Revista ARACÊ
ISSN: 2358-2472

ANALYSIS OF THE POPULATION DYNAMICS OF THE MUNICIPALITY OF BARÃO DE GRAJAÚ-MA USING THE MALTHUS AND VERHULST MODELS: A CASE STUDY

https://doi.org/10.56238/arev7n1-072

Submitted on: 06/12/2024 **Publication date:** 06/01/2025

João Vitor Silva Santos[1], Sandra Lima dos Santos[2], Guilherme Luiz de Oliveira Neto[3], Meldson Barros Rezende Lima[4], Carla Vanessa Silva Santos[5], Francislucia Abreu Silva Santos[6], Sônia Maria Santos da Silva[7], Marlos Félix de Azevedo[8] and Carlos Alberto Ferreira dos Santos Junior[9]

ABSTRACT

INTRODUCTION: The fundamental concept in the mathematical modeling of populations is the "population size". Depending on the type of population analyzed, this size can be expressed in various ways, such as the number of individuals, weight, volume, mass, density, or even a percentage of any of these measures (ALMEIDA, 2003). The analysis of population dynamics is extremely relevant to society, encompassing economic, political and sociocultural aspects (SETI et al., 1999).

To investigate population dynamics, a variety of mathematical models are used. The most prominent being Malthus's Theory (1798) and Verhulst's Statistical Model (1838).

Keywords: Population Dynamics. Models by Malthus and Verhulst. Barão de Grajaú-MA.

[1] Graduating in Mathematics, IFPI-CAFLO
E-mail: caflo.2022114lmat39@aluno.ifpi.edu.br
[2] Mathematics Student, IFPI-CAFLO
E-mail: caflo.2022114lmat34@aluno.ifpi.edu.br
[3] Dr. in Process Engineering, IFPI-CAFLO
E-mail: guilherme@ifpi.edu.br
[4] Graduating in Mathematics, IFPI-CAFLO
E-mail: caflo.2022114lmat02@aluno.ifpi.edu.br
[5] Undergraduate student in Geography, UESPI-PI
E-mail: carlavanessasilva095@gmail.com
[6] Degree in Mathematics, IFPI-CAFLO
E-mail: caflo.2020114lmat05@aluno.ifpi.edu.br
[7] Undergraduate student in Geography, UESPI-PI
E-mail: Soniamariasdas@aluno.uespi
[8] Degree in Mathematics, IFPI-CAFLO
E-mail: Marlos.100392@gmail.com
[9] Graduating in Mathematics, IFPI-CAFLO
E-mail: caflo.2022114lmat21@aluno.ifpi.edu.br

Revista ARACÊ
ISSN: 2358-2472

INTRODUCTION

The fundamental concept in mathematical modeling of populations is "population size." Depending on the type of population analyzed, this size can be expressed in various ways, such as the number of individuals, weight, volume, mass, density, or even a percentage of any of these measures (ALMEIDA, 2003). The analysis of population dynamics is extremely relevant to society, encompassing economic, political and sociocultural aspects (SETI et al., 1999).

To investigate population dynamics, a variety of mathematical models are used. The most prominent being Malthus's Theory (1798) and Verhulst's Statistical Model (1838).

The mathematical analysis of population dynamics began in 1798, with the publication of the article entitled "An Essay on the Principle of Population as it Affects the Future Improvement of Society", written by the British economist and demographer Thomas

Robert Malthus (RAFIKOV, 2003). Malthus's model, often called the Malthusian model, provoked controversy in the early nineteenth century, as Malthus argued that the world's population expanded in a geometric proportion, while the resources needed for survival increased only in an arithmetic proportion.

Thus, he stated that the population would be kept under control through misery, hunger, among other natural phenomena (Rocha and Botta, 2009). According to Rocha and Botta (2009), it is noteworthy that Verhulst's model, known as the logistic model, was introduced in 1837 and proposes that population growth is restricted by a logistic factor.

