Revista ARACÊ
ISSN: 2358-2472

THE EFFECTS OF CLIMATE CHANGE ON VULNERABLE WORKERS' PERFORMANCE AND INCOME

https://doi.org/10.56238/arev6n4-069

Date of submission: 05/11/2024 **Date of publication:** 05/12/2024

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ABSTRACT
This study models and analyzes how climatic variability impacts the productivity of vulnerable and non-vulnerable workers, empha sizing disparities driven by climate sensitivity. Vulnerable workers, heavily reliant on stable environmental conditions, face amplified pro ductivity losses due to deviations from optimal temperature, while non-vulnerable workers, shielded by technology, maintain stable out puts. A Cobb-Douglas production function with a CES framework quantifies these effects, revealing significant declines in wages and pro ductivity for vulnerable groups as climate variability increases. The research integrates theories of heat stress, psychosocial work environ ments, and climate justice to explain these impacts. Results highlight how firms adapt by substituting vulnerable labor with non-vulnerable workers, exacerbating employment and income inequalities. This dy namic aligns with climate justice concerns, as low-income, outdoor intensive roles suffer the most, underscoring the need for equitable adaptation policies.

Keywords: Climate Change. Vulnerable Workers. Productivity. Income Inequality. Adaptation Policies.

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ISSN: 2358-2472

INTRODUCTION

This article aims to model and investigate the differential effects of climatic factors on the productivity of vulnerable and non-vulnerable workers. Vulnerable workers, whose productivity is heavily contingent upon favorable environmental conditions, particularly temperature, face significant challenges as climate change intensifies. Deviations from ideal temperature ranges can lead to declines in productivity due to increased physical exertion, health risks, and reduced efficiency in outdoor tasks, such as those found in agriculture and construction (Zadow et al., 2017; Manapragada et al., 2019). Conversely, non-vulnerable workers, shielded by technological advancements and infrastructural support, maintain stable productivity levels despite climatic variations (Khan, 2016; Jameson & Parkinson, 2021). This dichotomy underscores the necessity of examining labor market dynamics through the lens of climate change, particularly as it relates to social inequalities and economic implications.

The theoretical framework for this investigation draws upon several key concepts, including heat stress models, psychosocial theories of work, and climate justice. Heat stress models elucidate the physiological and psychological impacts of temperature extremes on worker performance, highlighting how increased heat can exacerbate fatigue and reduce cognitive function (Cáceres et al., 2022; Lawrie et al., 2018). These models are particularly relevant for vulnerable workers, who may experience a pronounced decline in productivity as temperatures deviate from optimal levels. Moreover, the psychosocial theory of work and climate change posits that environmental stressors can adversely affect mental health, safety perceptions, and overall work engagement, further compounding the challenges faced by vulnerable workers (Dollard et al., 2017; Ulhassan et al., 2014). This interplay between climate stress and worker well-being necessitates a comprehensive understanding of how climatic factors shape labor productivity and the associated economic consequences.

In addition to the direct impacts of climate change on worker productivity, the concept of climate justice highlights the structural inequalities that exacerbate vulnerabilities among certain worker groups. Low-income workers, often employed in climate-sensitive sectors, are disproportionately affected by climatic fluctuations, leading to heightened job insecurity and income instability (Toropova et al., 2023; Akanni et al., 2021). This relationship between income and climate vulnerability underscores the need for policies that address these disparities and promote equitable adaptation strategies. Furthermore, the

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social capital and community resilience theory provides a framework for understanding how community networks can support vulnerable workers in coping with climatic adversities, thereby enhancing their overall resilience (Taibi et al., 2022; Seddighi et al., 2020). By integrating these theoretical perspectives, this article aims to elucidate the complex dynamics between climate change, labor productivity, and social inequalities.

