


DIGITAL GEOMETRY: MODELING THE REAL WORLD IN VIRTUAL SPACES

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ABSTRACT

The article investigated the educational possibilities of Augmented Reality (AR) and Virtual Reality (VR) in geometry education, highlighting their potential to transform abstract concepts into concrete, interactive experiences. The theme addressed how these technologies can promote more engaging learning, connecting theory and practice at different educational levels. The research aimed to understand and exemplify how digital tools can be used to improve students' academic performance and logical reasoning. To this end, a bibliographic research methodology was used, based on theoretical references that analyzed the impact of these technologies on education. According to Oliveira Netto (2006) and Ruiz (2006), this method allowed the selection of relevant data to contextualize and support the discussions presented. The article was structured in four sections: the first discussed the application of digital tools to connect the abstract to the concrete; the second explored the educational possibilities of AR and VR; the third addressed practical examples of the use of these technologies in the classroom; and the last presented activities applied in real contexts. The conclusion underscored that the integration of AR and VR into geometry teaching not only facilitates the understanding of complex concepts, but also promotes cognitive skills and broadens interdisciplinarity. Finally, the need for new research to overcome technological challenges and expand the educational impact of these practices was highlighted.

Keywords: Augmented Reality. Geometry. Digital Education. Active Methodologies. Interactivity.

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INTRODUCTION

The integration of technology into teaching has generated new possibilities to approach abstract concepts, such as those present in geometry, in a more accessible and interactive way. The relevance of this topic is evidenced by the growing adoption of digital tools in the educational context, especially in the use of Augmented Reality (AR) and Virtual Reality (VR). These technologies, by uniting the physical and virtual worlds, provide students with immersive experiences that connect geometric concepts to everyday life, promoting more meaningful and engaging learning.

The aim of this study was to investigate how AR and VR can be applied to geometry teaching to facilitate the understanding of abstract concepts and improve students' academic performance. The question that guided the research was: 'how can AR and VR technologies transform the teaching of geometry at different educational levels, connecting the abstract to the concrete?'

To answer this question, a bibliographic research was carried out, according to the methodological principles described by Oliveira Netto (2006) and Ruiz (2006). According to Oliveira Netto (2006), bibliographic research is an essential resource to investigate phenomena already studied, allowing for a deeper understanding and the construction of new approaches based on existing data. Ruiz (2006) highlights that this methodology is fundamental to organize and systematize knowledge in an efficient and grounded way. The analysis technique used involved the selection and interpretation of theoretical and practical references that dialogued directly with the investigated theme, with data collected from academic publications and experience reports.

The development of the article was structured in four main sections. The first addressed 'Digital Geometry in the Classroom: Connecting the Abstract to the Concrete in Virtual Spaces', highlighting how digital tools can make geometric concepts more tangible. The second explored 'The Possibilities of AR and VR in Geometry Teaching: An Educational Perspective', emphasizing the benefits and challenges of integrating these technologies into the school environment. Then, the third section entitled 'Virtual and Augmented Reality in Geometry Teaching: Possibilities and Practical Applications' presented innovative pedagogical resources and strategies. Finally, the fourth section, 'Practical Applications of AR and VR in Geometry Teaching: Contextualized Examples', detailed concrete activities that can be implemented in the classroom.

Therefore, this study sought to contribute to the reflection and deepening of the potential of AR and VR technologies in the teaching of geometry, promoting the transformation of learning into a more dynamic, attractive process aligned with the demands of contemporary education.

DIGITAL GEOMETRY IN THE CLASSROOM: CONNECTING THE ABSTRACT TO THE CONCRETE IN VIRTUAL SPACES

Geometry, as an essential branch of mathematics, has always challenged educators to translate abstract concepts into understandable and applicable forms in everyday life. In this sense, Skrodzki (2020, p. 1) states that "mathematical objects are generally abstract and not very accessible", indicating the difficulty faced by students and even specialists in understanding higher geometric concepts. However, the introduction of digital technologies, such as virtual reality (VR) and augmented reality (AR), has revolutionized the way these concepts are presented, making them more accessible and visually impactful.

In this context, Skrodzki (2020, p. 1) highlights that "illustrations and interactive visualizations help both students and professionals to understand mathematical material and work with it". A practical example would be the use of *software* that allows the construction of geometric solids in three dimensions, such as educational applications based on AR. Such tools allow students to manipulate prisms, pyramids and spheres virtually, providing a visual and tactile experience that transcends traditional teaching based only on books and boards. Thus, learning ceases to be static and becomes dynamic and engaging.

