describe the lack of appreciation in the teaching of geometry, and show that the lack of dedication to this subject in schools makes students not have a global view of mathematics.

In the search to overcome the problems of teaching this content, Matos and Serrazina, (1996) suggested a different model for the teaching of geometry, in which there should be an appreciation of this discipline, because the authors asked that the knowledge of geometry could be acquired gradually, using a geometric language, allied to reasoning

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and intuition. Geometry is thus identified as a "global" discipline because the figures or objects are not isolated from each other but correlate in simple or complex ways in order to be able to interact with other areas of mathematics. It is also a "constructive phenomenon" because students develop their concepts about geometry, thus becoming a learning that involves many agents, such as students, teachers, and the whole school. Still citing Matos and Serrazina (1996), the teaching of geometry can develop different skills in students, especially the ability to build and manipulate geometric objects, visualize, organize thought in a logical way, and have the possibility of applying geometry knowledge in everyday situations.

In this sense, the *National Council of Teachers of Mathematics* (NCTM, 2008) proposes norms that direct the teaching of mathematics from preschool to 12th grade, with emphasis on geometry, in which there are guidelines and expectations that must be followed. This document has an international reference and, according to these assumptions, the teaching of geometry allows the student to develop reasoning and argument naturally and, prior to that, it advocates the possibility of learning about geometric shapes and their relationships in space, allowing the understanding of other areas of life. Therefore, "from the first years of studies, students will be able to develop skills for geometric reasoning, with the help of the teacher, use of appropriate tools and targeted activities" (NCTM, 2008, p. 44).

Jones (2002, apud Abrahão, 2004, p.7) states that "the study of geometry must have the ability to stimulate students to develop their curiosity with the intention of exploration", that is, to allow better learning and mathematical knowledge. Speaking of curiosity, Freire and Faundez (2011) say that curiosity is taken as the starting point of knowledge, the origins of teaching, and pedagogy. Paulo Freire has always been in favor of a liberating education, centered on the egalitarian dialogue between teacher and student, on the question, feeding the student's curiosity, starting from the student's daily life in his society.

Thus, Tempera (2010, p. 8) showed that "geometry should not only be linked to geometric shapes themselves, but should be related to broader aspects". Therefore, the result of the development of geometric reasoning represents a set of actions carried out over time.

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DIFFICULTIES AND OBSTACLES TO THE TEACHING AND LEARNING OF GEOMETRY

Students have difficulties in learning geometry effectively and intensively, and this fact cannot be ignored. and it is necessary to understand the origin of these difficulties and obstacles that prevail in students. In the evaluations of the third quarter of the last three years at the national level, about 80% of the students' wrong answers were found in geometry (INDE, 2016). Thus, Abrahão (2004, p. 1) states that "children have difficulties learning geometry over the last twenty years, and these numbers have increased, and the problem may be related to the difficulty in the concepts of spatiality".

It is based on this logic that Pavanello (1989) states that geometry was not present in the school curriculum, and in other situations, it was taught formally. Removing geometry from the curriculum or treating it improperly may cause damage to training in the adult phase, as mentioned earlier.

In a study carried out by Perez, (1995, p. 57) with teachers of the 2nd grade, he concluded that "[...]they were able to diagnose a few hours a week for the teaching of mathematics, also making it impossible for them to fulfill the entire program of the school year and, inserting geometry at the end of the discipline, made it insufficient or almost null as learning". Corroborating this, Bishop (1986, p. 142) states that "in his general survey of mathematics in primary education, arithmetic remains the main subject in primary education". Thus, the study of geometry in schools is given little attention and, in many cases, is replaced by arithmetic or other curricular subjects.

São points out two main causes for the non-teaching of geometry by Lorenzato, (1995, p. 4): "many teachers do not have the geometric knowledge necessary to carry out their pedagogical practices [...] it is due to the exaggerated importance that the textbook plays among us, either due to the poor training of teachers, or due to the exhausting working hours to which they are subjected".