Mathematical modeling serves as a critical tool in this investigation, allowing for the quantification of the impacts of climatic factors on worker productivity. The proposed model employs a Cobb-Douglas production function, which incorporates labor inputs from both vulnerable and non-vulnerable worker groups, utilizing a constant elasticity of substitution (CES) function to analyze labor allocation shifts in response to climatic changes (Backström & Berglund, 2022; Saastamoinen et al., 2013). This modeling approach enables the exploration of how temperature deviations influence productivity and wages, particularly for vulnerable workers, while also examining the broader implications for firms reliant on these labor segments. The findings are expected to reveal significant declines in productivity and wages for vulnerable workers as temperature deviations increase, leading to shifts in labor allocation favoring non-vulnerable workers and exacerbating existing employment and income inequalities (Trott, 2019; Miller et al., 2023).

The economic implications of these dynamics are profound, particularly for firms that depend heavily on vulnerable workers. As productivity declines and wages decrease, firms may face increased costs associated with labor replacement and reduced overall productivity (Tyagi & Carley, 2021; Lehuluante et al., 2011). This deterioration in the relationship between profits and total labor costs may compel firms to reevaluate their labor strategies, potentially leading to a greater reliance on non-vulnerable workers and further entrenching social inequalities within the labor market (Kori, 2023; Ellehave, 2023).

This study adds to the expanding research on the relationship between climate change, labor markets, and economic dynamics, providing practical direction for policymakers and business leaders to address these multifaceted challenges. The article examines how climatic factors disproportionately impact vulnerable and non-vulnerable workers, exploring the complex connections between climate change, labor productivity, and social inequalities through a focused theoretical framework and mathematical modeling.

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MODEL

The proposed model investigates the differential impact of climate factors on the productivity of two distinct groups of workers: *vulnerable workers* (V) and *non-vulnerable workers* (N). Vulnerable workers (V) are those whose productivity is significantly affected by environmental conditions, particularly temperature deviations from an optimal range. Non-vulnerable workers (N), in contrast, maintain stable productivity regardless of external climate factors.

The productivity of vulnerable workers (V) depends heavily on conditions being within a specific range of temperature, T_0. As the actual temperature deviates from this optimal point, denoted as $T_d = |T - T_0|$, their productivity declines due to increased physical strain, health risks, or reduced efficiency in performing climate-exposed tasks. This group often includes workers in outdoor-intensive roles or sectors reliant on stable environmental conditions, such as agriculture, construction, and other weather-dependent industries.

Non-vulnerable workers (N), however, are shielded from the direct effects of temperature fluctuations due to the nature of their roles, which are typically performed in controlled or insulated environments. These workers benefit from technological and infrastructural protections that maintain their productivity irrespective of external conditions. Examples include workers in office settings, manufacturing processes conducted indoors, or roles with minimal physical exposure to the environment.

The model incorporates these differences by introducing a penalty on the productivity of vulnerable workers (V) as a function of the temperature deviation (T_d^2). As T_d increases, the penalty becomes more pronounced, reflecting the exponential nature of adverse effects from extreme environmental conditions. Non-vulnerable workers (N) experience no such penalty, maintaining a consistent level of output regardless of T_d.

Firms respond to these differences by adjusting their labor allocation between V and N. The elasticity of substitution between these two groups plays a critical role in determining how firms adapt to climate-induced changes in productivity. If substitution is relatively easy, firms may shift their reliance from vulnerable workers to non-vulnerable workers, amplifying inequalities in employment and income between the two groups. Conversely, if substitution is limited, firms may face higher costs associated with reduced productivity in climate-sensitive roles.

The detailed structure of the model, including its mathematical formulation, productivity penalties, firm decision-making processes, and labor allocation dynamics, will

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be presented in the following sections. This framework sets the stage for exploring how climate conditions influence labor markets and income distribution, particularly under the constraints of climate-induced vulnerabilities.

MATHEMATICAL FORMULATION

The model is structured to capture the dynamics between production, labor allocation, and the impacts of climate sensitivity on two groups of workers: *vulnerable workers* (V) and *non-vulnerable workers* (N). The goal is to formalize how deviations in temperature and firms' market strategies interact to determine labor market outcomes.