This concept is related to the carrying capacity of the environment and assumes that a population, upon settling in a given environment, will grow until it reaches a sustainable limit, thus reaching a state of equilibrium. The equation of the model includes the decrease in population growth, which is influenced by an inhibiting factor. The general objective is to analyze how the urban population dynamics of the city of Barão de Grajaú MA, according to the mathematical models of Thomas Malthus and Pierre-François Verhulst.

THEORETICAL FOUNDATION

The Brazilian Institute of Geography and Statistics (IBGE) is a federal public institution responsible for the production, analysis and dissemination of statistical and geographic information in Brazil. Founded in 1936, the IBGE plays a fundamental role in

Revista ARACÊ
ISSN: 2358-2472

the planning and elaboration of public policies, providing data on the Brazilian population, economy, territory and society.

Among its main activities, the IBGE carries out demographic censuses, economic surveys, surveys on the living conditions of the population and geographic mapping. The Population Census, carried out every ten years, is one of its best known and most comprehensive initiatives, providing detailed information on the composition and characteristics of the Brazilian population, and in addition, the IBGE offers essential information for researchers, companies and the government, contributing to the social and economic development of the country.

Ordinary differential equations (ODEs) are equations that involve an unknown function of a single independent variable and its derivatives. Differential equations emerged in the seventeenth century with the advances of Differential and Integral Calculus, through the works of Newton (1642–1727) and Leibniz (1646–1716), where numerous mathematical models could be noted.

The main motivation for the studies of differential equations came from the problems of modeling the phenomena of Classical Mechanics, such as the movement of the planets (Bassanezi and Ferreira, 1988).

They encompass certain quantities that vary over time, i.e., differential equations are directly linked to rates of change. Emphasizing, "Differential equations have wide application in solving complex problems about motion, growth, vibrations, electricity and magnetism, aerodynamics, thermodynamics, hydrodynamics, nuclear energy, and all kinds of physical phenomena involving rates of change of varying quantities" (Swokowski, 1994, p. 637).

METHODOLOGY

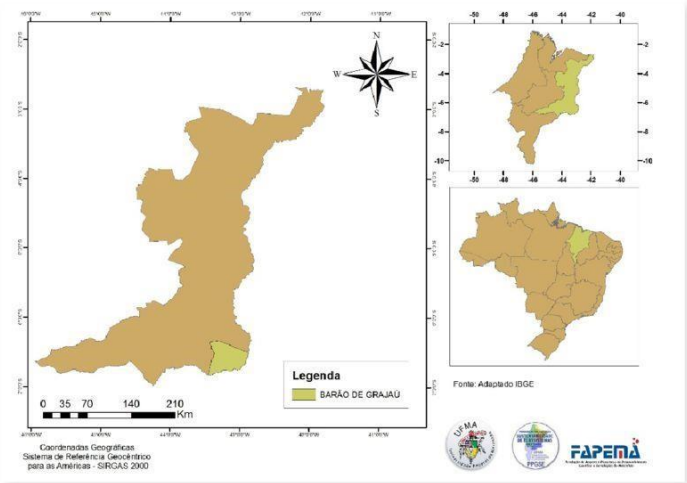
In this study, population data from the city of Barão de Grajaú, located in the state of Maranhão, were used. According to the Brazilian Institute of Geography (IBGE), in 2022 the population was 18,984 inhabitants and the demographic density was 8.59 inhabitants per square kilometer. In comparison with other municipalities in the state, it was in positions 92 and 183 out of 217. In comparison with municipalities throughout the country, it was in positions 1786 and 4526 out of 5570.

This city was founded on March 29, 1911 on the left bank of the Parnaíba River, with the city of Floriano-PI on the right bank. Its patron saint is Santo Antônio de Pádua, its

Revista ARACÊ
ISSN: 2358-2472

area is 2,247 km² representing 0.68% of Maranhão, 0.145% of the Northeast Region and 0.0264% of the entire Brazilian territory has a semi-arid climate, is at 108m altitude, and has the following geographic coordinates: Latitude: 6° 45' 25" South, Longitude: 43° 1' 29" West (Figure 1).

