

THE ETHNOGEOMETRIES PRESENT IN ARCHITECTURAL WORKS: A LOOK AT FRACTALS



<https://doi.org/10.56238/arev6n4-191>

Submitted on: 11/12/2024

Publication date: 12/12/2024

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ABSTRACT

This study arises from the convergence between two distinct research areas: Ethnomathematics and Fractal Geometry, thus originating Fractal Ethnogeometry. The objective is to investigate the presence of fractals in different cultures, especially in their architecture. The research is conducted qualitatively, using a bibliographic approach and a phenomenological method, with data analysis through the perspectives of Ideographic and Nomothetic Analysis. The investigative movement allowed the constitution of three Nuclei of Understanding, with which it was possible to articulate the presence that is questioned in the research question, namely: Fractal Ethnogeometry expressed in spontaneous architectural works of antiquity; Fractal Ethnogeometry expressed in architectural works, design, contemporary cities and natural landscapes and Fractal Ethnogeometry as the construction of interdisciplinary knowledge in architecture and in the classroom using technology. These nuclei of understanding intersect, highlighting the importance of Fractal Ethnogeometry.

Keywords: Ethnogeometry, Fractal Geometry, Architectural Works, Mathematics Education.

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INTRODUCTION

Ethnomathematics is presented by D'Ambrosio (2016) as being the different techniques and understandings about doing mathematics, as well as about teaching in natural, cultural and social environments, not limited only to Western Mathematics, but considering and valuing the different mathematical cultures developed by different peoples. With this, the relevance of Ethnomathematics as well as its complexity is understood here, as it goes against the teaching system that superimposes one mathematics on all others, imposing a conception of universality marked by European Mathematics, defined by D'Ambrosio (2016) as one among the many ethnomathematics.

The conception of ethnomathematics, in a more specific way, can be directed to themes and contents, as is the case of Geometry, which under the foundation of Ethnomathematics can be said to be Ethnogeometry, a field of knowledge that goes against the activity of Galileo, who by embracing the thesis of exact figures born of an alleged obviousness obtained by Logic, it left on the margins all the intuitiveness of practical surveying, which preceded ancient geometry (Detoni, Pinheiro, 2017). This intuitiveness was directed over time to spatial issues, which developed in different ways in different cultures; The triangles, the circumference and other geometric figures now instituted are mere words without the natural and cultural means (spaces) on which all human knowledge has developed and develops.

This knowledge, before being instituted as mathematical objects, is given to people in everyday life, in an experienced ethnogeometry; To be in the world is to spatialize, that is, to occupy oneself with spatiality, moving, and moving what is presented in it, following, or not, a logic of movement. In this daily work, knowledge is also learned, developed, which here is understood to be ethnogeometric, which is often not allowed in schools, because they prioritize what is in books and manuals, which have been guided by European mathematics since the first schools in Brazil.

Under the foundation of Ethnogeometry, we want to focus here on Fractal Geometry. "Fractals are geometric shapes that repeat their structure on smaller and smaller scales" (Stewart, 1996, p. 12). According to English (2007), the geometry of fractals explains natural phenomena, the self-similarity present in nature and in human constructions, such as architecture, which has been reconfiguring itself historically. It is with a focus on architecture, under the foundation of Ethnogeometry that this research will be carried out. Fractals in architecture are not isolated entities in themselves, or merely decorative items,

they expose ideas, characteristics, notions of aesthetics, design, which vary from culture to culture, which are constituted by geometric knowledge related to the natural and social space of each people.

In view of this theme, we want to understand here, *how fractal ethnogeometry is present in the architecture made by different peoples?* To this end, a qualitative research is carried out, of a bibliographic nature, seeking and articulating understandings expressed by researchers who work in the areas of Ethnomathematics and Architecture, which focus on fractals. The intertwining between Ethnomathematics, Architecture and Fractals constitutes the phenomenon of this research, that is: *Fractal Ethnogeometry expressed in architecture*, and ethno explains that the investigative look will be focused on different architectural cultures.

CONTEXTUALIZING ETHNOMATHEMATICS AND ETHNOGEOMETRY

According to D'Ambrosio (2016), although Mathematics is a science constituted in and by the diversity of cultures, it is taught in a universalized way. The teaching of Mathematics is adhered to all over the world under a single socioeconomic and political model and practically equal education systems. These systems are consolidated by disciplines, and among them is Mathematics, which is defined as a universal science, taught under the same theoretical bases, even in socioculturally different realities.

By attributing to Mathematics and its teaching a universal character, whether considering the world or the universe of a country, such as Brazil, it is understood that there are a series of conceptual problems that may occur. Issues such as numbers, shapes, measurements, inferences (many of them correlated to Geometry) are categories of thought that appear in all cultures, however, they diversify and expand when they become evident in these cultures as ways, ways and styles of explaining, knowing and dealing with reality.

When this diversity is disregarded to assert ideas, knowledge and universal teaching, there is a movement of overlapping and subjugation of what is born in the naturalness of the experiences of a people, what is different is imposed on them, without valuing or adding what they know, what they historically produce as truth. If other ways of being of knowledge are not understood, "there is a movement of exclusion, implying the delegitimization of social practices, of doing and perception, which are primacy of any and all human knowledge, among which the so-called mathematical knowledge" (Lima Junior et al., 2021, p. 4).

Going against this movement of exclusion and subjugation of cultures, Ubiratan D'Ambrosio thematized Ethnomathematics, understood as being "the art or technique of explaining, of knowing, of understanding in the various cultural and imaginary contexts" (D'Ambrosio, 1998, p. 5), which defends the valorization of knowledge and mathematical cultures of different peoples. Ethnomathematics is, then, the mathematics of a given civilization, in the culture of an ethno group, being made and remade at all times in different places (Barbosa, 2009). In this context, Western mathematics itself is one among many ethnomathematics. This conception of ethnomathematics can be redirected to themes and content, as is the case of Geometry, which under the foundation of Ethnomathematics can be called Ethnogeometry, whose discussion is based on this work.

Ethnogeometry is a field of knowledge that goes against the activity of Galileo, who by embracing the thesis of exact figures born from an alleged obviousness obtained by Logic left on the margins all the intuitiveness of practical surveying, which preceded ancient geometry (Detoni, Pinheiro, 2017). This intuitiveness began to be directed over time to spatial issues that developed in different ways in different cultures; Triangles, the circumference and all other geometric figures are mere words without the natural and cultural spaces on which all human knowledge has developed and develops. This knowledge, before being instituted as mathematical objects, is given to people in everyday life, in experienced ethnogeometries. In this daily work, knowledge is also learned and developed that is understood to be ethnogeometric, which is often not allowed in schools, because they prioritize what is in books and manuals, which have been guided since the first schools in Brazil by European mathematics.