In addition, hyperbolic geometries, mentioned by Skrodzki (2020, p. 1) as "particularly suitable for visual representations", offer a universe of pedagogical possibilities. In the classroom, they can be explored using *software* such as GeoGebra 3D or virtual reality simulations that illustrate curved spaces, such as the Poincaré disk model. This approach makes it easier to understand complex concepts, such as internal angles of hyperbolic triangles smaller than 180 degrees, something challenging to teach only with conventional methods.

However, according to Santana *et al.* (2021, p. 2084), "more than ever, significant training is essential, contextualized with the interests of a generation immersed in the digital world". This implies that the insertion of technologies in mathematics education requires adequate preparation of teachers, both in the field of digital tools and in the

pedagogical application of these innovations. For example, continuing education courses for teachers can include hands-on workshops on the use of AR and VR in geometry education, enabling them to incorporate these technologies effectively.

At the same time, Santana *et al.* (2021, p. 2086) argue that "opening up to digital pedagogical possibilities allows the teacher to achieve the profile of the new-age student". Thus, by using applications such as GeometriAR or VR resources, the teacher can align his practices with the profile of students who are used to interacting with digital devices. This not only facilitates learning, but also increases the motivation and engagement of students, who come to recognize themselves as active agents in the educational process.

Therefore, integrating digital technologies into geometry teaching represents not only a methodological advance, but also a response to contemporary demands. From practices that combine complex mathematical concepts with innovative digital resources, it is possible to build a more inclusive, dynamic and effective learning environment, promoting meaningful teaching that connects the abstract to the concrete. Thus, digital geometry consolidates itself as a bridge that transforms the classroom into a space where the real and virtual worlds coexist to enrich the educational experience.

THE POSSIBILITIES FOR GEOMETRY: INTEGRATION OF DIGITAL TECHNOLOGIES IN TEACHING

Geometry, with its foundations deeply rooted in abstract thinking, often challenges students to grasp concepts that, at first glance, may seem distant from everyday reality. However, as highlighted by Skrodzki (2020, p. 5), "Poincaré's model is widely used and was popularized beyond mathematics by the works of M.C. Escher". This popularization highlights the relevance of integrating visual representations in the teaching of geometry, particularly when combined with digital technologies.

In this sense, virtual reality (VR) emerges as a promising tool. According to Skrodzki (2020, p. 8), "virtual reality eliminates the coexistence between different geometric configurations by providing a fully immersive experience of non-Euclidean space". A practical example would be the use of applications that allow students to explore curved spaces, such as simulators based on *Jeffrey Weeks' Curved Spaces*, which, according to Skrodzki (2020, p. 25), "explore interesting visual effects in curved spaces". These immersive experiences enable students to visualize and interact with complex concepts, such as hyperbolic geometry, in a dynamic and engaging way.

In addition, digital games also play a crucial role in mathematical learning. According to Skrodzki (2020, p. 26), "games like 'Hyperrogue' combine hyperbolic geometry with innovative game mechanics to teach mathematical concepts". Such tools make learning more accessible and stimulating, especially for students who have difficulties in mathematics. Gamification, therefore, provides an environment in which students can learn while having fun, reducing the anxiety associated with learning content that is considered difficult.

In Brazil, the need for innovations in mathematics teaching is evident. Gomes *et al.* (2019, p. 405) point out that "only 4.5% of Brazilian students in the 3rd grade of high school reached the level of proficiency considered adequate in mathematics, according to the SAEB 2017". Faced with this scenario, digital technologies can play a transformative role. Narciso *et al.* (2024, p. 406) emphasize that the main objective in the use of digital technologies is to "positively influence the communication, socialization, and cognitive development of individuals". This perspective highlights the ability of technologies to expand learning opportunities and meet the needs of different student profiles.

Ademais, conforme Narciso *et al.*,

[...] digital technologies offer significant opportunities to improve people's quality of life [...] allowing them to express themselves and connect in innovative ways (Narciso *et al.*, 2024, p. 405).

In geometry education, this means providing students with tools that not only make learning more efficient, but also more meaningful and inclusive. Geometry apps, augmented reality simulators, and educational games are practical examples that can be integrated into the curriculum, helping to overcome learning difficulties and promoting student engagement.

