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HYPSOMETRIC EQUATIONS FOR KHAYA SENEGALENSIS (DESR.) A. JUSS

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ABSTRACT
Height is important to understand the behavior of a species over time and to assess the quality of the area for timber production which can be measured or estimated. Hypsometric models allow estimating height from tree diameter, reducing forest inventory costs. This research aimed to adjust a hypsometric model for Khaya senegalensis (Desr.) A. Juss., contributing to the study of the species in Brazil. Data were collected in a 9.8 ha stand, located at Rancho São Lucas in the municipality of Lajeado Novo-MA. Thirty-seven permanent circular plots with an area of 800 m² were installed and all diameters and heights of individuals in each plot were measured, totaling 721 individuals. Six hypsometric

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models were selected and adjusted by linear regression. The best equation was selected using the Adjusted Coefficient of Determination (R2adj.), Standard Error of Estimate (Syx%), Adjusted Standard Error of Estimate (SyxAdj.), Mean Percentage Deviation (MDP), F-test, graphical analysis of residuals and weighted value of statistical parameters. The Trorey model (H=-0.21461+0.98413 (DBH)+-0.02748 (DBH)^2+ε) and Prodan (H=(DBH)^2/(1.316862+0.769434(DBH)+0.057553(DBH)^2)) were chosen because they presented the best statistical parameters.

Keywords: Forest Measurement. Growth. Silviculture.

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INTRODUCTION

Planted forests are cultivated and harvested for industrial use, respecting the sustainable management plan, which aims to reduce environmental impacts and promote the economic and social development of neighboring communities. Management practices have been improved with respect for sustainability and the population, making planted forests a great ally of socioeconomic development in the countryside, contributing to reducing pressure on natural forests, protecting the soil, conserving biodiversity, and mitigating climate change (IBÁ, 2017). In 2019, the areas of planted forests increased by 2.4% compared to 2018. Among the many benefits of the sector, the opportunities for work and income generation for the population stand out, promoting local economic changes (IBÁ, 2020). In 2020, the estimated area of planted forests was 9.3 million hectares, of which 70.6% are concentrated in the South and Southeast regions of the country. Forestry production also increased in value, reaching R$18.8 billion, representing a growth of 21.35% compared to 2019. In primary forestry production, forestry expanded its share with 79.8%, with timber products continuing to dominate the sector with 90.1% of the value of forestry production (IBGE, 2020).

Due to its high commercial value, Brazilian mahogany (Swietenia macrophylla King) has been suffering from selective exploitation, which, coupled with the attack of the needle borer (Hypsipyla grandella Zeller), makes Brazilian mahogany an endangered species. The attack of the needle borer affects the growth of the tree in height and diameter, favoring the mortality of the species in monocultures and causing bifurcations in the plants (Maestre; Aquino; Rabelo, 2020).

The wood of African mahogany (Khaya ssp.) becomes the main substitute for Brazilian mahogany, in addition to being known worldwide as noble (Reis; Oliveira; Santos, 2019). African mahogany does not have a significant phenotypic difference from Brazilian mahogany, but it does have a difference in its reddish coloration, which in Brazilian mahogany is greenish (Falesi; Baena, 1999). The Khaya genus belongs to the Meliaceae family and has a high timber potential, with several forms of use, the main ones being the manufacture of luxury furniture, musical instruments, laminates, and shipbuilding. The investment in African mahogany is justified by the resistance to the needle borer and the physical and mechanical resistance of the wood (Reis; Oliveira; Santos, 2019).

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The first Khaya ssp. Plantations were in 1976 in northern Brazil, and since then the species has shown promise and commercial plantations have been increasing due to the growing demand for tropical timber in the country (Ribeiro et al., 2017). Several plantations have already been carried out in Australia, Asia, and Tropical America, but few studies have been published on silviculture, growth, and productivity, studies that can contribute to decision-making in the management of the species (Ribeiro et al., 2017).