It is important to highlight that the term Ethnogeometry appears in studies by Paulo Gerdes, where he brings contributions to the history of mathematics, and consequently to Ethnomathematics. D'Ambrosio (1998) states that Paulo Gerdes' studies on Ethnogeometry enable the reconstruction of geometric knowledge forgotten over time, becoming complementary to the studies carried out on Ethnomathematics.

Gerdes (2012) explains that the term Ethnogeometry is an expression used by Professor Donal Crowe of the University of Wisconsin. From this denomination, studies began to focus on Ethnogeometry, seeking to rescue cultural values of different peoples from geometry. Rios (2000, p. 3) explains that "Ethnogeometry is an intrinsic part of the daily experience of man and his natural environment", being the material that inspires Ethnomathematics, when studying history from Geometry.

Based on the assumption that "ethnogeometry can be understood as Ethnomathematics particularized to geometric contents" (Costa, 2020, p.11), we want to focus here on Fractal Geometry, which under the foundation of Ethnogeometry aims at the study of fractals. English (2007) tells us that the geometry of fractals explains natural phenomena, the self-similarity present in nature and in human constructions, such as in architecture, which has been reconfiguring itself historically. It is with a focus on architecture, under the foundation of Ethnogeometry that this research will be carried out.

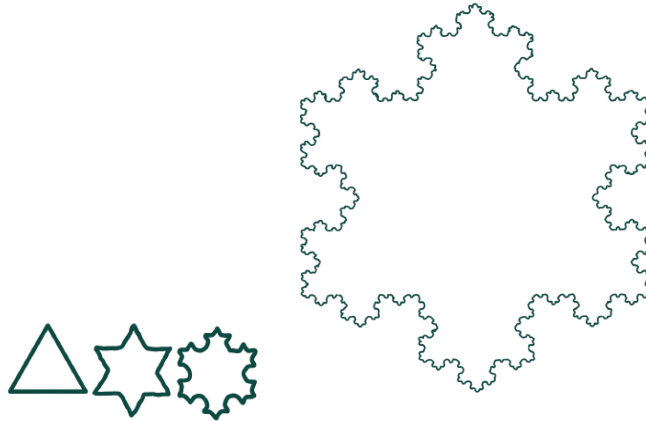
THEMATIZING FRACTAL GEOMETRY

Since Ancient Greece, it was considered that mathematical space would have only three dimensions, but over time the n dimensions of mathematical space were understood and made explicit (Souza, 2019), which consequently allowed the constitution of other geometries, among which Fractal Geometry (FG), conceptualized by Mandelbrot (1998, p. 207) as "The study of various objects, both mathematical and natural, which are not regular, but rough, porous, or fragmented, being so in the same degree at all scales".

Barbosa (2009) explains that Benoit defined the term fractal in the Latin language, where *fractals* mean to break, causing irregular fragments and, as a result, the term now known as fractal was originated. We can then say that "Fractals are geometric shapes that repeat their structure in smaller and smaller scales" (Stewart, 1996, p. 12), as can be seen in the examples of the Von Koch Curve, the Sierpinski Triangle and the David Hilbert Curve, presented below.

The Von Koch Curve or Von Koch Island, shown in Figure 1, is an example of a continuous curve.

Figure 1 – Von Koch Curve

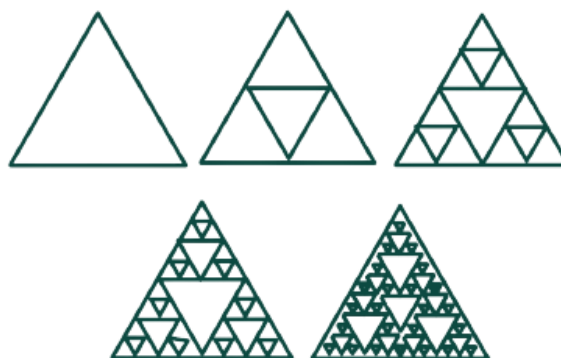


Source: Mandelbrot (1998, p.45). Adapted by the authors.

According to Mandelbrot (1998, p.44), the Von Koch Curve begins from an island in the shape of an equilateral triangle. Next, the central third of each of the sides of unit length "is replaced by a triangle-shaped cable, whose sides measure one third. We thus obtain a regular hexagon or Star of David, whose perimeter has a length of 4 units. The procedure is repeated for each of the 12 sides, and so on".

The Sierpinski Triangle - also called the Sierpinski Joint - "is one of the elementary forms of GF because it has some properties, such as: having as many points as the set of real numbers; have an area equal to zero, being self-similar (each of its parts is identical to the whole); not lose its initial definition as it is expanded" (Araujo et al, 2020, p. 288). As we can see in Figure 2.

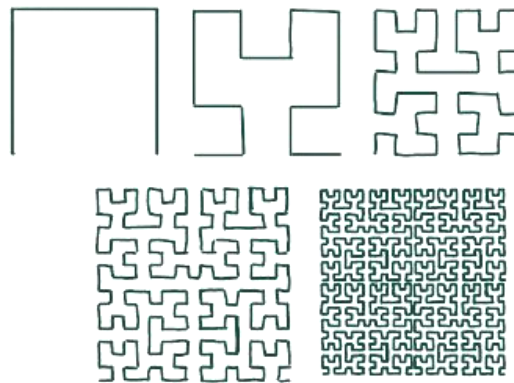
Figure 2 – Sierpinski triangle (Sierpinski joint)



Source: sferrerobravo.wordpress.com. Adapted by the authors

According to Barbosa (2009), David Hilbert (1862 – 1943) presented a curve on a quadrangular surface where each square followed a repetition pattern equal to the initial one, emphasizing the self-similarity of the fractals, as explained in Figure 3.

Figure 3 – David Hilbert curve (Hilbert curve)



Source: mkweb.bcgsc.ca. Adapted by the authors.

According to Barbosa (2009) the Geometry of Fractals makes it possible to interpret the phenomena that exist in the world, however, it is important to highlight that the natural universe does not exhibit fractals (graphic representations) but fractal structures, understood as a representation of observable fragments of infinity that is characteristic of a fractal, as a mathematical object. Such structures can be visualized in plans, in art, in natural phenomena, in models of terrestrial revelation, in architectural constructions and in other spaces and objects.