Therefore, the integration of digital technologies in geometry teaching opens up a range of possibilities that not only modernize pedagogical practice, but also democratize access to mathematical knowledge. Through interactive visual representations, immersive experiences, and gamified tools, it is possible to transform students' perception of mathematics, bringing them closer to concepts that previously seemed unattainable. Thus, geometry teaching becomes a bridge between the abstract and the concrete, allowing students to understand and apply these concepts in the real world.

THE POSSIBILITIES OF AR AND VR IN THE TEACHING OF GEOMETRY: AN EDUCATIONAL PERSPECTIVE

Geometry, often considered challenging due to its abstraction, finds in Augmented Reality (AR) and Virtual Reality (VR) technologies a transformative potential for the understanding of complex concepts. As highlighted by Skrodzki (2020, p. 1), "only virtual reality has recently made it possible to present immersive experiences of non-Euclidean geometries." This possibility of immersion allows students to visualize, interact and understand geometric properties in ways that would not be accessible in a traditional environment.

In this sense, "virtual reality illustration allows users to experience geometric properties and effects that are not present in their surrounding Euclidean world" (Skrodzki, 2020, p. 1). In the classroom, one can use hyperbolic geometry simulators, such as *Curved Spaces*, to explore how internal angles of hyperbolic triangles add up to less than 180 degrees. Such a practice not only broadens understanding but also stimulates students' critical thinking.

In addition, "altered depth perception in virtual reality allows users to experience the world as 'hyperbolic beings'" (Skrodzki, 2020, p. 18). In educational practice, this immersion can be used to demonstrate advanced concepts, such as geometric transformations, in an interactive way. Students can 'walk' within virtual environments that exemplify non-Euclidean spaces, which makes learning more meaningful and engaging.

At the same time, "augmented reality unites the real and the virtual on a single screen, promoting dynamic interaction with 3D objects" (Gomes *et al.*, 2019, p. 406). Tools such as GeometriAR, a free application for download, expand the accessibility to these technologies (Gomes *et al.*, 2019, p. 412). A practical example would be using the app to manipulate geometric solids in real-time, allowing students to observe properties such as areas, volumes, and symmetries directly on their mobile devices. This approach, in addition to stimulating active learning, favors digital inclusion, reaching students who may not have access to more complex technologies, such as VR glasses.

Therefore, the possibilities of AR and VR in geometry teaching go beyond the simple visualization of concepts. They represent a pedagogical innovation capable of uniting theory and practice in an interactive environment. By adopting these tools, teachers not only modernize their practices, but also respond to the needs of a generation immersed in digital, aligning learning with the competencies required in the twenty-first century. In this

way, geometry ceases to be an abstract content to become a concrete, accessible and engaging experience.

VIRTUAL AND AUGMENTED REALITY IN GEOMETRY TEACHING: POSSIBILITIES AND PRACTICAL APPLICATIONS

The use of Virtual Reality (VR) and Augmented Reality (AR) in geometry teaching has demonstrated significant potential to transform the learning of abstract concepts into interactive and engaging experiences. Skrodzki (2020) points out that only recently has VR enabled immersion in non-Euclidean geometries, allowing students to explore geometric properties in virtual environments. This innovation is key to making mathematics more accessible, especially in regions where teaching still faces significant challenges.

In this context, initiatives such as that of Inteceleri, highlighted by Gomes *et al.* (2019), exemplify the positive impact of these technologies in the school environment. Through devices such as MiritiBoard VR and applications such as Geometricando, the startup was able to gamify the learning of plane and spatial geometry. This approach combines theory and practice in a gamified environment, promoting greater engagement and understanding among students. Applying these technologies in the classroom means, for example, exploring geometric solids in 3D, allowing students to identify and manipulate figures such as cubes and pyramids directly on their mobile devices.

AR also plays a crucial role in bridging the real and the virtual on a single screen (Gomes *et al.*, 2019). Apps like GeometriAR allow students to visualize geometric shapes superimposed on the physical world, connecting theoretical concepts to everyday practice. In a practical example, students can observe a pyramid being moved over the Eiffel Tower, exploring its properties and applying mathematical knowledge in a contextualized way. This interactive experience, in addition to captivating students, reinforces active and meaningful learning.

Teacher training is also an essential element for the success of these initiatives. As highlighted by Santana *et al.* (2021), adapting to new technologies requires educators to be prepared to explore digital pedagogical possibilities. Inteceleri exemplifies this need by offering mathematics training workshops for teachers, integrating the use of AR and VR with active methodologies. This approach ensures that technology becomes an ally of the teacher, who remains irreplaceable in the educational process.