Likewise, Santos (2017, p. 24) states that "teachers presented the same difficulties as students regarding the principles of geometry. The greatest difficulties were related to the knowledge itself, and thus the importance for initial training for all teachers is emphasized". On the other hand, Lorenzato (1995) and Santos (2017) point out another problem with the teaching of geometry, which is the total dedication of teachers to textbooks caused by the long working day, lack of time to search for other didactic subjects, and ineffective training. Another factor is that geometry is only formally present in the last

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units of the textbooks, often at the end of the book, with little chance for its study, and above all unrelated to the daily life of the students, as already mentioned.

In accordance with these problems, Lorenzato (1995) also showed that there are problems directed to external factors in the classroom: inadequate teacher training and the separation of algebra and arithmetic from geometry. For teachers, it becomes indicative to follow the textbook, and in these books, there is a total separation of the disciplines of algebra and arithmetic from geometry.

In the development of the phases of learning geometry, the teacher's language is very important. For this reason, Crowely, (1996, p. 13) says that "in addition to the teacher's language, it is important to ask the student "how do you know?" Because, with this question, the student is forced to reflect on the relevant elements of the figure in question". From the informal language, "singing to designate angle; straight to designate a straight line or straight line segment; descent to oblique sides", the student must be led, little by little, to the most geometrically precise language.

According to Charlot (1995, apud Abrahão 2004, p. 2) "it is important that the teacher motivates the student to learn, using different teaching methodologies, because on the contrary there will always be a lack of interest on the part of the student".

Another problem is related to the divergence between school and family language, that is, "teachers need to be attentive to identify the cultural differences of each child, of their place of origin, country, to understand that each child will have particular experiences for exploration, inspiration of knowledge" (Abrahão, 2004, p. 2). It is worth mentioning that another problem is directed to the deficient training of teachers, with the absence of a program of updating and continuous training, warning that the proximity of teachers to the school unit is important. This is a real problem in teacher education and training in Mozambique, where continuing education is the responsibility of the individual teacher.

Last but not least, the problem is directed to the difficulty that teachers find in using didactic and appropriate materials in the classrooms, such as the geoplane. Due to the "lack of training in geometry, teachers feel uncomfortable creating and exploring teaching materials to be used in class, and in this way they could contribute to understanding and building ideas of geometry" (Abrahão, 2004, p.3).

In general, the traditional method taught in schools allows a block to the understanding of geometry as part of the mathematical area. As it is an important subject, it has to be evaluated by students and must be accessible to all.

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INCLUSION OF CLASSES

For what is discussed in this text, the concern is not to give definitions of each figure, but to know how the inclusion of the square in other classes of quadrilaterals is.

The study by Villers, (2010), and Van Hiele (1999, apud Pereira 2012, p. 10) shows that the inclusion of classes occurs at "level 3, identified as informal deduction, where students begin to establish the relationships of the properties of the figures, between and between them, including making deductions and recognitions of the classes of the figures."

Similarly, Hoffer (1996, p. 32), in the context of the inclusion of figures, reports that, in research carried out in Oregon, "a student who refused to include squares in the set of rectangles, but after being asked to describe the figures and reflect on their properties and definitions, decided to include squares in rectangles and rectangles in parallelograms". Hence, the study of the properties of the figures is emphasized as important for the inclusion of classes.

The authors Cruz *et al.* (1992) studied the properties of parallelograms of sides, angles, and diagonals to show class inclusion:

> Parallelogram as a quadrilateral that has the opposite sides equal and parallel; rectangle as a parallelogram that has the four right angles; rhombus as a parallelogram with all sides equal; square as a rectangle because it has the four right angles, and as a rhombus because it has the four equal sides (Cruz *et al*, 1992, p. 200).

This summary is defended in the definitions and systematizations of Murimo and Morgadinho (2006, p. 17), where the square is defined as a special quadrilateral because it is present in all classes of quadrilaterals, such as: "square can be called a rectangle because it is a quadrilateral that has four right angles and equal and parallel opposite sides; Finally, the square is also a rhombus because it is defined as a quadrilateral with all sides equal."

The authors Vieira, Gomes, and Burnay (1994) systematize the properties of quadrilaterals, showing the inclusion of classes by means of a diagram, in which it is clearly defined that the square belongs to rectangles and rhombuses simultaneously.