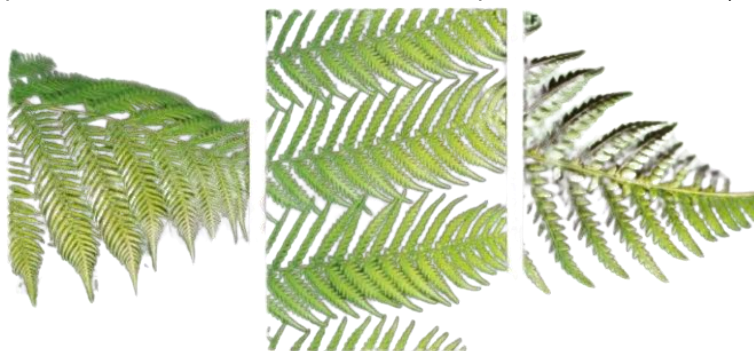
Fractal objects, according to English (2007), have five primordial characteristics that need further clarification, since we will admit only one of them specifically to conceptualize them and thus explain for educational purposes why there are fractal and non-fractal structures in nature themselves.

That said, the characteristics we will address here are: recursion, self-similarity, infinity, fractional dimension, and scalar.

Recursion consists of defining programmable functions to repeat itself. A function that uses recursion to divide a problem into smaller fragments, similar to the original problem, and thereby solves the smaller ones, combining the results obtained (Sedrez, 2019). These results repeated countless times is what we call recursion, and they can continue infinitely, but we can interrupt this recursive pattern within a scalar interval (Barbosa, 2009).

The fractal object has a *scalar* shape that "means that there are similar patterns on different scales within the interval" (Barbosa, 2009, p. 21). For example, the leaves of a fern illustrated in Figure 4 follow self-similar organization patterns at different scales.

Figure 4 – Approximation in the illustration of the fern plant at three scales (fractal structure).



Source: <https://br.freepik.com/vetores/samambaias>

When we discuss the concept of dimension in the studies of geometries, we immediately associate our answer with integers, however, when we approach contents belonging to Fractal Geometry, we have *fractional dimensions* that according to Mandelbrot (1998, p. 14) "is the measure of the degree of irregularity and fragmentation of fractal objects". An example of this is the Von Koch Curve, expressed in Figure 1, which has an increasing dimension in its curves, so that we can go so close to a dimensional line, already below the base the curve is so unusual that we could divide it in two, while in the middle of the curve we would need a "broken" dimension, that is, fractional (Barbosa, 2009).

Fractal objects have an *infinite complexity* that does not allow us to reproduce them completely since they exhibit infinite details. This characteristic is very important for this work since it does not appear in nature, as well as it is not exhibited in architectural works that will be addressed later. That said, we understand that nature and man do not exhibit fractals in themselves, since it would be impossible to expose the richness of infinite details of them. In view of this, the terminology *fractal structure is adopted*, to say of the representations expressed in the lived world.

Finally, *self-similarity* or *self-similarity*. Being self-similar grants the fractal the exact or similar reproduction of a part of itself, consisting of subdivisions equal to the original (Brito, 2020). In Figures 5 we can observe the fractal structure (self-similarity) exhibited in the construction of the Kandariya Mahadeva Temple in India.

Fig. 5 – Kandaria Mahadeva Temple (India)



Source: <https://www.istockphoto.com/br/foto/kjaruharo-tempos-%C3%ADndia-gm528050508-92886645>

With the above so far, we understand that fractals require extra attention when being studied, considering that their complexity requires an in-depth analysis, which is not the focus of this work. To justify the inclusion of fractal objects in this study, it is necessary to resort to the definitions and examples previously discussed, in order to allow the reader to understand the concept of fractal and how its infinite patterns do not manifest themselves in nature or in everyday life. We clarify that we base our analysis on Benoit's definitions, using the term "fractal structure" to explore self-similarity (fractal characteristics) in architectural works of various civilizations, in addition to using the definition of fractal objects itself.

RESEARCH METHODOLOGY AND PROCEDURES

In this research, the qualitative posture and the phenomenological method are assumed in the realization of a bibliographic study. With the phenomenological view, the understandings that are articulated are those that were shown in the investigative process, paying attention to the modes of presence of fractal structures in architectural works of different peoples.

The search process of the analyzed materials was carried out via Google Scholar and the Brazilian Digital Library of Theses and Dissertations (BDTD), in which works corresponding to the results of the search in this environment were taken for analysis by the keywords: ethnomathematics-fractals; ethnogeometry – fractals; architecture-fractals. On the *Google Scholar platform*, the results obtained on the first 15 pages were considered, and on the *BDTD*, productions from the last five (05) years were considered.

The texts thus highlighted will be analyzed from the Ideographic Analysis and the Nomothetic Analysis (Bicudo, 2011). In the first, in a constant return to the data, we turn to the punctual and singular aspects that are revealed when entering the research data (of each text). At this point, we opened ourselves to materializations that culminated in Significant Units (US), which represent highlighted passages in the texts that say about the research question





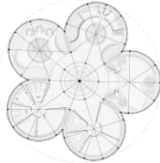

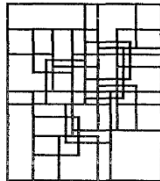

The process of Nomothetic Analysis is a moment to transcend the analysis of the individual data that were expressed in the Ideographic Analysis, taking into account the convergences and divergences of the HUs that, once articulated, point to "great convergences", which from the interpretations and articulations, bring perceptions, evidences and "efforts to express these articulations through language. Finally, it requests an understanding of the structure of the phenomenon interrogated, taking the individual ones as cases of more general understandings that now speak of structural ideas concerning the region of inquiry" (Bicudo, 2011, p. 59). Understandings about convergences through phenomenological analysis are possible paths for an open synthesis, in which the first analyses will be transcended, bringing reflections on the research question.

