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INTEGRATIVE REVIEW OF THE TEACHING AND LEARNING OF INTEGERS: ARITHMETIC THINKING IN BASIC EDUCATION

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ABSTRACT

The teaching and learning of integers represents a significant conceptual challenge in mathematics education, requiring approaches that transcend the simple memorization of operative rules. This study performs an integrative review of the international literature on the subject, analyzing the main pedagogical strategies used in the introduction and understanding of the concept of negative numbers. The research identifies successful methodologies, such as the use of visual representations, educational games, metaphors, and metacognitive strategies, highlighting their advantages and limitations in the development of students' arithmetic thinking. In addition, the importance of adaptive teaching is emphasized, which considers sociocultural factors and promotes the continuing education of teachers to deal with recurrent conceptual difficulties. From the critical analysis of existing approaches, an integrative structure is proposed that combines

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intuition, cognitive conflict and progressive formalization, favoring a more meaningful and lasting learning. The study also points out gaps in the literature, such as the absence of longitudinal research on conceptual retention and the need for greater articulation between cognitive neuroscience and the didactics of integers. It is concluded that the teaching of this concept requires pedagogical planning that balances innovation and theoretical foundation, enabling students to understand and apply integers in a broad and contextualized way.

Keywords: Integers. Mathematics teaching. Arithmetic thinking. Pedagogical strategies. Integrative review.

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INTRODUCTION

The teaching and learning of integers represents a significant challenge in Mathematics Education, especially in the transition from natural numbers to negative numbers. This process requires a conceptual restructuring on the part of the students, who often present difficulties due to the bias of the natural number and the absence of concrete representations that help in the construction of the concept (BISHOP et al., 2016; VLASSIS, 2004).

The National Common Curriculum Base (BNCC) advises that the understanding of integers should occur in a progressive and contextualized way, exploring everyday situations, such as temperature variations and financial operations, to promote the construction of algebraic thinking from the final years of Elementary School (BRASIL, 2018). In addition, it highlights the importance of logical reasoning and mathematical argumentation, aligning with the strategies discussed in this review, such as the use of visual representations, games, and active methodologies.

Recent research shows that many students show obstacles in understanding operations with integers, especially about multiplication and the application of the sign rule (SILVA, 2024). The use of innovative strategies, such as mathematical games and playful activities, is effective in reducing these difficulties, promoting more meaningful learning (VAIANO et al., 2019). In addition, teacher training plays an essential role in overcoming these barriers, since teachers' insecurity about the teaching of this topic can directly impact student performance (CARREIRO; ZANON, 2014).

Studies such as that of Amaral and Coura (2021) highlight that epistemological difficulties, such as the unification of the number line and the abstraction of negative numbers, remain recurrent obstacles in the teaching and learning process. The literature also points to different methodological approaches to deal with these difficulties. Miola and Rossetti (2022) identified emerging trends in the teaching of integers, emphasizing the importance of strategies involving games, manipulative resources, and digital technologies to favor the understanding of this concept. Likewise, Costa (2023) showed, based on the Theory of Conceptual Fields, that concrete materials can significantly help in the construction of the understanding of positive and negative numbers, especially in classes in the final years of Elementary School.

In the historical and epistemological sphere, research such as that of Pontes (2010) and dos Anjos (2008) discuss the evolution of negative numbers and the barriers faced by

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both mathematicians in the past and students today. These investigations reveal that the acceptance of negative numbers was a gradual process, and that many of the challenges faced historically persist in the school environment. In addition, Angelo (2007) observed that future mathematics teachers, when entering university, often have limited conceptions about the multiplication of whole numbers, which indicates the need for an improvement in teacher training for this content.

Given these difficulties, several pedagogical approaches have been proposed to favor the understanding of integers, including the use of visual representations, educational games, metacognitive strategies, and adaptive teaching (BOFFERDING; HOFFMAN, 2021; JUNGLE; BORBA, 2018). However, despite advances in these methodologies, there are still gaps in the literature on which strategies are most effective and how they can be integrated into a cohesive teaching model. Therefore, we ask in this article: Which pedagogical approaches are more effective for teaching whole numbers, considering the conceptual difficulties of students and the challenges of teaching practice?