The production process is modeled using a Cobb-Douglas function, aggregating capital and labor. Labor, however, is not homogeneous; it consists of V and N, whose productivity levels differ due to environmental factors. The aggregation of these two types of labor is done through a CES function, allowing for flexibility in substitution between V and N. Vulnerable workers, unlike non-vulnerable workers, face a productivity penalty as a result of temperature deviations from an optimal range.

The productivity of non-vulnerable workers is primarily determined by human capital and technological factors, making them largely unaffected by environmental conditions. In contrast, the productivity of vulnerable workers is directly penalized based on the square of the absolute deviation of the current temperature from the optimal temperature. This penalty grows exponentially as temperature deviates further from the ideal, reflecting how extreme conditions disproportionately harm climate-sensitive labor roles.

Firms, seeking to maximize profits, allocate labor between V and N based on their relative productivity and costs. Market power allows firms to set wages below workers' marginal productivity, which creates an additional layer of inequality. The elasticity of substitution between V and N influences how firms adjust their reliance on these groups in response to changes in climate or wage costs.

EQUATIONS OF THE MODEL

1. Production Function:

$$Q = A \cdot K^{\alpha} \cdot L^{1-\alpha},$$

where:

- Q is total production,
- A represents the technological level,

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- K is capital input,
- L is total effective labor input, and
- α is the elasticity of production with respect to capital.

2. Labor Aggregation (CES):

$$L = \left[\theta L_N^{\rho} + (1-\theta) L_V^{\rho}\right]^{\frac{1}{\rho}},$$

where:

- L_N and L_V are labor inputs of non-vulnerable and vulnerable workers, respectively,
- θ is the weight of non-vulnerable workers in the labor aggregation,
- $\rho = 1 - \frac{1}{\sigma}$, where $\sigma > 0$ is the elasticity of substitution between L_N and L_V.

3. Productivity of Non-Vulnerable Workers (N):

$$P_N = A_N \cdot X_N^{\phi_N},$$

where:

- A_N is the technological factor for N,
- X_N is the human capital of N,
- ϕ_N is the elasticity of productivity with respect to human capital.

4. Productivity of Vulnerable Workers (V):

$$P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2},$$

where:

- A_V is the technological factor for V,
- X_V is the human capital of V,
- ϕ_V is the elasticity of productivity with respect to human capital,
- $T_d = |T - T_0|$ is the absolute deviation of temperature from the optimal level (T_0),
- β is the sensitivity of productivity to temperature deviations.

5. Wage Determination:

$$w_N = \frac{P_N}{\eta}, \quad w_V = \frac{P_V}{\eta},$$

where:

- w_N and w_V are the wages of N and V, respectively,
- $\eta > 1$ represents the degree of market power, with $\frac{1}{\eta}$ indicating the fraction of productivity paid as wages.

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6. Profit Maximization: Firms maximize profits (Π) by choosing L_N, L_V, and K to maximize:

$$\Pi = P \cdot Q - w_N \cdot L_N - w_V \cdot L_V - r \cdot K,$$

where:

- P is the price of output,
- r is the cost of capital.

Labor Supply and Demand

In this subsection, we formalize the demand and supply functions for labor, distinguishing between *vulnerable workers* (V) and *non-vulnerable workers* (N). These functions determine the equilibrium levels of wages and employment for each group, based on their respective productivity, sensitivity to climate conditions, and firms' profit-maximizing behavior.

Labor Demand

Labor demand is derived from the firm's profit maximization problem. Firms choose the level of employment for L_V and L_N to minimize costs while maintaining a given level of production. The demand for labor is influenced by the relative productivity (P_V and P_N) and the wages (w_V and w_N).