The investment in African mahogany in tropical areas worldwide is justified by the similarity of the species to Brazilian mahogany, the price of the wood on the international market, the rapid growth, and the resistance to the pointer borer (Hypsipyla grandella Zeller). In 2018, the estimated area of planted African mahogany forests in Brazil was 37 thousand hectares, plantations that are not at the appropriate age for harvesting, but thinning has been carried out (Reis; Oliveira; Santos, 2019). In the state of Pará, the Khaya senegalensis species was one of the favorites of foresters. It is a species of great importance due to its rapid growth, helping to recover altered areas. However, commercial interest in African mahogany came from the exploitation of forests where African mahogany had been native for 70 years, reducing the concentration of the species, which led to organized plantations (Falesi; Baena, 1999). Height is an important variable for understanding how a species behaves over time. It can be measured or estimated and is important for determining the quality of the area for timber production. By adjusting a hypsometric model, it is possible to estimate height using only the diameter of the tree. This is a methodology that reduces inventory costs but can reduce the accuracy of height estimates (Soares; Neto; Souza, 2011). Hypsometric relationships are relationships between height and diameter, which allow estimating the height of trees. Different species have different hypsometric relationships and if performed on the same species, the relationships change according to the age of the stand. Hypsometric relationships are important in a forest inventory, as the costs of activities are reduced by reducing the number of heights measured in the field (Finger, 2006). In this context, the work aimed to adjust hypsometric models for Khaya senegalensis (Desr.) A. Juss., contributing to the study of the species in Brazil and reducing the time and cost of the inventory carried out by the forest producer.

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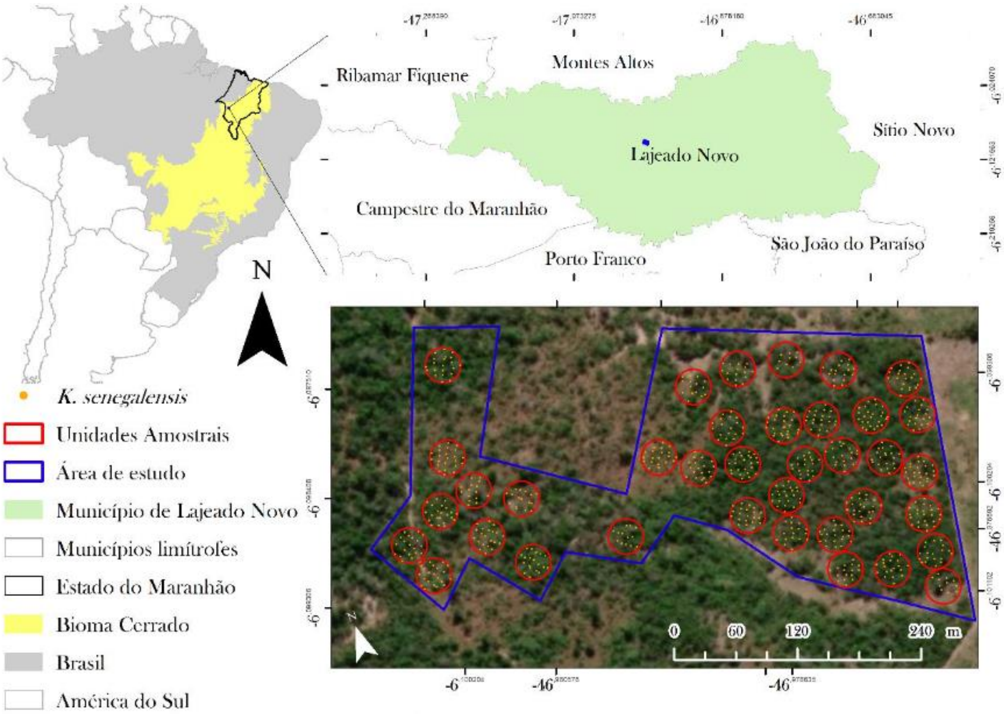
METHODOLOGY

STUDY AREA

The study was carried out at Rancho São Lucas, in the municipality of Lajeado Novo - MA, located at the geographic coordinates 6°05'58" S and 46°58'40" W, which belongs to the Imperatriz microregion - MA (Figure 1). According to the Köppen classification, the climate is Aw, hot and semi-humid tropical, with a dry season (Alvares et al., 2013). The average annual precipitation is 1220 mm, with the rainy season between November and May (Meneses, 2009). The microregion is bathed by the Tocantins River, which is the main source of water supply for the cities, and by the Cacau, Bacuri, Santa Teresa, Capivara, Barra Grande, Cinzeiro, Angical, Grotão do Basílio and Saranzal streams (Nascimento et al., 2015). The vegetation in the region is of the deciduous seasonal forest and cerrado type, characterized by the presence of trees with twisted trunks and branches. (IMESC, 2008).

Figure 1. The geographical location of the study area and experimental area in the municipality of Lajeado Novo – MA.



Source: IBGE, DATUM: SIRGAS 2000, UTM-Zone 23 S Projection. Authorship: Chaiane Rodrigues Schneider (Ferreira; Gomes; Schneider (2024).