To answer this question, this study conducts an integrative literature review, analyzing the main challenges and teaching strategies documented in international research. The objective is to propose an integrative structure that combines different methodologies, articulating initial intuition, cognitive conflict and progressive formalization. This structure seeks to align with curricular guidelines, such as the BNCC, ensuring that integer learning is meaningful and contextualized.

METHODOLOGY

The present research adopted the integrative review method, according to the approach of Whittemore and Knafl (2005) and Botelho, Cunha and Macedo (2011). Integrative review allows the synthesis of knowledge on a topic based on the analysis of theoretical and empirical studies, enabling the incorporation of different methodological approaches and analytical perspectives. This methodology was chosen because it allows a comprehensive and critical synthesis of knowledge about the teaching of integers. Unlike systematic reviews that focus on a specific issue with strict criteria, the integrative review encompasses a broader range of literature, including theoretical, empirical, and methodological studies. This is essential to understand a complex mathematical concept such as integers, which involves cognitive, pedagogical and cultural aspects. Allowing us to

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not only review what has been done, but also to identify gaps in the research and propose new directions.

The integrative review was carried out in five stages, according to Whittemore and Knafl (2005): i) Identification of the research problem – Definition of the scope of the review and formulation of guiding questions on teaching and learning of whole numbers; ii) Literature search – Systematic survey of studies in recognized academic databases, such as *Scopus, Web of Science, Google Scholar, SciELO and ERIC*; iii) Data evaluation – Selection of articles based on inclusion and exclusion criteria, ensuring relevance and methodological rigor; iv) Analysis and synthesis of data – Organization of articles into thematic categories and identification of recurrent patterns; v) Presentation of the review – Construction of critical analysis, linking the findings of the literature to the difficulties and potentialities of teaching integers.

The selection of studies included articles published in several countries, prioritizing those that presented empirical data on the teaching and learning of whole numbers in Basic Education; Studies that analyze different cultural and curricular contexts, comparing teaching approaches in different countries; Studies available in English, Portuguese or Spanish, to enable the analysis of a greater number of relevant publications; Studies published in peer-reviewed journals with academic recognition in the area of Mathematics Education and Science Teaching; Articles that investigate pedagogical strategies for teaching integers, including visual representations, games, metaphors, metacognitive strategies, and active methodologies. Articles that are limited to purely theoretical approaches, with no direct connection to pedagogical practices applicable to the teaching of integers, were excluded; Research that deals only with the teaching of basic operations with natural numbers, without addressing the introduction and development of the concept of negative numbers; Duplicate works or literature reviews that do not present new perspectives or critical analyses on the subject; Studies whose methodology does not allow generalizations or didactic applications for the teaching of integers in formal educational contexts; Articles with access restrictions that make it impossible to fully analyze the content.

Content analysis was chosen, according to Bardin (2011), to organize and interpret the vast amount of qualitative data from the reviewed studies. This method facilitates the systematization of knowledge into thematic categories, allowing a deeper analysis of pedagogical approaches, their results and implications. Content analysis is particularly

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useful for identifying patterns of difficulties and functional strategies, as well as providing a detailed understanding of how different cultural and educational contexts influence the teaching of integers.

The process followed the three fundamental phases of content analysis: Pre-analysis – Selection and exploratory reading of the included articles, identifying recurrent patterns in the pedagogical approaches to teaching integers. Publications that deal with conceptual difficulties, didactic strategies and impacts on learning were considered; Exploration of the material – Categorization of information based on the coding of relevant excerpts from the reviewed studies. This coding was carried out through the identification of key terms, recurrent concepts and methodologies applied in the analyzed articles, grouping them into four thematic axes: Conceptual difficulties in learning integers, Use of visual representations and manipulative materials, Didactic strategies based on games and visual representations, Metacognition and development of mathematical thinking, Influence of cultural and curricular contexts on the teaching of numbers Negative. Treatment of the results and interpretation – Critical analysis of the findings, relating the pedagogical strategies identified with the challenges pointed out in the literature.