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Figure 1 - Diagram of inclusion of quadrilateral classes.

Source: Nacarato *et al* (2016)

In this diagram, it is noted that it is easy to analyze how the inclusion of quadrilateral classes works and is well justified through the properties because each figure is framed in the class.

METHODOLOGICAL PROCEDURES

The research is qualitative in nature with an interpretative approach, as Ludke and André (2017) highlight that it accepts that the perspectives of those involved in the research are portrayed, emphasizing and valuing the process more than the product. For Flicks (2009, p. 25), corroborating the same perspective, he says that such methods "[...] consider the researcher's communication in the field as an explicit part of the production of knowledge instead of simply seeing it as a variable to interfere in the process" and the subjectivity of those who make up the research becomes part of it.

Still on the qualitative approach, Creswell (2007) presents the following characteristics:

> The research takes place in a natural setting. The researcher always goes to the place where the participant is to conduct the research, thus allowing them to develop a level of detail about the person and the place and to be highly involved in the participants' experiences. Qualitative research uses multiple methods that are both interactive and humanistic. The researchers seek the active involvement of participants in data collection and try to establish harmony and credibility with the people in the study. The most real methods of data collection are based on open observation, interview, questionnaire, and documents. The data collected involves data in texts and photos or images (Creswell, 2007, p. 186).

This research is also exploratory, because it aims to "provide greater familiarity with the problem, to make it more explicit. But it also has a descriptive direction insofar as it "aims to describe the characteristics of a given population or phenomenon, or to establish a relationship between variables" Gil (2021, p. 42).

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The literature review was the first step in data collection, which, according to Minayo (2017, p. 40), "[...] It is based on the studies already carried out, helping to map the questions elaborated in that area of knowledge, allowing us to identify what has been most emphasized and what has been little worked on".

The teaching and learning process requires a dialogue from its direct actors in the classroom, as Freire has referred to. We believe that dialogicity in educational processes, both among those involved in teaching and learning and between knowledge, is a fundamental practice and can lead to the development of essential skills for autonomous action in school and in society.

In this sense, we opted for Action Research, which at its core brings "the active involvement of the researcher and the action on the part of the people or groups involved in the problem" (Gil, 2021, p. 55). With an empirical character, Thiollent (1986, p. 14) argues that it is "[...] conceived and carried out in close association with an action or with the resolution of a collective problem and in which researchers and participants are involved in a cooperative or participatory way".

Also commenting on educational action research, he points out that,

> [...] The conception of pedagogical and educational activities is not seen as the transmission or application of information. Such a conception has an awareness-raising dimension. In the investigation associated with the reconstruction process, elements of awareness are taken into account in the situations investigated, [...] in the teacher/student relationship (Thiolent, 1986, p. 75).

Together with intentions, professor/researcher and student focus on reality and "[...] find themselves in a task in which both are subjects in the act, not only of unveiling it and, thus, critically knowing it, but also in the act of recreating this knowledge" (Freire, 2011a, p. 68).

The research was carried out at the Maragra Complete Primary School, a public institution, which is located in Maputo province, Manhiça district in Mozambique. The school has a population of 370 5th-grade students, divided into 7 classes with an equal number of teachers, with an average attendance of 52 students per class, with classes taking place from 6:40 am to 10:10 am (the school operates in a 3-day shift regime due to insufficient classrooms). A sample of 7 teachers and 140 students, 70 men and 70 women, was used, in a random selection of 20 students per class. According to Lakatos and Marconi (2021, p. 108), "probabilistic sampling is based on the random choice of those

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surveyed, meaning that each member of the population has the same probability of being chosen".

Starting Freire's Action Research, we anchored ourselves in Freire and Faundez (2011) and Freire (2003) who highlight that, when we put into practice our ability to inquire, compare, doubt, we become more curious and more critical. Curiosity, inherent to the human being, makes us "[...] to ask, to know, to act, but to ask, to re-acknowledge" (Freire, 2003, p. 86).