DATA COLLECTION AND ANALYSIS

The experimental area covers 9.8 ha, where 37 permanent circular plots of 800 m² each were installed, and distributed using a random sampling method. The planting was

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carried out with seedlings averaging six months of age in January 2020, with spacing of 5m × 8m and 5m × 7m, originating from seminal planting. Data were collected in January 2022.

In the plots, the diameter at breast height (DBH) and Total Height (Ht) of all trees were measured, totaling 721 individuals. Their distribution by class is shown in Table 1. Diameters were measured using a mechanical caliper, and heights were measured with a Haglöf electronic clinometer.

Table 1. Distribution of sampled individuals by diameter and height classes in the Khaya senegalensis (Desr.) A. Juss. Stand in the municipality of Lajeado Novo - MA.

Diameter Classes (cm)	**Height Classes (m)**	**Total**
	0⊢1	1⊢2
0⊢2	9	42
4⊢6	1	14
6⊢8		
8⊢10		
10⊢12		
12⊢14		
Total	26	94

Source: Authors (2022).

The height and diameter data were adjusted using linear regression, testing six hypsometric models, as shown in Table 2.

Table 2. Hypsometric relationship models for adjustments in Khaya senegalensis (Desr.) A. Juss. plants in Lajeado Novo - MA.

No.	**Hypsometric Models**	**Authors**
1	H=β_0+β_1 (1/DBH)+ε	Assmann (1970)
2	H=β_0+β_1 (DBH)+β_2 (DBH)^2+ε	Trorey (1932)
3	H=(DBH)^2/(β_0+β_1 (DBH)+β_2 (DBH)^2)+ε	Prodan (1965)
4	Ln(H)=β_0+β_1 Ln(DBH)+β_2 (1/DBH)+Ln(ε)	Azevedo et al. (1999)
5	Ln(H)=β_0+β_1 (1/DBH)+Ln(ε)	Curtis (1967)
6	Ln(H)=β_0+β_1 Ln(DBH)+Ln(ε)	Stoffels (1953)

Where: β_n = estimated coefficients, H = total height (m), DBH = diameter at 1.30 m above the ground (cm), Ln = natural logarithm.

The best equation (Table 3) was selected based on the following statistical parameters: adjusted coefficient of determination (R^2Adj), standard error of estimate (Syx%), adjusted standard error of estimate (SyxAdj), mean percentage deviation (MPD),

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and significant F-value. Additionally, a graphical residual analysis was conducted, and a weighted value (WV) of the statistical parameters was calculated.

The weighted values (WV) were computed for each equation, ranking each parameter according to efficiency, with a weight of 1 assigned to the most efficient equation, 2 to the second most efficient, and so on. The WV is the sum of the weights assigned to the different parameters for each equation, and the best equation is the one with the lowest WV.

Table 3. Criteria for selecting the best equation to estimate the height of Khaya senegalensis in Lajeado Novo - MA.

Criterion	Formula
Coefficient of Determination	$R^2 = (SS\ Regression) / (SS\ Total)$
Adjusted Coefficient of Determination	$R^2\ Adj. = 1 - [(1 - R^2)((n - 1) / (n - p - 1))]$
Standard Error of Estimate %	$Syx\% = \sqrt{(MS\ Error)} / \bar{h} \times 100$
Adjusted Standard Error of Estimate %	$SyxAdj.\% = \sqrt{(\sum(h - \hat{h})^2 / (n - p - 1))} / \bar{h} \times 100$
Mean Percentage Deviation	$MPD = (\sum((\hat{h} - h) / h)) / n \times 100$

Where: SS = sum of squares, MS = mean square, n = number of observations, p = number of regression coefficients, h = observed height, $\hat{h}$ = estimated height, $\bar{h}$ = mean observed height.

VALIDATION

For validation of the adjusted equations, 144 trees (20%) representing all diameter classes were selected from the dataset. These data were not used in the equation adjustments. Validation was conducted using the Chi-square test (χ^2) at a 5% significance level, testing the following hypotheses:

- **H_0:** There is no significant difference between the actual height and the estimated height from the selected model.
- **H_1:** There is a significant difference between the actual height and the estimated height from the selected model.

RESULTS AND DISCUSSION

The F-values ranged from 1366.90 to 627.12. The highest F-value was for the Stoffels equation (1366.90), while the Trorey equation had the second highest value (1286.15). The lowest value was for the Assmann equation (627.12). The F-values were significant at 5% for all models, indicating regression between dependent and independent variables. These values suggest that at least one regression coefficient is significant for all equation models. All results are presented in Table 4.