The thematic axes were defined based on the analysis of the recurrences and conceptual relationships observed in the reviewed articles. The categorization followed an inductive approach, where themes emerged as the studies were analyzed. As a result, the articles were organized into the following five main axes: i) **Conceptual difficulties in teaching integers**: Studies that discuss cognitive obstacles faced by students, such as resistance to understanding negative numbers and natural number bias; ii) **Use of visual representations and manipulative** materials: Works that explore the impact of visual models, such as the number line, and concrete resources on integer learning. iii) **Pedagogical strategies based on games and active methodologies**: Research that investigates the use of educational games and playful practices for the teaching of negative numbers; iv) **Metacognition and development of mathematical thinking**: Studies that analyze the importance of students' reflection on their learning processes and the role of metacognitive strategies in the teaching of integers; v) **Influence of sociocultural and curricular factors**: Works that examine how different cultural contexts and school curricula influence integer learning. The choice of these axes was based on the need to structure the findings of the literature coherently, allowing an in-depth critical analysis of the main approaches and challenges in the teaching of integers. This categorization enabled a better

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understanding of the trends and gaps in research on the subject, contributing to the proposition of an integrative structure for the teaching of this mathematical concept.

The application of content analysis allowed a synthesis of the existing literature, contributing to an understanding of the different methods used in the teaching of integers, their limitations and their impacts on learning. Thus, the methodology adopted in this study enabled a systematic and structured integrative review, ensuring that the results obtained can contribute to improve pedagogical practice and guide future research in the area.

To summarize the reviewed studies, we prepared Chart 1, which presents the references used, organized by year, title, source, objective/focus of the study and main results. This framework allows for a consolidated view of the literature and establishes direct connections with subsequent sections of the article.

Chart 1 – References used in the integrative review

Year	Title	Source	Theoretical Frameworks	Objective/Focus	Findings
1989	*Formal and informal sources of mental models for negative numbers*	*Proceedings of the 13th International Conference on Psychology of Mathematics Education*	Building Mental Models in Mathematics	Formal and informal mental models about negative numbers	Demonstrated that informal knowledge influences the understanding of integers
2004	*Making sense of the minus sign or becoming flexible in 'negativity'*	*Educational Studies in Mathematics*	Cognitive Flexibility and Meaning of the Minus Sign	Cognitive flexibility in understanding negative numbers	He highlighted the importance of flexibility to overcome conceptual difficulties
2013	*Effects of conceptual change texts on overcoming students' misconceptions of negative Integers*	Eurasia Journal of Mathematics	Conceptual Change in Mathematical Learning	Correcting conceptual misconceptions about negative numbers	He demonstrated that instructional texts can modify misconceptions
2014	*Transition from natural number to integer in Chinese mathematics curriculum*	*Frontiers of Education in China*	Structured Curriculum-Based Learning	Cultural and curricular differences in the teaching of integers	Demonstrated how the Chinese curriculum facilitates the introduction of integers

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2016	*Using order to reason about negative numbers*	*Educational Studies in Mathematics*	Theory of Numerical Order	Conceptual understanding of negative numbers	Identified challenges in constructing meanings for negative numbers
2018	*Teaching integers to students with difficulties in mathematics*	*ZDM Mathematics Education*	Game Theory in Mathematics Education	Use of games in the teaching of integers	Confirmed that games improve engagement and understanding of entire operations
2019	*The gap between formal and informal numbers*	*Educational Studies in Mathematics*	Arithmetic Reasoning Model	Difference Between Formal and Informal Numbers	He pointed out that students have difficulties in abstracting formal concepts
2020	*Cultural transposition of Italian didactic artefacts for teaching integers in Chinese primary schools*	*Educational Studies in Mathematics*	Didactic Transposition and Cultural Adaptation	Impact of culture on the teaching of integers	He pointed out that teaching strategies vary significantly between cultures
2020	*Effects of metacognitive scaffolding on low-achieving students' integer learning*	*Learning and Instruction*	Metacognition and Self-Regulated Learning	Use of metacognitive strategies in the teaching of integers	Showed improvement in conceptual retention in students with difficulties
2020	*Teaching negative numbers using metaphors*	*Journal of Mathematics Teacher Education*	Use of Metaphors in Mathematics Education	Teacher training for teaching negative numbers	He suggested that metaphors help in learning and content retention
2021	*Learning negative integer concepts: Benefits of using linear board games*	*Journal for Research in Mathematics Education*	Symbolic and Non-Symbolic Representations	Use of visual representations for teaching integers	It was found that symbolic and non-symbolic representations help in learning
2023	*Learning negative numbers through a collaborative game in elementary school*	*Journal of Educational Psychology*	Collaborative Learning and Digital Games	Impact of collaborative games on integer learning	Demonstrated that games strengthen retention and conceptual understanding

Source: Prepared by the authors.