Figure 1: Location of the Municipality of Barão de Grajaú – MA



Source: Adapted from IBGE

MATHEMATICAL MODELS

The mathematical models in which they were used to estimate the population calculation of the city of Barão de Grajaú-MA in the years 2010 and 2022, consist of using the applications of the models of Thomas Malthus and Pierre François Verhulst.

MALTHUS'S MODEL

Malthus's theory holds that population expands at a geometric rate, while resources for its survival increase only at an arithmetic rate. This change in population size is exclusively influenced by the number of births and deaths, disregarding aspects such as immigration or other elements that may restrict growth over a period. This dynamic can be illustrated as follows: $\frac{dP}{dt} = rP(t) \Rightarrow P(t) = P0ert$, where P(t) is the population as a function of time, P0 is the initial population, and r is the constant of proportionality.

MODELO DE VERHULST

Verhulst's proposal suggests that, in situations of low population density, growth occurs exponentially. However, as resources become more limited, this growth begins to slow due to an element known as environmental carrying capacity. This element acts as a

Revista ARACÊ
ISSN: 2358-2472

limiting factor over time, evidencing the maximum population that a region can sustain. Below, we present the corresponding equation.

$$\frac{dP}{dt} = rP(t)\left(1 - \frac{p(t)}{K}\right),$$

where P(t) is the population as a function of time, r is the constant of proportionality, and K is the maximum sustainable population capacity of the environment.

RESULTS AND DISCUSSION

To arrive at the results in Table 1, the values of the real population between the years 2000 and 2022 were necessary, in which these values obtained through the logistic formulas of both mathematical models: Malthus and Verhulst, which are being demonstrated below, were applied.

- Malthus logistic equation:

$$P(t) = P_0 e^{rt} \implies P(t) = 15349.\, e^{ln\left(\frac{17493}{15349}\right).t} = 15349.\ (17493)^{t}.\ 15349$$

- Verhulst's logistic equation:

$$P(t) = rP(t)\left(1 - \frac{p}{K(t)}\right) \implies P(t) = \frac{K}{1+\left(\frac{K-P_0}{P_0}\right).e^{-rt}}.$$

Based on the results of Table 1, it is noted that the Malthus and Verhulst models adequately report excellent data when compared with the real values of the urban population dynamics of the city of Barão de Grajaú during the period from 2000 to 2022.

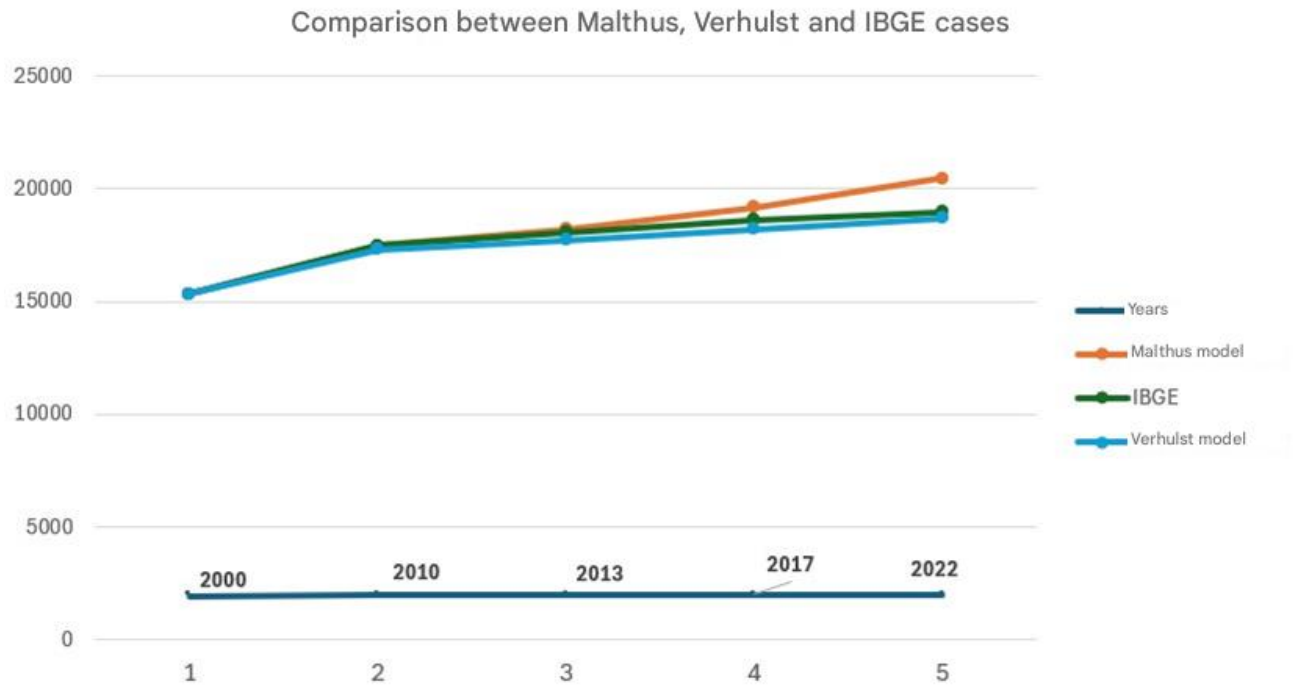
Table 1: Population of the Municipality of Barão de Grajaú - MA in the period between 2000/2022