The 5th grade was chosen because the mathematics program of this class requires that all the convex quadrilaterals and some properties related to sides, angles, and diagonals have been studied. For data collection, class observation, interview, a questionnaire carried out at a time when they had learned the quadrilaterals and some of their properties, were used. In front of a sheet of plane figures and geoplane, the students answered the researcher's questions by performing the proposed activities, as described in the use of the geoplane as a strategy for teaching quadrilaterals.

For data analysis, Freirean Discourse Analysis was used, which considers discourse as an essential and transformative form of communication, which can reveal ideological conditioning and can also help to overcome limited views of the problems faced (Freire, 1982b). Converted into texts for analysis and reflection, they must be investigated and systematized in order to understand the reality and levels of self-awareness of individuals in their interactions with the world (Freire, 1982a; 1982b; 1997).

Thus, supported by the Freirean Discourse Analysis, proposed by Araújo (2023), we focus on the discourses of students and teachers. As the author points out, this is an alternative to analyze qualitative data; a form of analysis that recognizes students as co-constructors of knowledge, promoting the interaction of discourses with the theoretical framework, enriching discussions about the results (Araújo, 2023).

Thus, Freirean Discourse Analysis allows a deeper understanding of students' perceptions and experiences, enriching the analysis, in addition to valuing the role of students in the construction of knowledge.

As Araújo (2023, p. 60) argues, his proposal for analysis is structured in the following phases:

> 1. Knowing the material: organization of the data constituted with the study. Process in which the researcher knows, transcribes and organizes the corpus of the research using different resources, having a panoramic view, identifying the

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subjects addressed, the audience, the context in which the data were constituted and the themes that will be analyzed;
2. Coding the data: the material is read, fragmented, treated, separated into units, sequenced, structured into initial categories that can be defined before or emerged in the research process;
3. Finding associations: relationships are identified in the analyzed material that can be by thematic or analysis categories, finding the similarities, the explicit and implicit meanings evidenced by the

Taking into account the proposed structure, in the first phase we organized the material for analysis, separating the data from each instrument used.

Next, we identified the survey participants, and created codes to name them. We chose to identify them with the letter A (A1a A 140) for students, "P" for teachers (P1 to P7), group 1 to group 28 for groups in geoplane activities.

THE USE OF GEOPLANES AS A QUADRILATERAL TEACHING STRATEGY

The geoplane is one of the most practical instruments and facilitates the understanding of the properties of quadrilaterals. It allows the construction of figures by means of properties and subsequent conclusive comparison.

With the geoplane, the activities were carried out in small groups as a way of valuing cooperation. As Fernandes (1997) says, cooperation is about creativity and is very necessary in mathematics because students help each other to understand the basic concepts. Also working cooperatively, students gain confidence and confidence in their individual abilities, thus facilitating the understanding of mathematical concepts due to greater student interaction.

Five groups of four elements were formed for each sample of twenty students per class, as a way to help understand some properties of some quadrilaterals. The work took place in the form of an interview, in a quiet environment, showing students that there are other ways that can be used in the teaching and learning process of quadrilaterals.

In question 1, each group should build quadrilaterals that have parallel and equal sides, two by two. Each group built what they thought was right, but over the time of discussion in the middle of the group, they erased and wrote again. Then we asked questions to each group to understand what was actually happening as work.

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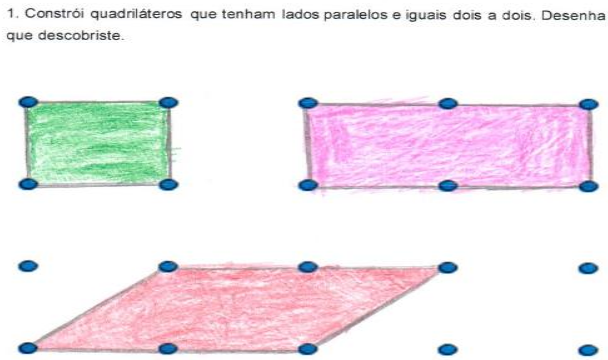
Figure 2 – Photo of the quadrilaterals built by group 5.