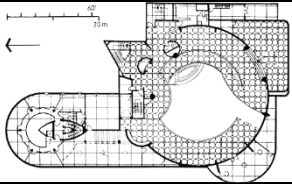


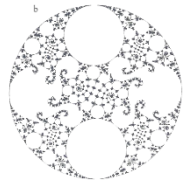

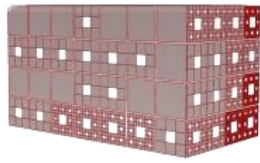
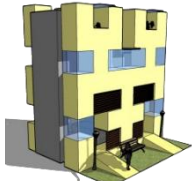

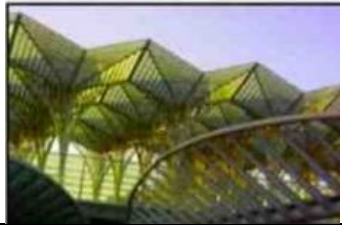
DATA DESCRIPTION AND ANALYSIS: WEAVING UNDERSTANDINGS ABOUT THE RESEARCH QUESTION



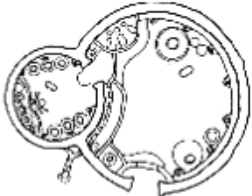

The search on *Google Scholar* and *BDTD*, with the aforementioned criteria, left us two hundred and seven (207) productions, one hundred and eighty-nine (189) on *Google Scholar* and eighteen (18) on *BDTD*. Disregarding texts such as those related to Fractal Geometry, not grounded by Ethnomathematics, and texts on Fractal Geometry applied to other areas such as neural networks. Preserving only productions that focus on Ethnogeometry from the fractal perspective, twenty-one (21) productions remained for analysis: ten (10) articles in scientific journals and two (02) in annals of events, three (03) undergraduate Course Completion Papers (TCC), four (04) master's dissertations and two (02) doctoral theses.

Chart 1, which follows, shows these productions, as well as the fractal structure present in architectural works to which they were the object of study by the researchers.

Chart 1 – List of articles, theses, monographs and dissertations for the analysis of the presence of fractal Ethnogeometry.

Text	Title	Author(s)	Publication	Fractal Structure
1	Fractal form in the teaching of computer-aided architectural design.	Maycon Ricardo Sedrez	UFSC Repository	
2	Parametric design process of façade elements with fractal geometry characteristics considering their luminous behavior	Pedro Oscar Pizzetti Mariano	Repository UFSC	
3	Intentions in Fractal Architecture: An Analysis of Form in Two Works by Steven Holl-Sarphatistraat and Loisiun.	Leonardo de Oliveira Brito	Repository UFSC	
4	Framing concepts of urbanism in contemporary architecture	Carolina Barros and Eduardo Rocha	The U.S. Department of Homeland Security: The Future of the U.S.	
5	Living art, refuge and nature as products of the same experience (URDIR)	Ana Lia Lopes de Azevedo	Unichristus Repository	
6	Geometry: Fractal and Euclidean in African historical buildings	Diego Lourenço Francisco de Souza	Ânima University Repository (RUNA)	
7	Fractal geometry and activities for teaching mathematics: fractal steps and Menger's sponge	Marcus Vinícius Oliveira Lopes da Silva	UFBA Repository	
8	About Fractal Dimensions of Built and Natural Environments	Natalia Naoumova, Andrei Bourchtein and Lioudmila Bourchtein	USP Magazine	

9	Fractal geometry and its applications in architecture and urbanism	Ana Maria Sala Minucci Martins and André Felipe Henrique Librantz	Exacta Magazine	
10	Art and culture: from the past to the future.	Carlos Eduardo Silva Leite and Paula Valéria Coiado Chamma	Vérticefib Magazine	
11	The (geometric) dialogues that Gehry establishes with the city of Bilbao	Luciana Sandrini Rocha and Adriane Borda Almeida da Silva	V!RUS Magazine	
12	Fractal geometry in an ethnomathematical approach.	Paulo Cesar Fassini Barbosa	USP Magazine	
13	Architecture and complexity: fractal geometry with generative system.	Maycon Ricardo Sedrez	UNICAMP Repository	
14	Compositional and luminous organization of a fractal façade: development and evaluation.	Pedro Oscar Pizzetti Mariano and Alice Theserinha Cybis Pereira	USP Magazine	
15	CAAD and creativity, an experience with Fractal architecture.	Maycon Ricardo Sedrez and Alice Theserinha Cybis Pereira	Risco Magazine	
16	Mathematics and architecture: use of fractals in urban furniture.	Rosângela Salles dos Santos, Mirian Caras, Rosana Maria Luvezute Kripka and Regis Alexandre Lahm	Revista Scientia Plena	
17	Research of patterns and their applications in architecture and urbanism: emphasis on fractal geometry.	Lorena P. Waihrich, Rosângela S. Santos, Acácio D. Rosalen, Bruna Z. Comin, Paola K. Scheidmandel, Paulo R. Pasquetti, Taciana Dôro	Journal of the Argentine Association of Computational Mechanics	

18	Ruskin Digital: A Discussion of the Nature of Ornament in Contemporary Architecture	Diogo Ribeiro Carvalho	Architecture and Urbanism Notebooks	
19	Geometry, Geometrization and Afro-Islamic Art	Henry Cunha Junior	Teias Magazine	
20	Earth architecture and different ways of building.	Maria Estela R. Ramos Penha, Ilana Ramos C. Santos and Israel Jonatas Veloso dos Santos	Annals of congress - terra Brasil network	
21	African presence in Brazilian architecture and education: a decolonial perspective under the aegis of Ethnomathematics	Valdirene Rosa de Souza	UNESP Repository	

Source: authors

After completing the exploratory research phase, which provided a considerable collection of texts on the subject, we revisited each one of them with a focus on the approach linked to the fractal structures present in architecture. We analyze the present and underlying dialogue in the citation of the theme, incessantly asking: how does fractal Ethnogeometry manifest itself?

Through the interpretation carried out, we present in Chart 2 the ways in which each text, from the perspective of architectural works, approaches fractal Ethnogeometry. Each of these modes, within the scope of Ideographic Analysis, is interpreted as data from the unitarization process, which are relevant to the understanding of the research question. With this, significant units are constituted, expressed as Fractal Ethnogeometry Mode of Presence, with abbreviation: MPEF1, MPEF2, MPEF3, and so on, aiming to bring evidence about the presence questioned in the research question.

Chart 2 – Modes of Presence of Fractal Ethnogeometry in works with architectural works.

Text 1 – fractal ethnogeometry manifests itself in the spontaneous constructions based on religious beliefs of Hindu temples, respecting the diverse cultural origins (*MPEF1*).