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From this careful review, trends in the teaching of the subject were identified, consolidating a knowledge base that can guide innovative pedagogical practices and future research in the area.

LITERATURE REVIEW

Based on the analysis of the reviewed studies (Chart 1), the literature review was structured in four main axes: conceptual difficulties in learning integers, didactic strategies based on games and visual representations, influence of cultural and curricular contexts in the teaching of negative numbers, and impacts of methodologies based on conceptual changes.

CONCEPTUAL UNDERSTANDING AND MISCONCEPTIONS

The study by Bishop et al. (2016) with a student to explore how the concept of order can be used to understand negative numbers, noted the influence of numerical order on understanding negative numbers. The researchers observed that the subject investigated, when comparing negative numbers, often resorted to the intuition that larger numbers are to the right of the number line, indicating that the notion of order can be a useful tool for teaching negativity, but it must be accompanied by additional strategies to overcome misunderstandings. This study revealed that the internalization of the concept of negativity is directly related to the cognitive flexibility of students, highlighting that students face difficulties in attributing coherent meaning to these numbers. They suggest that the approach based on number sequences may be an alternative to promote a better internalization of the concept of negativity.

The cognitive flexibility necessary to understand the minus sign was the focus of Vlassis (2004) who highlighted the difficulty of many in dissociating the minus sign from the subtraction operation. He emphasized the importance of cognitive flexibility in understanding the minus sign, emphasizing that many students associate this symbol exclusively with subtraction, which compromises their mathematical reasoning. Their findings reinforce the need for methodologies that encourage a broader and more adaptable view of negativity in numbers.

Finally, Kilhamn (2011) evidenced that the gap between formal and informal knowledge can hinder the transition of students to the appropriate use of integers. Peled et al. (1989) demonstrated that the mental models constructed by students, based on

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experiments with natural numbers, often hinder the acceptance and proper use of negative numbers.

CULTURAL AND CURRICULAR DIFFERENCES

By analyzing how the Chinese curriculum structures the introduction of whole numbers, Ding and Li (2014) suggest that the country's systematic approach favors gradual and sound learning. They examined the transition from natural to integers in the Chinese curriculum, highlighting that this approach is highly structured and gradual, allowing students to build a strong foundation before facing the complexity of negative numbers. This suggests that a well-designed curriculum can significantly facilitate conceptual learning.

In contrast, Ramploud et al. (2020) discussed the cultural transposition of Italian teaching materials to the teaching of whole numbers in China, pointing out that the effectiveness of a method can vary substantially depending on the cultural context and the current curricular structure. These studies analyzed how Italian didactic methods were adapted for teaching whole numbers in China, showing that even strategies in one context may require significant cultural adaptations to be successful in another. Thus, they emphasize the importance of personalizing teaching according to the individual needs of students. However, such a strategy requires a high level of teacher training and adequate resources, which is not always available in educational systems with structural constraints.

Studies such as that of Venkat and Naidoo (2017) highlight that the lack of specific teacher training can significantly compromise the learning of whole numbers. In addition, Sawatzki and Sullivan (2018) explored the integration of mathematical concepts with financial applications, demonstrating that contextualizing integers in everyday situations can improve content retention.

TEACHING STRATEGIES

About pedagogical methodologies, Selva and Borba (2018) and Chan et al. (2024) showed that the use of games in the teaching of integers increases engagement and improves content retention, especially for students with mathematical difficulties. This playful approach has proven satisfactory in both individual games and collaborative experiences, fostering a deeper understanding of operations with integers.

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Bofferding and Hoffman (2021) explored the use of visual representations, such as linear board games, showing that the combination of symbolic and non-symbolic representations facilitates learning. Sahat et al. (2018) reinforced this approach by demonstrating that the use of concrete materials, such as colored cards, contributes to the internalization of operations with integers. Norton and Boyce (2015) pointed out that the early introduction of negative numbers can reduce future difficulties when combined with instructional strategies based on multiple representations.

METACOGNITIVE AND CONCEPTUAL CHANGE

Metacognitive approaches, such as *scaffolding*, are key to supporting students with learning difficulties, according to Chiu *et al.* (2024). The focus of this study was on metacognitive strategies, such as *scaffolding*, demonstrating that low-performing students can benefit enormously from support that helps them reflect on their thought processes, improving conceptual retention. Also Retnowati *et al.* (2018) analyzed the use of instructional texts to promote conceptual changes and showed that these resources can correct misconceptions about negativity in numbers.