Source: Researchers' fieldwork

The group drew a square, a rectangle, and a parallelogram and called them parallelograms. Asked about the existence of other figures with the same properties as the figures drawn, one of the members of the group said that the other figures could not be part of this group because they only have parallel and not equal sides. We asked the student to draw the figure in question, hoping to see a rhombus, the student drew an isosceles trapezoid.

Certainly, the trapezius does not have the properties described in the exercise. But we insisted with the group until a girl said "*I already know what the missing figure is, parrot*". When drawing that parrot we discovered that it was actually talking about a rhombus.

Because the objective of this activity is to help students understand some properties of quadrilaterals, parallelograms were discussed in plenary. During the dialogue, the students were able to realize that in the parallelograms class, we all have quadrilaterals that have parallel sides two by two, and that the parallel sides are equal. Thus, it was evident that the parallelogram, the rectangle, the rhombus, and the square are parallelograms.

In activity 2, the students had to build all quadrilaterals with all right angles. The groups had no difficulty in representing the quadrilaterals with all right angles. But during the group analyses something very interesting was discovered in the figures constructed with group 11.

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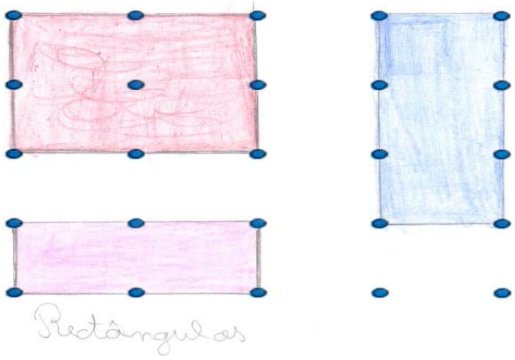
Figure 3 - Rectangles constructed by group 11.



Source: Researchers' fieldwork.

This group, when asked about the three figures, said that they are all rectangles.

> Researcher*:* We're seeing that the first one is a square.
> Student*: non-teacher is a rectangle.*
> Researcher*:* Friends, this first figure has three dots by three dots on each side.
> Pupil*: It's a large rectangle*
> Researcher*:* Friends This figure is a square.
> Student*: it may be, but it has right angles.*
> Researcher*:* Yes
> Student*: It's true the square also has right angles.*

In fact, this group 11 drew the square as a figure with all the right angles in order to draw a large rectangle. But they were on the right track, just noticeable lack of attention during the drawing, the truth is that they were sure that the figure had all right angles. Another problem noticed in this group is the fact that they do not want to change, since they were taught that the rectangle has right angles. They did not expect another figure to have these characteristics, even when they are faced with clear evidence. This fact has been discussed in previous circumstances, where teachers continue to give inconclusive definitions, but that the teacher's manual has clear methodological guidelines. There is a resistance on the part of teachers to make a change in attitude taking into account that they have been taught or trained by traditional methodologies and prefer to continue with them, despite the fact that there are new effective methodologies for these contents.

The next activity consisted of building quadrilaterals with all sides equal. It was the activity that all the groups got right both in the drawings as well as in the justifications. Group 23 drew a rhombus and two squares and said that they are the only figures with all sides equal. It was noticed that there was already a clarity in the interviewees, to the point of correctly designing and justifying their choices. This group called these figures rhombuses and squares. They were only able to give these figures the name of diamonds

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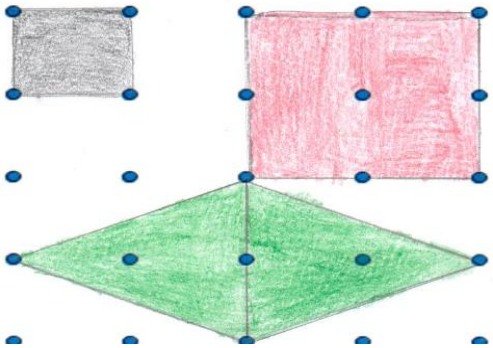
after listening to the plenary. The plenary debate was one of the greatest gains of this activity because the students themselves were able to leverage the others, without the researcher performing this task.