Text 2 – fractal ethnogeometry emerges as an expressive element, manifesting itself continuously in

architectural works from antiquity to contemporaneity (MPEF2).
<i>Text 3</i> – fractal ethnogeometry is revealed in Holl's contemporary works, where fractal patterns present a limit of interactions that contribute to the uniqueness of his architectural works (MPEF3).
<i>Text 4</i> – fractal ethnogeometry is manifested in his contemporary works, which are characterized by creative freedom and incorporation of external values. (MPEF4).
<i>Text 5</i> – fractal ethnogeometry to be present in the composition of contemporary buildings that were born and conceived in the search for fractal representation (MPEF5).
<i>Text 6</i> – fractal ethnogeometry manifests itself in centennial constructions, honoring and preserving cultures and beliefs through the spontaneous productions of the people (MPEF6).
<i>Text 7</i> – fractal ethnogeometry expressed as activities and workshops that rescue, through architecture, customs of different cultures excluded over the centuries (MPEF7).
<i>Text 8</i> – fractal ethnogeometry is expressed in the landscapes and built or natural environments of different cities (MPEF8).
<i>Text 9</i> – fractal ethnogeometry shows itself in the contemporary city as a fractal multiple, exhibiting fractal structures in street compositions (MPEF9).
<i>Text 10</i> – fractal ethnogeometry shows itself as a design for the recovery, valorization and encouragement of contemporary cities (MPEF10).
<i>Text 11</i> – fractal ethnogeometry expressed in the contemporary architecture of the Museum of Bilbao, permeating both its external and internal construction (MPEF11).
<i>Text 12</i> – fractal ethnogeometry is expressed in the spontaneous constructions of a people, exempt from the rigor present in contemporary architecture (MPEF12).
<i>Text 13</i> – fractal ethnogeometry emerges as an approach to conceive complex and distinct contemporary works, taking advantage of the benefits of generative systems (MPEF13).
<i>Text 14</i> – fractal ethnogeometry is exposed as an element of a façade, with fractal characteristics (MPEF14).
<i>Text 15</i> – fractal ethnogeometry is exposed as didactic content for architectural projects that address the potential of fractals in modernity using a computer (CAAD) (MPEF15).
<i>Text 16</i> – fractal ethnogeometry in the construction of interdisciplinary knowledge, where the use of these patterns offers a practical application of mathematical theory in Architecture and Urbanism projects (MPEF16).
<i>Text 17</i> – fractal ethnogeometry is exhibited with traces of fractal structures in works of modernity, seeking an innovation (MPEF 17).
<i>Text 18</i> – ethnogeometry is expressed in contemporary times in works by John Ruskin (MPEF18).
<i>Text 19</i> – fractal ethnogeometry is present in ornamentations of different ancient cultures, from architecture to design (MPEF19).
<i>Text 20</i> – the authors' research reveals fractal structures in the ancient constructions of distant civilizations, highlighting the presence of fractal ethnogeometry (MPEF20).
<i>Text 21</i> – ethnogeometry reveals itself as a proposal to give visibility to distinct knowledges that have been historically excluded (MPEF21).

Source: authors

Some MPEFs, when resumed, are convergent with broader ideas. The movement of identification and registration of these convergences constitutes Nomothetic Analysis, from which structuring ideas are constituted from what expresses groups of HUs that converge with each other. The three (3) categories constituted by the convergences were named Nuclei of Understanding (NC), as expressed in Table 3:

Chart 3 – Organization of the Nuclei of Comprehension

Modes of Presence of Fractal Ethnogeometry	Centers of Understanding
(MPEF1) – (MPEF2) – (MPEF6) – (MPEF12) – (MPEF19) – (MPEF20)	NC1 - Fractal Ethnogeometry expressed in spontaneous architectural works of antiquity.

(MPEF3) – (MPEF4) – (MPEF5) – (MPEF8) – (MPEF9) – (MPEF10) – (MPEF11) – (MPEF11) – (MPEF14) – (MPEF17) – (MPEF18)	NC2 – <i>Fractal ethnogeometry expressed in architectural works, design, cities and contemporary natural landscapes.</i>
(MPEF7) – (MPEF13) – (MPEF15) – (MPEF16) – (MPEF21)	NC3 – <i>Fractal Ethnogeometry as the construction of interdisciplinary knowledge in architecture and in the classroom, using technology.</i>

Source: authors

It is emphasized that the Nuclei of Understanding (NC) are shown in the convergence movement, which reinforces the investigative process, in which no prior judgment is made about the research question. The research methods allow us to affirm that both the Modes of Presence of Fractal Ethnogeometry and the Nuclei of Understanding refer to the guiding question and, with this, it is understood that articulated assertions about each of the NCs, bringing the *MPEF (in bold and italics)* is a way of expressing understandings about the investigation. We then started the articulated assertions, starting from the Core of Understanding 1 (NC1).

NC1 - FRACTAL ETHNOGEOMETRY EXPRESSED IN SPONTANEOUS ARCHITECTURAL WORKS OF ANTIQUITY

The longevity of the millennial architectural techniques of different cultures persists, even in the face of the advent of more technologically advanced and productive construction systems. This occurs because these practices incorporate, in their essence, a cultural perspective intrinsic to the relationship with the metaphysical (Penha et al, 2018). The meaning and meaning of these techniques are intrinsically linked to the fusion between form and content, process and result, function and form, past and future, object and subject, natural and social (Santos, 2006).

The millennial constructions of earth architecture represent accumulations of knowledge from different peoples, many of whom were unfamiliar with the rigorous mathematical principles of Fractal Geometry. However, these cultures already employed fractal structures in their natural constructions. It is imperative that this knowledge, applied to architecture, is not relegated to oblivion, since valuing different customs expands knowledge.

The buildings designed by various civilizations not only aimed to provide housing, but also constituted essential means to ensure survival from the resources offered by the land itself. The structures intended for protection and accommodation, designed to accommodate possible increases in population within the villages, were not expressions of

disorganization, but rather manifestations of conscious organization. The uniform and repeated arrangement of these constructions within the space reflected a careful ordering, evidencing a refined understanding of the importance of organization in optimizing the environment for local communities, as expressed in Figure related to Text 6, the Fractal structure of the village of Ba-ila (village organization), expressed in Chart 1.

According to Sala (2003), builders incorporated the characteristics of fractal geometry in two ways: unconscious or conscious. In the first approach, **fractal features are integrated into the old construction unintentionally**, being used for various reasons and identified only after the completion of the work. In the second form, these characteristics are incorporated into the project in a deliberate way, according to the conscious decision of the builder (Mariano, 2018).

Fractals "are present in architecture in its general context or in different details and singular volumes" (Mariano p.62, 2018). Thus, **fractal ethnogeometry emerges as an expressive element, manifesting itself continuously in architectural works from antiquity to contemporaneity**. Through the centuries, architects and builders have explored complex geometric patterns, often inspired by cultural and religious conceptions. From the pyramids of Egypt to the Gothic cathedrals of Europe, and more recently, in modern skyscrapers and futuristic structures, the presence of fractal geometry is undeniable. This interplay between culture, art, and mathematics not only provides aesthetic beauty but also reflects the depth of human aspirations and the quest to understand the universe through form and structure.