Anos	tempo t	Modelo Malthus	IBGE	Modelo Verhulst
2000	0	15349	15349	15349
2010	10	17493	17493	17291
2013	23	18192	18074	17722
2017	27	19169	18619	18203
2022	32	20464	18984	18675

Revista ARACÊ
ISSN: 2358-2472

In view of this, we determined the parameters of the models in which we better see what best describes the population dynamics of the city of Barão de Grajaú in the period considered, projections of population growth for both years were made in the models of Malthus and Verhulst is illustrated in Figure 2.

Figure 2: Projections of the population of Barão de Grajaú-MA between 2000 and 2022



The data obtained through the calculations for the years worked, using the population growth models of Malthus and Verhulst, it became possible to carry out the projection of the population of Barão de Grajaú-MA from the last data for 2022, in which it represented an estimated population for the Malthus model of 20,464 inhabitants, and for the Verhulst model 18,675 inhabitants.

CONCLUSIONS

Therefore, the analysis of the population dynamics of the city of Barão de Grajaú-MA using the models of Malthus and Verhulst, allows a deeper understanding of the demographic characteristics and trends of the region. This study showed that the Verhulst model is more adequate to predict the population growth of Barão de Grajaú-MA, since it takes into account the limitation of resources and reflects a future stabilization of the population.

Revista ARACÊ
ISSN: 2358-2472

These results can be useful for the city's strategic planning, especially with regard to the allocation of public resources.

For future studies, it is recommended to include factors such as migration and public policies for population control, which can make the model even more accurate.

Revista ARACÊ
ISSN: 2358-2472

REFERENCES

1. Almeida, I. A. (2003). Aplicação dos modelos de Malthus e Verhulst de dinâmica populacional à população do Brasil. Catalão: GO.

2. Zill, D. G., & Cullen, M. R. (2001). Equações diferenciais (3ª ed.; A. Zumpano, Trad.). São Paulo: Pearson Makron Books.

3. Boyce, W. E., & DiPrima, R. C. (2010). Equações diferenciais elementares e problemas de valores de contorno (9ª ed.; V. de Magalhães Iório, Trad.). Rio de Janeiro: LTC.

4. Santos, R. J. (2011). Introdução às equações diferenciais ordinárias. Belo Horizonte: Imprensa Universitária da UFMG.

5. Brasil Escola. (n.d.). IBGE (Instituto Brasileiro de Geografia e Estatística). Disponível em: https://brasilescola.uol.com.br/geografia/ibge.htm. Acesso em: 20 de outubro de 2024.

6. Guitarrara, P. (n.d.). IBGE (Instituto Brasileiro de Geografia e Estatística). Brasil Escola. Disponível em: https://brasilescola.uol.com.br/geografia/ibge.htm. Acesso em: 20 de outubro de 2024.