Demand for Vulnerable Workers (L_V):

$$L_V = \left(\frac{(1-\theta)}{w_V^{\sigma}}\right) \cdot (P_V \cdot Q)^{\frac{\sigma}{1+\sigma}},$$

where:

- Q is the total production,
- $P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}$ is the productivity of vulnerable workers, and
- w_V is the wage paid to vulnerable workers.

Demand for Non-Vulnerable Workers (L_N):

$$L_N = \left(\frac{\theta}{w_N^{\sigma}}\right) \cdot (P_N \cdot Q)^{\frac{\sigma}{1+\sigma}},$$

where:

- $P_N = A_N \cdot X_N^{\phi_N}$ is the productivity of non-vulnerable workers, and
- w_N is the wage paid to non-vulnerable workers.

The demand equations incorporate σ, the elasticity of substitution between L_V and L_N. Higher values of σ imply that firms can more easily substitute one type of labor for the other in response to changes in wages or productivity.

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Labor Allocation:

The relative allocation of L_N and L_V is determined by:

$$\frac{L_N}{L_V} = \left(\frac{\theta}{1-\theta}\right) \cdot \left(\frac{w_V}{w_N}\right)^{\sigma}.$$

This system of equations captures the interplay between production, climate conditions, labor allocation, and wage-setting behavior under firm market power. In the subsequent sections, we will analyze the implications of these relationships under different climatic and economic scenarios.

Labor Supply

The supply of labor for each group depends on the willingness of workers to participate in the labor market, which is influenced by their wages. The supply functions assume an elasticity of labor supply (ϵ), which measures how responsive workers are to changes in wages.

Supply of Vulnerable Workers (L_V):

$$L_V = L_{V0} \cdot \left(\frac{w_V}{w_{V0}}\right)^{\epsilon},$$

where:

- L_{V0} is the baseline supply of vulnerable workers,
- w_V is the current wage, and
- w_{V0} is the reference wage for vulnerable workers.

Supply of Non-Vulnerable Workers (L_N):

$$L_N = L_{N0} \cdot \left(\frac{w_N}{w_{N0}}\right)^{\epsilon},$$

where:

- L_{N0} is the baseline supply of non-vulnerable workers,
- w_N is the current wage, and
- w_{N0} is the reference wage for non-vulnerable workers.

The parameter ϵ reflects the elasticity of labor supply. A higher ϵ implies that workers are more responsive to changes in wages.

Labor Market Equilibrium

Equilibrium in the labor market occurs when the demand for labor equals the supply, for both vulnerable and non-vulnerable workers. The equilibrium conditions are given by:

Equilibrium for Vulnerable Workers (L_V):

$$\left(\frac{(1-\theta)}{w_V^{\sigma}}\right) \cdot (P_V \cdot Q)^{\frac{\sigma}{1+\sigma}} = L_{V0} \cdot \left(\frac{w_V}{w_{V0}}\right)^{\epsilon}.$$

Equilibrium for Non-Vulnerable Workers (L_N):

$$\left(\frac{\theta}{w_N^{\sigma}}\right) \cdot (P_N \cdot Q)^{\frac{\sigma}{1+\sigma}} = L_{N0} \cdot \left(\frac{w_N}{w_{N0}}\right)^{\epsilon}.$$

These equations determine the equilibrium wages (w_V, w_N) and employment levels (L_V, L_N) as functions of productivity, climate conditions (via T_d^2), and the elasticity parameters (σ, ϵ).

This framework provides the foundation for analyzing how climate-induced productivity changes influence labor demand and supply dynamics, and how they propagate through wages and employment outcomes in the labor market.

THEORETICAL RESULTS

Lemma 1: The Productivity of Vulnerable Workers Decreases as T_d Increases

Statement: The productivity of vulnerable workers (P_V) decreases when the temperature deviation (T_d) increases. This relationship is modeled through an exponential penalty term $e^{-\beta T_d^2}$, which captures the non-linear effects of temperature deviations on productivity. The greater the deviation from the optimal temperature, the sharper the decline in productivity.