In the architecture of Hindu temples, **fractal ethnogeometry reveals itself as an intrinsic and transcendently woven element in the structures that emerge from devotion and spirituality**. The intersection between fractal geometry and religious beliefs gives rise to a singularity, where every pattern and proportion is imbued with sacred meanings. These constructions are also tangible representations of the metaphysical conceptions of Hindu cosmology. At the same time, they embody a cultural richness, respecting and honoring the multiple origins and traditions that make up the tapestry of the Hindu faith. Thus, fractal ethnogeometry manifests itself not only as an aesthetic expression, but also as a testimony to the diversity and unity that characterize Hindu spirituality.

By exploring the presence of fractal ethnogeometry throughout ancient cultures, its influence is observed not only in architecture, but also in design and

ornamentation. Civilizations such as the Aztecs, Mayans, and Incas, for example, incorporated fractal patterns into their pyramids, temples, and ceremonial artifacts, symbolizing their understanding of the connection between the microcosm and the macrocosm. Similarly, the intricate decorations found on Persian carpets, and in Islamic and Aboriginal Australian art reveal an intuitive understanding and practical application of fractal geometry in different cultural contexts. This cross-cutting presence of fractal ethnogeometry highlights its importance as a universal language, connecting people and societies throughout history.

Fractal ethnogeometry expresses in centennial constructions honors and preserves cultures and beliefs through the spontaneous productions of the people.

These ancestral structures are living guardians of a society's wisdom and identity. By incorporating fractal patterns into their buildings, communities express their understanding of the cosmos and its relationship to the divine, and weave a tapestry of meanings that spans generations. It is in this link between the sacred and the secular, between the ancient and the contemporary, that fractal ethnogeometry transcends the boundaries of time, inspiring us to appreciate and preserve the cultural richness of humanity.

In an increasingly globalized and interconnected world, the importance of fractal ethnogeometry today becomes undeniable. In addition to its aesthetic beauty and cultural value, this form of architectural expression serves as a bridge between the past and the present, connecting us to the forgotten architectural memories of societies whose knowledge has been excluded. By rescuing and valuing such architectural works, from different cultures, we not only recognize the diversity and richness of human heritage, but also learn important lessons about resilience, innovation, and the ability to adapt over the centuries. More than simply historical monuments, these structures carry with them the accumulated wisdom of past civilizations, offering us valuable insights into contemporary challenges. Therefore, by preserving and studying fractal ethnogeometry and its manifestations in architecture, we are not only celebrating cultural diversity, but also enriching our understanding of the world and enriching our legacy for future generations.

NC2 – FRACTAL ETHNOGEOMETRY EXPRESSED IN CONTEMPORARY ARCHITECTURAL WORKS, DESIGN, CITIES AND NATURAL LANDSCAPES

In the context of contemporary architecture, practitioners are constantly looking for ways to transcend conventional boundaries, exploring new approaches that connect

cultural roots with the cutting edge of design. In this sense, fractal ethnogeometry is inserted in architectural works as a unique and captivating expression. Imagine yourself immersed in a cityscape where buildings are not mere structures, but rather tangible manifestations of a deep cultural identity. Here, ***fractal ethnogeometry manifests itself in contemporary works, which are characterized by a creative freedom and incorporation of external values.*** These visionary architects not only design functional spaces, but also tell stories through lines, shapes, and patterns that echo ancestral traditions. Every detail is meticulously crafted to evoke a sense of belonging and respect for cultural diversity, resulting in environments that transcend time and space, connecting past, present, and future in a harmonious dance of geometry, fractals, and creativity.

In the relentless pursuit of innovation in modern architecture, fractal ethnogeometry emerges as a powerful source of inspiration. Present in contemporary works, this approach challenges traditional paradigms by incorporating traces of fractal structures into its design. Imagine walking through a futuristic city, where skyscrapers no longer follow the rigidity of straight lines, but rather the fluidity and complexity of fractals. Here, ***fractal ethnogeometry reveals its striking presence, reinventing urban spaces with a bold and innovative aesthetic.*** Each building is a work of art in itself, with shapes that repeat at different scales, creating a sense of harmony and continuity. This integration between ethnogeometry and culture not only elevates the concept of architectural design, but also enriches the human experience by providing environments that transcend the ordinary, inviting us to explore and contemplate the beauty of fractal complexity in our built environment.

The integration of ***fractal ethnogeometry in the contemporary urban scenario transcends the mere concept of design, revealing itself as a powerful tool for recovering, valuing and encouraging cities.*** Here, fractal ethnogeometry is not just an aesthetic, but an invitation to the rediscovery and renewal of urban spaces. The intricate geometric patterns that adorn facades and squares are symbols of identity and belonging. They echo the past while pointing to the future potential of cities, inspiring a new generation of architects, urban planners and citizens to engage in the positive transformation of their urban environments. In this context, fractal ethnogeometry beautifies cities and strengthens, promoting inclusion, diversity, and sustainability at its core.

Fractal ethnogeometry shows itself in the contemporary city as a fractal multiple, exhibiting fractal structures in street compositions. In cities such as Barcelona and its famous Passeig Avenue (Gràcia), the outline of which is shown in Figure

6, we can observe a striking example of this integration. The sidewalks, designed with mosaics of tiles that follow complex geometric patterns, reflect the cultural richness of the region and the practical application of fractal ethnogeometry (Martins and Librantz, 2006). This is just one example of how fractal ethnogeometry is present and active in the streets of contemporary cities, enriching the built environment and also the human experience in urban space.

Figure 6 – Passeig Avenue (tiles with complex geometric patterns)



Source: <https://www.pinterest.ch/pin/350154939752431908/>

The presence *of fractal ethnogeometry in the composition of contemporary buildings reveals itself as a testimony of the incessant search for fractal representation in the architectural scenario*, as well as *emerging as a striking element in the facades of buildings*. In cities such as Singapore, the iconic Marina Bay Sands exhibited in Figure 6 emerges as a striking example of this concept, as well as the Burj Khalifa building (Figure 7), in Dubai, whose façade features a series of geometric patterns inspired by Arab culture and fractal geometry.