Figure 4 – Quadrilaterals with all sides built by group 23



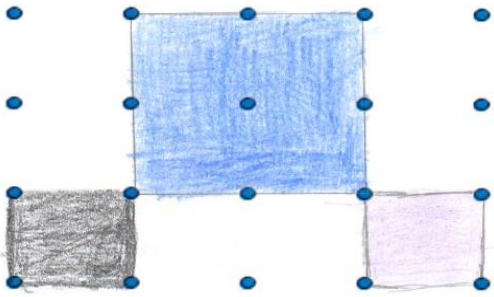
Source: Researchers' field work.

Once again it became clear that letting students discuss the activities they do has an applaudable yield, instead of dictating conclusions that they themselves cannot prove.

Activity four had the following content: to build quadrilaterals that have all sides equal and all right angles.

This was the most fundamental activity of the work, because it aimed to prove that there are other methodological strategies that can facilitate the understanding of the properties of quadrilaterals and lead the student to understand the inclusion of classes, especially the square as a rectangle. The square is a special quadrilateral because it has properties of all the convex quadrilaterals and trapezoids.

Figure 5 – quadrilaterals with all sides equal and all right angles, constructed by group 2



Source: Researchers' field work.

The researcher left each group alone to discuss and draw what they actually discovered. Some doubts arose, but the researcher told them to notice the previous activities and that they would find out the right one. All groups drew the single square,

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some drew a square, rectangle and rhombus, and this group chose to draw a square supported by two. Asked about the design, he said that it was to show that only the square, whether large or small, fit the characteristics described.

Finally, the original geoplane was placed so that the students could recapitulate the activities exercised.

Figure 6 – photo of the students building quadrilaterals on the geoplane using rubber bands.

Source: Researchers' work in the field.

These geoplane activities showed that the process of teaching and learning quadrilaterals is not as difficult as the questionnaires revealed. The identification and classification of quadrilaterals by means of geoplane or dotted paper was an easy and exciting process, where the students enjoyed, worked as a team, helped each other, learned a lot and overcame the difficulties they had. It was very clear that the use of this methodology is a step that teachers should embrace because it facilitates student learning.

SUMMARY OF THE WORK WITH TEACHERS AND STUDENTS

Of the classes attended by these 7 teachers, in none of them was the relationship of inclusion of classes of figures established, possibly because the teacher's and the student's manuals do not clarify when this relationship should be effective. It was observed that there were many difficulties in the definitions of the rectangle and square, even giving unsatisfactory definitions with a lack of rigor in the terminology.

For P6, with the theme: study of the rectangle and the square. He defined a rectangle as "a geometric figure with all right angles and four closed sides. And, square as being a geometric figure with four equal sides". (See the photo below of the class on the board)

Although this teacher has presented an accepted definition of rectangle, at no time did she put the square as a rectangle, although she has the conditions placed as a

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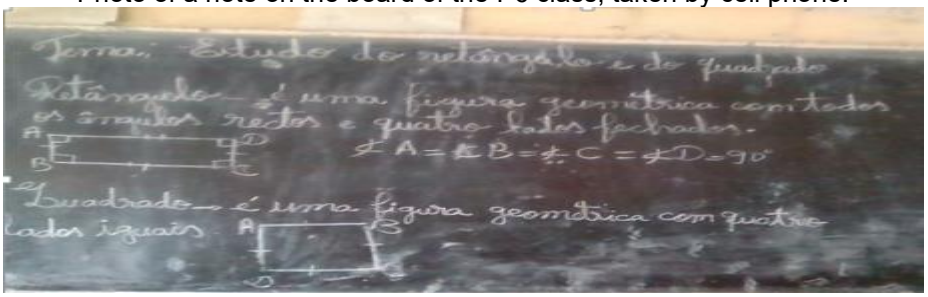
rectangle. He did not ask any questions to the students to see if they explored the topic under study further, he did not give any example of any figure similar or equal to the rectangle. Learning only has meaning when it is connected to real life, hence abstract concepts have little attention in students at this level.

Photo of a note on the board of the P6 class, taken by cell phone.