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Table 4. Precision statistics.

Model	β_0	β_1	β_2	F	R²Adj	Syx%	SyxAdj	MPD
Assmann	5.886521	-7.56436	-	627.12	0.52	32.16	-	14.92
Trorey	-0.21461	0.98413	-0.02748	1286.15	0.82	19.88	-	9.36
Prodan	1.316862	0.769434	0.057553	808.57	0.74	-	20.87	-0.58
Azevedo et al.	-0.74783	1.165153	0.88684	715.55	0.71	-	21.37	5.91
Curtis	1.870125	-2.45813	-	708.27	0.55	-	27.64	8.77
Stoffels	-0.11654	0.898377	-	1366.90	0.70	-	20.50	6.06

Source: Authors (2025).

The Adjusted Coefficient of Determination (R²adjust) is used to compare equations with different numbers of coefficients when necessary (Sanquetta et al., 2014). The equations presented good results for the adjusted coefficient of determination (R²Adjust), with values ranging from 0.52 to 0.82; the highest values were for the Trorey and Prodan equations with 0.82 and 0.74, and the lowest R^2 values for Assmann and Curtis with 0.52 and 0.55, respectively. According to Campos and Leite (2006), it is normal for the R^2 values found for hypsometric equations to be no greater than 0.80, since the relationship between height and diameter is not so strong. Silva et al. (2016) adjusted the hypsometric equations for the species Khaya ivorensis, in the region of Pirapora, Minas Gerais, aged between 30 and 54 months, also in a seminal plantation, and found values for R^2 close to 0.70, which is close to the values found in this study. Stolle et al. (2018) adjusted hypsometric equations in a stand of Khaya ivorensis A. Chev from seedlings by seeds, three years old, and found values for the coefficient of determination that ranged from 0.48 to 0.46, values lower than those found in this study.

The Standard Error of the Estimate demonstrates the accuracy of the adjustment of a mathematical model; the smaller the error, the better the estimates obtained with the equation (Thomas et al., 2006). Of the six adjusted equations, the Trorey equation obtained the lowest Syx% with 19.88%. For the adjusted estimate error (Syxajust), the lowest value was that of Stoffels with 20.50% and the highest value was that of Curtis with 27.64%. In the research by Sanqueta et al. (2017), in a young forest restoration stand, Syx% values close to 15% were found. Stolle et al. (2018) found a value close to 11%, which differs from the present study, where the lowest Syx% is 19.88%. However, values similar to the present study were found by Gama (2018) who obtained a standard error percentage for

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the Trorey equation of 19.85%, however, the lowest error in the study by Gama (2018) was for the Prodan equation with 19.84%, approaching the values found in this study, where the Prodan equation obtained 20.87% for Syxajust. The Mean Percentage Deviation (MSD) represents the average of the deviations between the real and estimated values, measuring the trend that the values represent; the closer to zero, the lower the trend. The Prodan equation underestimates the height by 0.58%, while the other models overestimate the height. Observing the graphical analysis of the residuals (Figure 2), it is possible to analyze that all models fit the data, presenting a homogeneous dispersion. The Chi-Square test applied in the validation found that there is no significant difference between the real height values and the height values estimated by the six adjusted models. However, the equations that presented the lowest Chi-Square value were the equations of Trorey, Stoffels, and Azevedo et al.(1999).