Proof: The productivity of vulnerable workers is defined as:

$$P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2},$$

where:

- A_V is the technological factor specific to vulnerable workers,
- X_V is the human capital of vulnerable workers,
- ϕ_V is the elasticity of productivity with respect to human capital,
- $T_d = |T - T_0|$ is the absolute deviation of temperature (T) from the optimal temperature (T_0),
- $\beta > 0$ is the sensitivity of productivity to temperature deviations.

To determine the behavior of P_V as T_d increases, we calculate the partial derivative of P_V with respect to T_d:

$$\frac{\partial P_V}{\partial T_d} = A_V \cdot X_V^{\phi_V} \cdot \frac{\partial}{\partial T_d}\left(e^{-\beta T_d^2}\right).$$

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Using the chain rule to differentiate $e^{-\beta T_d^2}$:

$$\frac{\partial}{\partial T_d}\left(e^{-\beta T_d^2}\right) = e^{-\beta T_d^2} \cdot \frac{\partial}{\partial T_d}(-\beta T_d^2).$$

The derivative of $-\beta T_d^2$ is:

$$\frac{\partial}{\partial T_d}(-\beta T_d^2) = -2\beta T_d.$$

Substituting back, we get:

$$\frac{\partial P_V}{\partial T_d} = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2} \cdot (-2\beta T_d).$$

Simplifying:

$$\frac{\partial P_V}{\partial T_d} = -2\beta T_d \cdot A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}.$$

Analysis of the Derivative:

- The term $-2\beta T_d$ is negative for $T_d > 0$, indicating that $\frac{\partial P_V}{\partial T_d} < 0$.
- Therefore, as T_d increases, P_V decreases exponentially due to the penalty term $e^{-\beta T_d^2}$.

The productivity of vulnerable workers (P_V) decreases with increasing temperature deviations (T_d). This decline becomes more pronounced as T_d grows larger, reflecting the compounding effects of extreme environmental conditions.

Lemma 2: Temperature Deviations Negatively Impact the Ratio of Non-Vulnerable to Vulnerable Workers

Statement: An increase in the temperature deviation (T_d) negatively impacts the ratio of non-vulnerable to vulnerable workers ($\frac{L_N}{L_V}$). As T_d increases, the wage of vulnerable workers (w_V), which depends on their productivity, decreases exponentially, reducing the relative attractiveness of vulnerable workers for firms.

Proof: The ratio of non-vulnerable to vulnerable workers is given by:

$$\frac{L_N}{L_V} = \left(\frac{\theta}{1-\theta}\right) \cdot \left(\frac{w_V}{w_N}\right)^{\sigma},$$

where:

- θ is the share of non-vulnerable workers,
- w_V and w_N are the wages of vulnerable and non-vulnerable workers, respectively,
- $\sigma > 0$ is the elasticity of substitution between L_N and L_V.

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The wage of vulnerable workers (w_V) depends on their productivity (P_V), which is affected by temperature deviations (T_d):

$$w_V = \frac{P_V}{\eta}, \quad \text{where} \quad P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}.$$

The wage of non-vulnerable workers (w_N) is assumed to be constant, as it is not affected by T_d. Substituting w_V into $\frac{L_N}{L_V}$, we have:

$$\frac{L_N}{L_V} = \left(\frac{\theta}{1-\theta}\right) \cdot \left(\frac{\frac{P_V}{\eta}}{w_N}\right)^{\sigma}.$$

Simplifying:

$$\frac{L_N}{L_V} = \left(\frac{\theta}{1-\theta}\right) \cdot \left(\frac{A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}}{\eta \cdot w_N}\right)^{\sigma}.$$

To analyze the impact of T_d on $\frac{L_N}{L_V}$, we compute the derivative of $\frac{L_N}{L_V}$ with respect to T_d:

$$\frac{\partial}{\partial T_d}\left(\frac{L_N}{L_V}\right) = \frac{\partial}{\partial T_d}\left[\left(\frac{\theta}{1-\theta}\right) \cdot \left(\frac{A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}}{\eta \cdot w_N}\right)^{\sigma}\right].$$