Source: Researchers' work in the field

As for the concept of the square that this professor wrote, it does not satisfy the definition of the square, because the equality of the sides is not the only preponderant factor for a figure called a square. The question of right angles was not mentioned here. The 5th grade student handbook presents two definitions of the square. According to Langa and Paulo (2014, p. 32), square is "a rectangle with all sides equal". The same authors later on, on page 95, define squares as being "parallelograms in which all sides have the same length and the four angles are right" (Langa; Paulo, 2014, p. 95). On the same page of this book, the relationship of inclusion of the square as a rectangle and as a rhombus is very clear

Based on these aspects, it was possible to see that these teachers have many difficulties in the treatment of rectangles, these difficulties may be linked to the lack of mastery of the same contents and to the consideration of non-relevant elements as determinants in the identification and classification of quadrilaterals, especially rectangles.

To overcome this difficulty, it would be possible with a continuing education program for teachers. The continuing education of teachers has a great contribution to improving the professional quality of teachers as it aims to improve the professional level of teachers, improve the performance of the classroom to the community in general, overcoming the difficulties that were not solved during initial training.

The behavior of the students questioned during the tasks of identification, classification and inclusion of classes shows that they also have many difficulties in this

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theme, these students use non-relevant definitions and properties when they are faced with activities that require the application of one of these components.

The reaction of these students was not very different from the revelations shown by the teachers assisted, interviewed and questioned, perhaps because they are students of these teachers.

The teachers do not compare the properties and definitions of the figures, they do not exercise, nor do they systematize the classes, although the manuals already have systematization exercises and methodological suggestions for these classes of identification, classification and inclusion of quadrilateral classes.

In general, it can be said that the students questioned have the notion of some geometric shapes studied in the classroom. They have difficulties in establishing the relationship of inclusion of classes of quadrilaterals, they were not able to understand that a square is a trapezoid, it is a parallelogram, it is a rectangle, it is a rhombus, in addition to being a quadrilateral, that is, it belongs to every class of convex quadrilaterals studied. There are some students who have shown that they have acquired the basic skills defined for a 5th grade student in quadrilateral learning. It should be noted that certain students are still far from reaching these skills and that it is not possible to understand what ideas they have about rectangles, also facing the problem of verbalization or language.

The greatest difficulty that the students questioned and interviewed presented is that they have little knowledge of the properties of the rectangle and the square, if not of the quadrilaterals. This fact may be the result of the lack of visualization, manipulation, and figure construction activities in the classroom. Teachers do not dialogue with their students, which brings us to Freire (2011a) when he says that students are considered as empty containers where they will pour content and knowledge instead of exercising them to know the properties of each figure, which would facilitate the inclusion of classes. Freire (2011a, p. 116), says that "for the educator -student, the dialogic, the problematizer, the programmatic content of education is not a donation or an imposition – a set of information to be deposited in the students -, but the organized, systematized and added return to the people of those elements that they delivered to them in an unstructured way."

Still speaking of dialogic education, Freire (2011a, p. 116) says that it allows for an authentic education, and it "is not made of A to B or A on B, but of A to B, mediated by the world".

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Several students could not say that "it is a square because it has four equal sides and right angles", they were only concerned with quantifying the sides and describing their characteristics without referring to the type of angles.

One of the main objectives of the study of the properties and definitions of figures in education in Mozambique is to help the student interpret and solve geometry problems. Students are not required to memorize these definitions and properties, but rather they must know them to apply them when solving real-life geometric issues.

The use of traditional teaching methodologies by teachers is one of the major concerns that we observe during class assistance. According to Freire (2011a), one of the characteristics of this method is memorization for future reproduction when questioned. The student must copy as the teacher wrote or drew and must reproduce as he saw on the board, not allowing the student to draw his conclusions or interpretations about something, because the teacher is the authority and transmitter of the teaching and learning process.

The difficulties that the teachers presented during the consultations and verified in the questionnaires are the same as those that the students presented during the questionnaires. This overcoming of the difficulties from the teacher to the student was originated by the methodological strategies that teachers use during the teaching of classes. Education as a transmission of culture, using traditional teaching methodologies. It was made very clear that the difficulties that students present are originated by the methodological strategies used by teachers in the teaching and learning process.