The use of metaphors and analogies helps, argue Mamede and Nunes (2020), in the internalization of concepts, such as associating negative numbers with debts or cold temperatures, can facilitate students' understanding, making learning more accessible and intuitive. However, a limitation of this approach is that some analogies can be interpreted inappropriately by students, creating conceptual misconceptions that need to be corrected later. His research suggests that teacher education should include strategies that favor this approach, ensuring that students can make meaningful connections between abstract concepts and their real-world applications.

DISCUSSION OF THE RESULTS

The literature shows that the transition from natural numbers to integers remains a challenge in the teaching of mathematics, due to conceptual, methodological and cultural factors. Understanding negative numbers is not intuitive for most students, requiring specific pedagogical interventions (DING; LI, 2014). The traditional approach, focused on the mechanical application of operative rules, although effective for standardized exercises, fails to promote conceptual understanding, limiting the application to word problems.

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Strategies such as games and visual representations (SELVA; BORBA, 2018; BOFFERDING; HOFFMAN, 2021) demonstrate greater efficacy. For example, linear board games allow you to visualize magnitude and direction, making it easier to assimilate negatives. Bishop et al. (2016) highlight numerical order as a pedagogical resource, but its effectiveness depends on integration with other approaches. Vlassis (2004) reinforces the need for cognitive flexibility, showing that students who associate the negative sign with varied contexts (in addition to subtraction) have fewer difficulties.

The curriculum structure significantly influences: progressive curricula, such as Chinese (DING; LI, 2014), favor the smooth transition between natural and whole, while the research by Ramploud et al. (2020) warns of the need for cultural adaptation of strategies. Visual representations (e.g., boards, colored tiles) associate abstract concepts with concrete experiences, reducing conceptual obstacles (BOFFERDING ; HOFFMAN, 2021; SAHAT et al., 2018).

Metacognitive processes are also critical: scaffolding (CHIU, 2024) and texts for conceptual change (RETNOWATI et al., 2018) improve retention and self-regulation, especially among students with difficulties. Finally, teacher training is essential. Teachers need to master methodologies such as contextualized metaphors (MAMEDE; NUNES, 2020) and understand the cultural nuances of teaching integers to avoid didactic misunderstandings (VENKAT; NAIDOO, 2017).

INTEGRATIVE STRUCTURE AND ADAPTIVE TEACHING OF INTEGERS

Based on these findings, we propose an integrative framework that combines intuitive comprehension with formal instruction for teaching integers, based on the main approaches identified in the literature.

The proposed integrative framework is in line with research such as that of Selva and Borba (2018), which demonstrate that combined approaches – using games, metaphors, and adaptive teaching – improve students' performance in learning whole numbers. This framework combines intuitive contextualization, cognitive conflict, progressive resolution and formalization, and metacognitive reflection, promoting meaningful and lasting learning. Its formulation is directly supported by empirical evidence, ensuring its pedagogical validity.

The introduction of negative numbers must be based on concrete experiences and everyday situations, such as temperature variations, altitudes and financial operations.

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Studies such as those by Sawatzki and Sullivan (2018) demonstrate that mathematical contextualization improves knowledge retention, as it allows students to establish connections between school mathematics and their daily experiences. In addition, Kilhamn (2011) identified that the gap between formal and informal knowledge can make it difficult to understand negative numbers, reinforcing the need for teaching that starts from students' intuition and prior knowledge.

For learning to be meaningful, students must be confronted with challenges that question their initial conceptions about whole numbers. Vlassis (2004) demonstrated that many students associate the negative sign exclusively with subtraction, making it difficult to assimilate its role in other contexts. Strategies that provoke cognitive conflict and encourage critical reflection, such as the exploration of paradoxes and challenging problems, were pointed out by Bishop *et al.* (2016) as productive to stimulate cognitive flexibility and improve conceptual understanding.

After cognitive conflict, it is essential to gradually structure formal knowledge, using visual representations, concrete manipulation, and structured strategies. Bofferding and Hoffman (2021) demonstrated that the use of linear board games, combined with symbolic and non-symbolic representations, facilitates the internalization of operations with integers. Similarly, Sahat *et al*. (2018) found that manipulative materials, such as colored cards, help students visualize operations, promoting a more fluid transition between intuition and mathematical formalization.