The results of these initiatives have been remarkable. Walter dos Santos Oliveira Júnior, CEO of Inteceleri, says that the *Matematicando* application, which combines gamification and neurolinguistic techniques, promoted significant increases in the Basic Education Development Index (IDEB) and in students' school grades. This improvement is attributed to the interaction of students with innovative technologies, which facilitate the development of mental calculation, computational thinking, and logical reasoning (Gentil, 2020).

Thus, AR and VR experiences, such as Inteceleri's, exemplify how these technologies can be integrated into the school curriculum to transform the way geometry is taught. By allowing students to experience the mathematical world in an immersive and interactive way, these tools not only make learning more accessible but also promote digital inclusion and the development of essential skills for the twenty-first century. Thus, the possibilities for teaching geometry become limitless, connecting theory, practice, and innovation in a single learning space.

PRACTICAL APPLICATIONS OF AR AND VR IN GEOMETRY TEACHING: CONTEXTUALIZED EXAMPLES

The use of Augmented Reality (AR) and Virtual Reality (VR) in geometry teaching offers a wide range of pedagogical possibilities to make mathematical concepts more accessible, interactive, and meaningful. Technological tools, such as *Geometricando*, *GeometriAR*, *Curved Spaces*, *Google Expeditions*, *GeoGebra 3D* and *CoSpaces Edu*, provide resources that integrate theory and practice, promoting more engaging learning. Examples of application in different educational stages are detailed below, based on the references already explored.

GEOMETRICATING IN ELEMENTARY SCHOOL 1 AND 2

The *Geometricando* tool, which uses AR and VR to gamify the learning of geometric shapes, can be used in activities that connect mathematics to students' daily lives. For example, students can explore geometric shapes present in everyday objects, such as balls (spheres) or boxes (cubes). Using AR, the app highlights vertices and edges, helping students understand the three-dimensional properties of figures. As Skrodzki (2020) points out, interactive visualizations help both students and professionals understand and work with mathematical material. The practice reinforces the importance of exploring

technologies that capture students' interest and stimulate active learning (Santana *et al.*, 2021).

GEOMETRY IN ELEMENTARY SCHOOL 2 AND HIGH SCHOOL

In Elementary 2 and High School, GeometriAR can be used to teach advanced concepts, such as calculating the lateral area and volume of geometric solids. Students manipulate a triangular prism in 3D, observing each face separately and connecting theory to practical applications. Gomes *et al.* (2019) emphasize that AR promotes dynamic interaction with three-dimensional objects, allowing students to establish direct relationships between plane and spatial geometry. This activity not only improves academic performance but also prepares students to tackle real-world math challenges.

CURVED SPACES IN HIGH SCHOOL

The *Curved Spaces* app allows students to experiment with concepts of non-Euclidean geometries in an immersive way. In class, students can simulate light trajectories in curved spaces, exploring properties that do not exist in the Euclidean world. Skrodzki (2020) points out that VR eliminates the coexistence between different geometric configurations, providing deep experiences in hyperbolic environments. This practice can be connected to astronomy and theoretical physics, broadening interdisciplinarity in learning and allowing students to understand scientific applications of geometry.

GOOGLE EXPEDITIONS FROM KINDERGARTEN TO HIGH SCHOOL

With Google Expeditions, teachers can organize virtual trips to iconic buildings, such as the Leaning Tower of Pisa, to teach concepts such as symmetry and tilt. This approach is particularly useful for introducing geometric shapes in the context of architecture and design. According to Gomes *et al.* (2019), these immersive experiences make learning more concrete and engaging, connecting abstract concepts to the reality experienced by students.

GEOGEBRA 3D IN ELEMENTARY SCHOOL 2 AND HIGH SCHOOL

3D geometric modeling is another powerful tool for mathematical learning. With GeoGebra 3D, students can create three-dimensional graphs of rotated parabolas and explore their intersections with other planes. This practice aids in understanding complex

concepts such as conic sections and properties of curved surfaces. According to Skrodzki (2020), interactive visualizations facilitate the understanding of higher mathematical concepts, providing a solid foundation for progressive learning.

COSPACES EDU IN ELEMENTARY 2 AND HIGH SCHOOL

CoSpaces Edu allows students to create interactive virtual environments that integrate geometric shapes into projects. A practical activity would be to design a 'geometric city', where students calculate areas, volumes and dimensions of the buildings created with basic figures such as prisms and cylinders. This activity not only promotes the learning of mathematics but also encourages creativity and collaborative work. Santana *et al.* (2021) argue that opening up to digital pedagogical possibilities allows the teacher to achieve the profile of the new-age student.