Figura 6 – Manina Bay Sands



Source: <https://litemerarosa.com/wp-content/uploads/2019/08/Foto-2019-05-28-09-57-23.jpg>

Figura 7 – Burj Khalifa



Source: <https://cdn.culturagenial.com/es/imagenes/burj-khalifa-og.jpg>

Imagine yourself in front of a building with a singular appearance, where every detail of the façade seems to follow a geometric pattern that is repeated at different scales, creating a sense of harmony and beauty. This is the manifestation of fractal ethnogeometry, expressed in the figures above, where elements such as lines, textures, and shapes intertwine to form a visually captivating composition. The sinuous lines and repetitive patterns create a unique aesthetic that stands out in the urban landscape, reflecting the cultural richness of the region and demonstrating the practical application of fractal ethnogeometry in contemporary architecture. Thus, fractal ethnogeometry reveals itself as a decorative element, but also as a powerful tool for the creation of urban spaces where the beauty of geometry merges with the richness of culture, history and man's capacity for creation.

The expression of ***fractal ethnogeometry in the landscapes and built or natural environments of different cities reveals itself as a synthesis between culture, nature and design***. Imagine yourself walking through the winding streets of Kyoto, where Zen gardens and ancient temples coexist harmoniously with modern skyscrapers, creating a cityscape that echoes the fractal patterns found in the surrounding nature. Similarly, in Brasilia, the Brazilian capital designed by Oscar Niemeyer, the fluid geometry and sinuous curves of public buildings dialogue with the cerrado that stretches as far as the eye can see, forming a unique symbiosis between the artificial and the natural. In each of these cities, fractal ethnogeometry manifests itself in different ways, but always as a link between cultural traditions, natural beauty, and urban design innovation.

The Guggenheim Museum Bilbao exhibited in Figure 8, designed by renowned architect Frank Gehry, is an undisputed landmark in contemporary architecture and an

inspiring example of how **fractal ethnogeometry can be expressed in both the external and internal construction of a building**. Located on the banks of the Nervión River, the museum stands out for its organic and fluid forms, which resemble a dynamic and futuristic sculpture. This iconic architecture incorporates fractal elements into its structure, with curved lines and irregular surfaces that create a sense of movement and flow, evoking patterns found in nature and different cultural traditions (Rocha and Silva, 2017).

Figure 8 – The Guggenheim Museum in Bilbao



Source: <https://wallhaven.cc/w/lq5l6l>

According to Rocha and Silva (2017), upon entering the museum, this same fractal aesthetic permeates the internal spaces, creating an immersive and engaging experience for visitors. The galleries and exhibition halls feature a variety of shapes and textures, with undulating walls and cascading ceilings that provide a sense of continuity and connection to the surrounding environment. This integrated approach to fractal ethnogeometry in the architecture of the Guggenheim Museum Bilbao enriches the aesthetic experience and also underscores the importance of recognizing and celebrating cultural and natural diversity in our contemporary architectural creations (Rocha e Silva, 2017).

The contemporary works of architect Steven Holl are true manifestations of fractal ethnogeometry, where fractal patterns play a crucial role in the uniqueness and expression of his architectural creations. By exploring the limits and interactions of fractal patterns, Holl challenges established conventions, creating spaces that are both innovative and immersive, such as the Nanjing Museum of Contemporary Art (Figure 9) in China and Simmons Hall (Figure 10) at the Massachusetts Institute of Technology (MIT). The first has a façade adorned with geometric patterns that are repeated at different scales, creating a sense of movement and fluidity (Brito, 2020). The second is ethnogeometry, incorporating itself into the very structure and function of buildings. Fractal patterns are

used to maximize energy efficiency and natural lighting, creating spaces that are both sustainable and aesthetically captivating (Brito, 2020).

Figure 9 – Nanjing Museum of Contemporary Art



Source: <https://imagens-revista-pro.vivadehora.com.br/uploads/2018/03/steven-holl-nanjing.jpg>

Figura 10 – Simmons Hall



Source: <https://imagens-revista-pro.vivadehora.com.br/uploads/2018/03/steven-holl-nanjing.jpg>

Like Steven Holl, the renowned nineteenth-century art critic, writer, and philosopher, John Ruskin, left a significant legacy that echoes to the present day, especially in the field of architecture and design, introducing concepts that are now considered fundamental to understanding the relationship between art, culture, and society. Among these concepts is the idea of ***ethnogeometry, which is expressed in a striking way in his works***. For Ruskin, geometry was not just a matter of proportion and form, but rather a symbolic language that carried with it the cultural traditions and values of a society (Carvalho, 2017). In his critical analyses of Gothic architecture, for example, Ruskin identified complex geometric patterns that reflected the builders' technical skill, spirituality, and worldview of a bygone era. In addition, in his own artworks and writings, Ruskin incorporated geometric elements inspired by diverse cultures and traditions, creating a rich and multifaceted visual language that challenged the boundaries between the local and the global, the ancient and the modern (Carvalho, 2017).

With the foregoing here, it is understood that fractal ethnogeometry expresses and contributes to an innovative synthesis between tradition and avant-garde, where it is employed not only as a decorative element, but also as a guiding principle that contributes to the uniqueness and relevance of architectural creations.

NC3 – FRACTAL ETHNOGEOMETRY AS THE CONSTRUCTION OF INTERDISCIPLINARY KNOWLEDGE IN ARCHITECTURE AND IN THE CLASSROOM, USING TECHNOLOGY

Over the centuries, many cultures around the world have seen their architectural traditions and cultural customs being suppressed or marginalized due to a variety of influences, including colonization, modernization, and globalization. However, fractal ethnogeometry offers a way to resist this cultural homogenization, allowing communities to rebuild and revitalize their cultural practices through architecture. This approach ***manifests itself in a dynamic and inclusive way through activities and workshops that seek to rescue, through architecture, the customs and traditions of different cultures that have been historically excluded over the centuries.*** These initiatives represent a powerful tool for the preservation and appreciation of cultural diversity, while promoting inclusion and intercultural dialogue. In these activities, fractal ethnogeometry can be used as a common language that transcends cultural barriers, allowing each participant to contribute their knowledge and experiences. These activities and workshops can rescue the customs and traditions of marginalized cultures, as well as promote the empowerment and self-expression of the communities involved.

Fractal ethnogeometry emerges as an innovative and inspiring approach to conceiving complex and distinctive contemporary works in architecture and design. By taking advantage of the benefits of generative systems, this methodology combines geometric elements inspired by different cultures and traditions with computational algorithms, resulting in creations that enhance human creativity. The interplay between the local and the global, the traditional and the modern, allows for the design of architectural spaces that visually impress and fosters a deeper and more meaningful connection between architecture and the communities that experience it.