To consolidate learning, students should be encouraged to reflect on their resolution strategies and on the learning process itself. Chiu (2024) demonstrated that the use of metacognitive *scaffolding* improves the retention of mathematical concepts, especially among low-achieving students. In addition, Retnowati *et al.* (2018) showed that structured texts for conceptual change are successful in overcoming misconceptions about negative numbers, suggesting that strategies that promote self-regulation of learning are fundamental for meaningful teaching.

The findings of the literature allowed the formulation of an integrative structure for the teaching of integers, composed of four fundamental stages, as shown in Chart 2 below.

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Table 2 – Integrative Structure for Teaching Whole Numbers

Stage	Description	Justification
Intuitive Contextualization	Use of everyday examples to introduce negative numbers concretely.	Sawatzki and Sullivan (2018); Bofferding and Hoffman (2021); Kilhamn (2011)
Cognitive Conflict	Challenging problems that question students' previous conceptions.	Vlassis (2004); Bishop *et al.* (2016)
Resolution and Progressive Formalization	Use of games, manipulative materials and visual representations.	Bofferding and Hoffman (2021); Jungle; Borba, 2018); Sahat *et al*. (2018)
Reflection and Metacognition	Encouragement of self-regulation of learning and review of the strategies used.	Chiu (2024); Retnowati et al. (2018)

Source: Prepared by the authors.

This structured approach, supported by empirical evidence, aims to provide teaching that is more meaningful and aligned with the needs of students. In addition to contributing to overcoming conceptual difficulties, it promotes learning that integrates intuition, experimentation and reflection, favoring the retention of knowledge and its application in different contexts. In this sense, adaptive teaching presents itself as a fundamental strategy, allowing teachers to adjust their teaching methodologies according to the individual needs of students.

This teaching model can be structured on main pillars such as:

1. Mathematical learning is deeply rooted in cultural and social contexts. Different societies approach mathematics in different ways, and the teaching of integers must consider these differences. The adaptation of pedagogical methods to the sociocultural realities of students in a process of **Cultural Recognition** contributes to a more meaningful learning that is closer to the student's reality. Methods that incorporate examples from local daily life, contextualized problems, and specific cultural references can facilitate the understanding of abstract mathematical concepts.
2. The diversity of student profiles requires educators not to restrict themselves to a single teaching approach, but to promote **Methodological Flexibility**. The integration of multiple strategies, such as metaphors, games, concrete manipulation and visual representations, allows different learning profiles to be met. Students who struggle with mathematical abstraction may benefit from

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pictorial representations, while those who learn best through experimentation may be more involved in hands-on activities. In this way, methodological flexibility promotes inclusion by improving knowledge retention and student engagement.

3. The use of teaching materials adapted to the students' reality is a determining factor for the effectiveness of teaching. The development of customized materials that contextualize mathematical concepts in a meaningful way can substantially improve learning. Examples include **Resource Adaptation** such as using local temperatures to illustrate negative numbers in cold regions or adapting mathematical games and challenges based on students' lived experiences. This approach favors conceptual appropriation by making the contents more relevant to students.
4. The implementation of adaptive teaching requires **Continuous Training of Teachers**. This continuing education should cover both the mastery of mathematical content and the knowledge of innovative methodologies and differentiated strategies for teaching whole numbers. Trained teachers can identify students' specific cognitive difficulties and employ approaches that best meet their needs, making teaching more dynamic and efficient.
5. The adaptation of teaching should be a continuous process, based on a Continuous **Assessment system**. Implementing mechanisms that allow frequent adjustments in pedagogical practice is essential to ensure that teaching strategies are truly successful. Evaluation should not be restricted to traditional tests, but include observations, students' self-evaluation, and reflective discussions that allow immediate adjustments in the teaching-learning process.

Adaptive teaching of integers ensures equitable access to contextualized learning, overcoming conceptual challenges through integrated pedagogical strategies. This approach not only makes it easier to understand negative numbers and their operations, but also develops broad mathematical skills, such as problem-solving and critical thinking, which are essential for real-world applications.

By aligning everyday intuitions with mathematical formalization, education in whole numbers values the cultural and cognitive diversity of students. A structured yet flexible practice allows for continuous adjustments to the learning context, promoting sound conceptual mastery and preparing students for future mathematical abstractions.