Since $\frac{\theta}{1-\theta}$ and $\frac{A_V \cdot X_V^{\phi_V}}{\eta \cdot w_N}$ are constants with respect to T_d, the derivative focuses on the exponential term:

$$\frac{\partial}{\partial T_d}\left(\frac{L_N}{L_V}\right) = \sigma \cdot \left(\frac{\theta}{1-\theta}\right) \cdot \left(\frac{A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}}{\eta \cdot w_N}\right)^{\sigma-1} \cdot \left(-2\beta T_d \cdot e^{-\beta T_d^2}\right).$$

Simplifying further:

$$\frac{\partial}{\partial T_d}\left(\frac{L_N}{L_V}\right) = -2\beta\sigma T_d \cdot \left(\frac{\theta}{1-\theta}\right) \cdot \left(\frac{A_V \cdot X_V^{\phi_V}}{\eta \cdot w_N}\right)^{\sigma} \cdot e^{-\sigma\beta T_d^2}.$$

Analysis of the Derivative:

- The term $-2\beta\sigma T_d$ is negative for $T_d > 0$, indicating that $\frac{\partial}{\partial T_d}\left(\frac{L_N}{L_V}\right) < 0$.
- Therefore, as T_d increases, the ratio $\frac{L_N}{L_V}$ decreases due to the exponential penalty on w_V.

As T_d increases, the productivity of vulnerable workers declines exponentially, leading to a proportional reduction in their wages (w_V). This makes vulnerable workers less attractive to firms relative to non-vulnerable workers, reducing the ratio $\frac{L_N}{L_V}$.

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Lemma 3: The Firm's Total Productivity Decreases as T_d Increases

Statement: The firm's total productivity (Q) decreases as the temperature deviation (T_d) increases, provided that the employment of vulnerable workers (L_V) is greater than zero ($L_V \neq 0$). This decline is caused by the exponential reduction in the productivity of vulnerable workers (P_V) due to increasing T_d, which lowers the aggregate labor input (L).

Proof: The firm's production function is given as:

$$Q = A \cdot K^{\alpha} \cdot L^{1-\alpha},$$

where:

- Q is the total production,
- A is the technological factor,
- K is the capital input,
- L is the aggregate labor input,
- α is the elasticity of production with respect to capital.

The aggregate labor input (L) is defined as a CES function of non-vulnerable (L_N) and vulnerable (L_V) workers:

$$L = \left[\theta L_N^{\rho} + (1-\theta) L_V^{\rho}\right]^{\frac{1}{\rho}},$$

where:

- θ is the weight of L_N,
- $\rho = 1 - \frac{1}{\sigma}$, and $\sigma > 0$ is the elasticity of substitution between L_N and L_V.

The productivity of vulnerable workers (P_V) decreases with T_d as:

$$P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}.$$

Since $L_V > 0$, the reduction in P_V reduces the effective contribution of vulnerable workers to the aggregate labor input L. Substituting P_V into L, we see that L_V's effective weight diminishes as T_d increases:

$$L = \left[\theta L_N^{\rho} + (1-\theta)(L_V \cdot P_V)^{\rho}\right]^{\frac{1}{\rho}}.$$

Differentiating Q with respect to T_d, we find:

$$\frac{\partial Q}{\partial T_d} = A \cdot K^{\alpha} \cdot (1-\alpha) \cdot L^{-(\alpha)} \cdot \frac{\partial L}{\partial T_d}.$$

Since $P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}$, its derivative with respect to T_d is:

$$\frac{\partial P_V}{\partial T_d} = -2\beta T_d \cdot A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}.$$

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This implies:

$$\frac{\partial L}{\partial T_d} < 0,$$

as $T_d > 0$ reduces P_V, which in turn lowers L.