The following table details the main applications of Augmented Reality (AR) and Virtual Reality (VR) to work on geometry, including practical examples of classroom application:

Table 1 - AR and VR Applications

Application	Description	Application Contexts
Geometricating	It gamifies the learning of geometric shapes in AR and VR.	Elementary School 1 and 2: Used to teach basic geometric shapes (circle, square, triangle). Proposal: Students identify shapes in everyday objects and accumulate points in the game.
GeometriAR	Platform that uses AR to work geometric solids.	Elementary 2 and High School: Teach volume and area of geometric solids. Proposal: Students manipulate 3D objects in the app to calculate properties and test hypotheses.
Curved Spaces	Curved space simulator, focused on hyperbolic geometry.	High School: Explore properties of non-Euclidean geometry. Proposal: Students simulate light trajectories in curved spaces and discuss scientific applications.
Google Expeditions	Exploration of virtual environments for interdisciplinary learning.	Kindergarten to High School: Introduce geometric shapes in nature and architecture. Proposal: Students visit famous structures, such as the Eiffel Tower, and discuss their properties.
GeoGebra 3D	Tool for modeling and visualization of geometric concepts in 3D.	Elementary 2 and High School: Study of conic sections and intersections of planes. Proposal: Students use the app to build 3D graphics and analyze their characteristics.
CoSpaces Edu	Creation of interactive virtual environments for mathematics teaching.	Elementary 2 and High School: Build environments that integrate geometric shapes into projects. Proposal: Students create a "virtual city" by applying concepts of area

Source: The authors.

These practical applications demonstrate that the integration of AR and VR into geometry teaching not only facilitates the learning of complex mathematical concepts, but also promotes skills such as logical reasoning, critical thinking, and interdisciplinarity. Tools such as Geometricando, GeometriAR, *Curved Spaces*, *Google Expeditions*, GeoGebra 3D, and CoSpaces Edu are concrete examples of how emerging technologies can be powerful allies in educational development. By adopting these practices, teachers not only

modernize their pedagogical approaches, but also contribute to the engagement and integral formation of students.

CONCLUSION

The incorporation of digital technologies in geometry education, especially through Augmented Reality (AR) and Virtual Reality (VR), has proven to be a powerful strategy to transform the learning of abstract concepts into concrete and interactive experiences. This study was able to achieve the proposed objectives by investigating the educational possibilities that these tools offer and by proposing practical examples of their application at different levels of education. Through an analysis of the theoretical references and the organization of contextualized activities, it was possible to understand how technology can expand the reach of geometry teaching, promoting interaction, engagement and cognitive development of students.

Throughout the article, it was highlighted that the use of AR and VR in teaching is not just a matter of innovation, but a necessity in the face of the demands of a digitally connected generation. The tools presented, such as Geometricando, GeometriAR and *Curved Spaces*, proved to be effective in building bridges between theory and practice, allowing students of different age groups and contexts to understand geometric concepts in a more accessible and dynamic way. In addition, the immersive and interactive experiences provided by these technologies have demonstrated potential to improve academic performance, stimulate logical reasoning, and promote interdisciplinarity.

However, for these technologies to be effectively incorporated into the educational environment, significant challenges need to be overcome. Among them are the continuing education of teachers, the technological infrastructure in schools and the need for content adapted to local realities. Despite this, the practical examples discussed show that it is possible to integrate these tools into the curriculum effectively, as long as there is planning, creativity and adequate technical support.

Thus, the study also points to the need for further research that explores the long-term impact of these technologies on academic performance and the integral education of students. Future investigations can address, for example, how AR and VR practices can be adapted to different disciplines, in addition to geometry, or how these technologies can be used to meet the demands of students with special needs. Studies that analyze the cost-

benefit ratio of such initiatives and strategies to overcome technological limitations in public schools are also essential to expand the reach of these practices.

Finally, it is believed that this article contributed to fostering reflection and dialogue on the transformative role of technology in education. By emphasizing the use of AR and VR as innovative pedagogical tools, the way is paved for a more inclusive, dynamic teaching that is connected with the demands of the twenty-first century. Therefore, it is encouraged that more research be done on the subject, with the aim of expanding knowledge and deepening the integration of technologies in the educational context, ensuring that more students have access to meaningful and transformative learning.

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