In addition, ***fractal ethnogeometry also reveals itself as an essential didactic content for educational projects in architecture and mathematics.*** By introducing students to the world of fractals and their application in modern architecture, especially

through Computation Applied to Architecture and Design (CAAD) tools, fractal ethnogeometry enriches the educational curriculum. Allowing students to virtually experiment with fractal shapes, patterns, and structures, the approach provides a deeper and more comprehensive understanding of fractal principles, preparing future professionals to address the challenges and opportunities of contemporary architectural practice in a more informed and creative way.

However, ***fractal ethnogeometry emerges as a bridge between different fields of knowledge, promoting the construction of interdisciplinary knowledge by uniting mathematical theory with the practice of architecture and urbanism.*** The use of these complex geometric patterns offers a practical application of mathematical concepts, enriching the aesthetic aspect and the functionality and sustainability of architectural and urban projects.

Thus, ***ethnogeometry reveals itself as an essential proposal to give visibility to distinct knowledges that, throughout history, have been systematically excluded and marginalized.*** This approach recognizes the importance of traditional and cultural knowledge, often disregarded in academic and institutional contexts, and seeks to rescue and value these forms of wisdom. By incorporating geometric and cultural elements from diverse communities and traditions, ethnogeometry can broaden understanding of the world around us and promote inclusivity and recognition of the diversity of perspectives and experiences. In this way, ethnogeometry constitutes a tool for the analysis and understanding of geometry in different cultures and as a powerful instrument for the promotion of epistemic justice and equity in the field of human knowledge.

In this way, by incorporating the principles of Fractal Ethnogeometry into Architecture and Urbanism education, we create opportunities to promote inclusion and diversity in our teaching and learning spaces. It is through this dialogue between mathematics, architecture, and cultural diversity that we can build a fairer and more equitable environment for all, understanding that skin color, accent, and native language are just details in a complex field of shared identities.

Fractal ethnogeometry offers an innovative and rich perspective for teaching mathematics and geometry, integrating geometric concepts with cultural practices and the fractal nature of patterns found in various cultures around the world. This approach recognizes cultural diversity as well as the intrinsic complexity of geometric patterns, connecting abstract concepts with tangible experiences and specific ethnic contexts.

By exploring the fractal patterns present in cultural manifestations, such as indigenous art, traditional architecture, and weaving, students can understand how geometry is intrinsically linked to the identity and cultural expression of different peoples. This can make mathematics more relevant and meaningful to students, as well as it can foster a deeper appreciation of cultural diversity.

In addition, fractal ethnogeometry broadens perspectives on what geometry is, challenging traditional and Eurocentric notions. By recognizing and valuing the different ways of understanding and representing shapes and patterns, students are encouraged to think more critically and creatively about geometric concepts. They can explore how different cultures interpret fractal patterns present in nature, expanding their understanding of geometry.

The integration of cultural practices related to fractal ethnogeometry into classroom activities provides an opportunity for students to realize their mathematical knowledge in real-world contexts. Through projects that involve creating fractal art inspired by specific cultures or analyzing geometric patterns in traditional architectural structures, students develop problem-solving and mathematical reasoning skills while exploring and valuing cultural diversity.

FINAL CONSIDERATIONS

The analysis of the three nuclei of fractal ethnogeometry shows elements with which we weave an understanding of *how fractal ethnogeometry is present in the architecture made by different peoples*. It reveals a journey that crosses different eras and contexts, highlighting its relevance throughout history and in contemporaneity. From the spontaneous architectural works of antiquity to the architectural creations, urban design, and contemporary natural landscapes, the presence of fractal ethnogeometry is remarkable, reflecting the intricate relationship between culture, geometry, and the built environment. In addition, the role of fractal ethnogeometry in building interdisciplinary knowledge in architecture, especially through the integration of technology in the classroom, stands out as an important advance in architectural education, and can provide students with a deeper and more practical understanding of geometric concepts in real contexts. Thus, fractal ethnogeometry not only enriches our understanding of geometries and cultures, but also inspires innovation and interdisciplinary dialogue, strengthening the bonds between past,

present, and future in the quest for more inclusive, sustainable, and meaningful built spaces.

In view of this, this work is relevant, because by focusing on ethno geometries, on the ways of showing the variety of fractals in the architectural works carried out by peoples of different cultures, in different spaces and times, it can be possible for the reader of this research, constituted by all the experiences that make him a historical-cultural being, to understand the presence of the other in the various possible perspectives: another subject, another culture, another space, another historicity, other objectivities and subjectivities. This understanding consolidates in the subject an expansion of himself, as a being in the world.

Fractals, and they inserted in architectural works, are evidences of the cultural diversity mentioned above, they are creations that encompass space and time and, when studied outside this spatial-temporal context, allow a glimpse of aspects of the societies in which they originated, of their cultural practices.

The perception of the other is structuring of Ethnomathematics, as it configures the realization that the other does or can do mathematics in different ways. If this doing involves fractals in architecture, there may also be differences in evidence that, when focused, can show rules and techniques for the production of mathematics correlated to the socio-historical-temporal environment of the creation of this architecture. In the exercise of a subject's attention to diversity, to multiculturalism, becoming aware of himself and his historicity, in order to understand what is different, without subjugating it, an a priori movement of intentionality takes place, of being focused on the possibilities of mathematics, geometries, ways of making and practicing ethno geometries, as in architectural projects and constructions.

Thus, reinforcing the relevance of this work, it is understood that it is of a decentralizing character by allowing a redirection of previously hegemonic European pedagogical proposals, for the resignification of approaches having fractals as one of the instruments. Thus, to assume the presence of ethno geometries, detaching oneself from the affirmation of a universal mathematics or geometry, is to build a path of decoloniality, going against tacitly placed standards and concepts.

It is important to think about the teaching of mathematics from this perspective, of decoloniality, because the school period and what is learned in it constitute the person who, later, will make decisions related to their surroundings, to society, from the simplest to the most complex, depending on the profession. Many of these decisions can also go against

prejudice and racism, because the work with fractals in architectural works around the world, from the perspective of Ethnogeometry, can constitute in students the understanding that they are people immersed in various cultures, due to the characteristic of the construction of Brazilian society. Thus, they can understand that in this cultural immersion, which is shared every day, the color of the skin, the accent, the native language, are just details and that, therefore, anti-black racism, prejudices against indigenous people and other groups must be fought, so that the spaces, including those for the production and dissemination of knowledge, are democratic, of access to all, without distinction.

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