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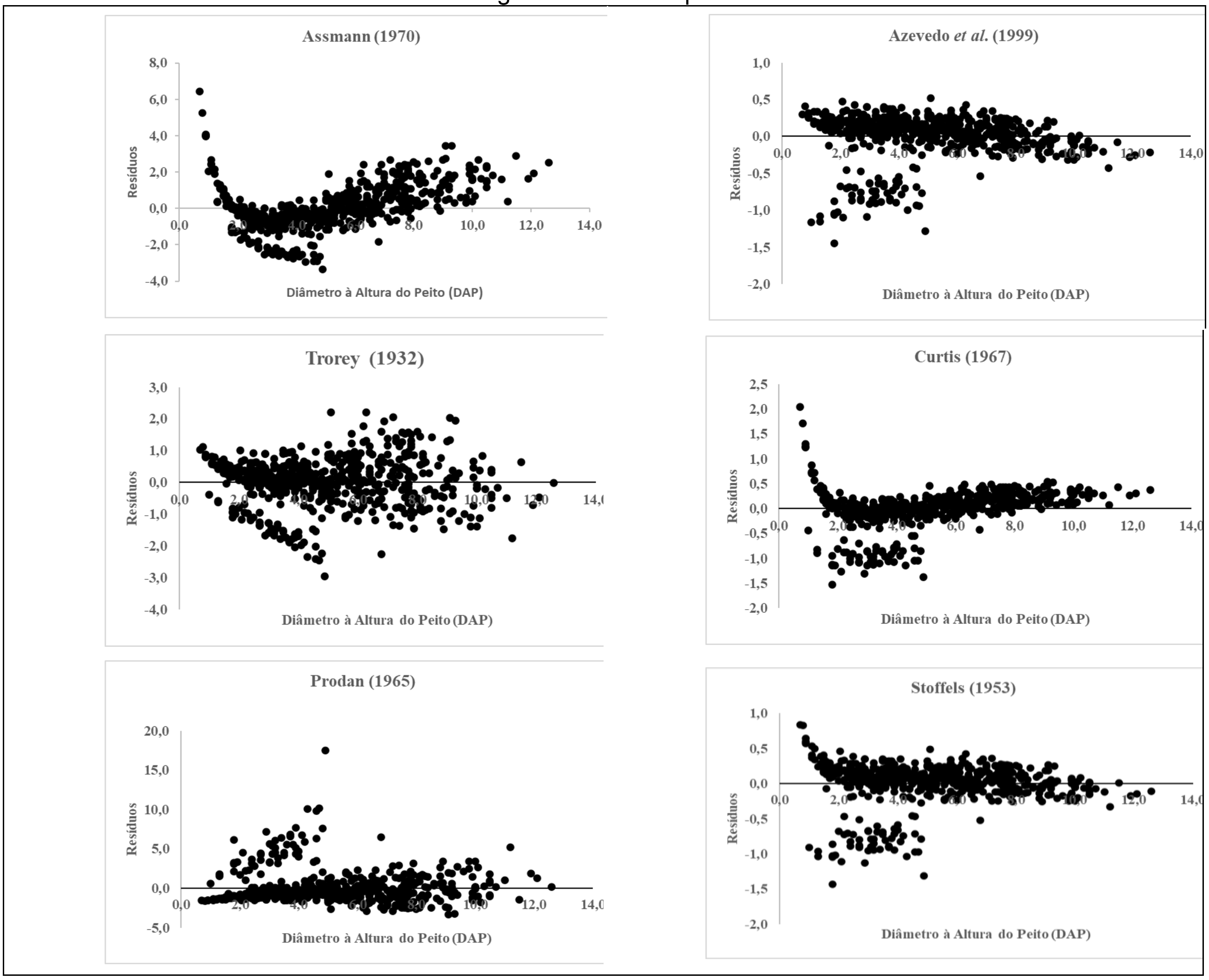
Figure 2. Residual plot.



Source: Authors (2025).

Analyzing the weighted value of statistical parameters (Table 5), the most suitable equations for estimating the height of *Khaya senegalensis* (Desr.) A. Juss were those of Trorey and Prodan, as they presented higher precision compared to the other equations.

Gama (2018) also fitted six models of hypsometric equations to a 2.9-year-old *Khaya senegalensis* plantation located in the southern part of the state of Tocantins and determined that the most appropriate model for estimating height is the Trorey model due to its ease of adjustment, similar to the model found in the present study. In the study by Stolle et al. (2018) with the species *Khaya ivorensis* A.Chev, the Curtis model yielded the best results. Sousa et al. (2019) determined that the Naslunde model provided the highest accuracy in equation adjustment.

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Table 5. Weighted value of statistical parameters.

Authors	F	Adjusted R²	Syx%	Syxajust	DMP	VP
Assmann	6	6	6		6	24
Trolley	2	1	1		5	9
Prodan	3	2	3	1	9	
Azevedo et al.	4	3	4	2	13	
Curtis	5	5	5	4	19	
Stoffels	1	4	2	3	10	

Source: Authors (2022).

CONCLUSION

The most suitable models for estimating the height of *Khaya senegalensis* (Desr.) A. Juss are those of Trorey ($H = -0.21461 + 0.98413 \times (DAP) + (-0.02748) \times (DAP)^2$) and Prodan ($H = (DAP)^2 / (1.316862 + 0.769434(DAP) + 0.057553(DAP)^2)$) as they provide the best statistical parameters for precision.

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