The use of the geoplane showed that the representation of quadrilaterals is not as difficult as the students imagined, who even liked to manipulate and palpate figures. It was evident the comparison of figures using the overlay as a way to confirm whether the figures are the same or not, based on visualization and experimentation.

This didactic instrument served for the students to mentally represent the figures, identify the common properties of the quadrilaterals, thus favoring the inclusion of classes. The valuable discussions about the characteristics of the quadrilaterals that the researcher gave to be able to draw or build the figure on the geoplane were also notorious. The use of the geoplane is one of the strategies that should be encouraged for the teaching of geometry of plane figures, since it has numerous advantages for the teaching and learning process, especially of quadrilaterals.

The methodological strategies that teachers use in teaching quadrilaterals do not help students in learning these contents, nor in identifying and classifying, nor in the

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inclusion of classes, consequently they do not help in the application of these contents in the resolution of everyday problems.

In summary, we can conclude that the difficulties that 5th grade students present in the teaching and learning of quadrilaterals, especially in the inclusion of the square as a rectangle, are related to the methodological strategies used by teachers in the classroom.

FINAL CONSIDERATIONS

In the context of class attendance, it was clear that teachers use traditional, expository methods, which devalue the intellectual capacities of students. They consider the student as an empty container that needs to be filled, as they write, speak and the students copy, according to Freire (2011a), without any exercise mechanism, which is why they face enormous difficulties in identifying, classifying and including classes because they do not know the properties of the quadrilaterals. The teacher is the center of the teaching and learning process instead of the student, as governed by the curriculum of primary education.

The teachers assisted during the teaching of classes showed a poor preparation of classes, poor knowledge of definitions and properties of the quadrilaterals. They also revealed that they were not concerned with this theme, since it was not of any importance to the students, which is why it is part of the last thematic units of the course.

They presented unclear, incomplete and sometimes unfit definitions to the students, who only limited themselves to copying them without the power of questioning

The use of the classification of quadrilaterals by partition and not by hierarchy was very notorious, which is why at no time did the teachers come to include classes of figures, thus showing the students that each figure is isolated.

In the teachers' questionnaire, we found that 6 of the 7 teachers did not accept that the square is a rectangle, they identified the rectangles without establishing the inclusion relationship, that is, they identified the rectangles as rectangles and the squares as squares. The justifications that were given by those professors made it clear that they lacked rigor in defining or enunciating some properties of the same figures. In these teachers, a certain lack of mastery of the definitions and properties of rectangles was noted, something that is reflected in the formation of concepts in their students.

Analyzing the teachers' justifications, there is a great similarity with those that the students used when they were asked to talk about the criteria they use in the identification

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and classification of rectangles. This aspect shows the extent to which the role of the teacher is in the formation of geometric concepts in primary school students, i.e. the progress of teaching and learning of geometric concepts depends entirely on the methodological strategies that the teacher uses in the teaching-learning process in the classroom.

The behavior of the students questioned during the activities of identification and classification of rectangles showed that they also have immense difficulties in identifying rectangles in particular and quadrilaterals in general. These students use non-relevant definitions and properties when they are faced with activities that require the application of one of these components. The reactions of the students were not very different from the revelations given by the teachers assisted and questioned, which was to be expected, because they are students of these teachers.

Faced with a flat picture sheet, only 12 students out of 140 made more than one grouping, thus representing 8.6% of the correct students. No student identified the rectangle including the square, just as no student correctly defined the rectangle and square. There were 140 students directly involved in the study and no one included the square as a rectangle, which was notorious during class attendance.

The work with teachers and students leads us to the conclusion that there are enormous difficulties, both for teachers and students, in the identification and classification of rectangles and inclusion of classes, areas in which this research was most focused.

Given this situation, we conclude that the methodological strategies used by primary school teachers, particularly in the 5th grade, in the teaching of quadrilaterals are related to the difficulties that the student has in not being able to include the square as a rectangle. This is because the difficulties that the teachers presented are almost identical to those that the students also presented during the questionnaire. And, as they used traditional methodologies, the transfer of them to the students is shown here.

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