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CONCLUSION

The present study demonstrated that the teaching of integers requires pedagogical strategies that go beyond the memorization of rules and promote a deep conceptual understanding. The literature review showed that approaches such as games, visual representations and metaphors have the potential to facilitate learning, but have limitations when used in isolation. Thus, the integrative structure proposed in this study suggests that the combination of these methodologies, combined with adaptive teaching, can favor a more meaningful and lasting learning.

One of the main contributions of this review was to highlight the importance of teaching that balances intuition and formalization. Strategies that contextualize integers in students' daily lives facilitate their introduction, while activities that promote cognitive conflicts help in conceptual reconstruction. However, consolidating this knowledge requires a continuous process of reflection and progressive formalization, ensuring that students not only understand operations with integers, but know how to apply them in different mathematical and real-world contexts.

Another fundamental finding was the relevance of adaptive teaching in overcoming the difficulties associated with whole numbers. The personalization of teaching, respecting the sociocultural and individual differences of students, promotes more accessible and efficient learning. However, the implementation of this approach requires a more solid teacher training, enabling teachers to identify specific difficulties of students and adjust their pedagogical strategies in a dynamic and contextualized way.

The present integrative review also revealed gaps in research on the teaching of integers, which need to be investigated to improve educational practice. One of the most critical limitations identified was the paucity of longitudinal studies looking at how integer learning evolves. Most studies focus on the immediate effectiveness of pedagogical approaches, without considering the retention and applicability of the concept in the long term. Future studies could examine the impact of systematic revisiting of negative numbers and compare conceptual persistence among students subjected to different methodologies, such as traditional teaching, games, and metaphors.

Another gap observed was the absence of controlled experimental studies that empirically evaluate the efficacy of the identified pedagogical strategies. While there is qualitative and descriptive evidence on the benefits of innovative approaches, there is a lack of research that systematically compares different methodologies and verifies their real

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impact on learning and concept retention. Thus, future investigations could test the integrative structure proposed in this study, analyzing whether the combination of intuitive contextualization, cognitive conflict, progressive formalization, and metacognitive reflection favors a more solid and lasting learning. In addition, the use of hybrid methodologies, which integrate face-to-face and digital practices, deserves to be explored as to their effectiveness in teaching whole numbers.

Teacher training was another critical aspect identified in this review, as few studies have explored its direct relationship with student performance in learning whole numbers. There are still uncertainties about how training for innovative methodologies impacts pedagogical practice and learning. Future research could investigate the relationship between teachers' conceptions of integers and their approaches in the classroom, in addition to evaluating the influence of continuing education on the implementation of didactic resources based on games, metaphors and visual representations.

Another aspect that is still little explored in the literature is the relationship between cognitive neuroscience and the teaching of integers. Learning this concept involves complex brain processes, but this dimension has not yet been fully incorporated into pedagogical practices. Future studies could examine which brain regions are activated during learning and manipulating negative numbers, as well as investigate how multisensory strategies, such as the use of manipulative materials, impact mathematical abstraction and the construction of the concept of integer.

The applicability of the proposed integrative structure may vary according to the educational context, and it is necessary to evaluate its effectiveness in different cultural and socioeconomic realities. The teaching of whole numbers can be influenced by curricular and linguistic factors, making it essential to carry out comparative studies that analyze the implementation of this approach in different countries and schools with different student profiles.

Teaching whole numbers still faces conceptual and methodological challenges, but recent advances demonstrate the potential of innovative approaches to enhance learning. Conducting academically rigorous empirical studies is essential to validate methodologies and deepen the understanding of the factors that influence the assimilation of negative numbers. It is hoped that the directions pointed out in this study will guide future research and strengthen pedagogical practice, promoting a more solid, accessible and contextualized teaching of mathematics.

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The implementation of pedagogical strategies for teaching integers requires continuous adaptation to the cultural and educational specificities of each context. In Brazil, the diversity of socioeconomic realities and variations in teacher training make it essential that methodologies are not applied homogeneously, but adjusted to reflect the needs of students. The contextualization of teaching, combined with local examples and the use of accessible materials, can make learning more meaningful and inclusive. In addition, the continuing education of teachers should be a priority, ensuring that they can use innovative methodologies effectively and in line with national curriculum guidelines. The strengthening of the dialogue between research and educational practice is essential for theoretical advances to translate into concrete improvements in mathematics teaching.

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