Combining these results:

$$\frac{\partial Q}{\partial T_d} < 0.$$

Thus, as T_d increases, the firm's total productivity (Q) decreases due to the compounded reduction in the productivity of vulnerable workers, provided that $L_V \neq 0$.

Theorem 1: Unemployment Increases as T_d Increases (Given $L_V > 0$)

Statement: Unemployment increases as the temperature deviation (T_d) rises, provided that the employment of vulnerable workers ($L_V > 0$). This occurs because the aggregate labor input (L) decreases when T_d increases, leading to less effective labor being utilized in production and a corresponding increase in unemployment.

Proof: The aggregate labor input is defined as:

$$L = \left[\theta L_N^\rho + (1-\theta) L_V^\rho\right]^{\frac{1}{\rho}},$$

where:

- L_N and L_V are the labor inputs of non-vulnerable and vulnerable workers, respectively,
- θ is the weight of non-vulnerable workers,
- $\rho = 1 - \frac{1}{\sigma}$, and $\sigma > 0$ is the elasticity of substitution between L_N and L_V.

The productivity of vulnerable workers (P_V) is given by:

$$P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}.$$

Since P_V decreases exponentially with T_d, the effective contribution of L_V to L diminishes:

$$L_V^{\text{effective}} = L_V \cdot P_V.$$

Substituting $L_V^{\text{effective}}$ into the expression for L, we have:

$$L = \left[\theta L_N^\rho + (1-\theta)(L_V \cdot P_V)^\rho\right]^{\frac{1}{\rho}}.$$

To analyze the effect of T_d on L, we take the derivative of L with respect to T_d, ensuring $L_V > 0$:

$$\frac{\partial L}{\partial T_d} = \frac{\partial}{\partial T_d}\left[\theta L_N^\rho + (1-\theta)(L_V \cdot P_V)^\rho\right]^{\frac{1}{\rho}}.$$

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Using the chain rule:

$$\frac{\partial L}{\partial T_d} = \frac{1}{\rho} \cdot \left[\theta L_N^\rho + (1-\theta)(L_V \cdot P_V)^\rho\right]^{\frac{1}{\rho}-1} \cdot (1-\theta) \cdot \rho(L_V \cdot P_V)^{\rho-1} \cdot \frac{\partial(L_V \cdot P_V)}{\partial T_d}.$$

The term $\frac{\partial(L_V \cdot P_V)}{\partial T_d}$ is given by:

$$\frac{\partial(L_V \cdot P_V)}{\partial T_d} = L_V \cdot \frac{\partial P_V}{\partial T_d}.$$

Since $P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}$, we have:

$$\frac{\partial P_V}{\partial T_d} = -2\beta T_d \cdot A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}.$$

Thus:

$$\frac{\partial(L_V \cdot P_V)}{\partial T_d} = -2\beta T_d \cdot L_V \cdot A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}.$$

Substituting back:

$$\frac{\partial L}{\partial T_d} = \frac{1}{\rho} \cdot \left[\theta L_N^\rho + (1-\theta)(L_V \cdot P_V)^\rho\right]^{\frac{1}{\rho}-1} \cdot (1-\theta) \cdot \rho(L_V \cdot P_V)^{\rho-1} \cdot \left(-2\beta T_d \cdot L_V \cdot A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}\right).$$

Since $L_V > 0$, $\frac{\partial L}{\partial T_d} < 0$, indicating that L decreases as T_d increases. A decrease in L represents a reduction in the effective labor utilized in production. Since total labor supply remains constant, this reduction in utilized labor implies an increase in unemployment. Thus, unemployment increases as T_d rises, provided $L_V > 0$.

Theorem 2: The Ratio Between w_N and w_V Increases with T_d

Statement: The ratio between the wages of non-vulnerable workers (w_N) and vulnerable workers (w_V) increases as the temperature deviation (T_d) increases. Formally, it must be shown that:

$$\frac{\partial}{\partial T_d}\left(\frac{w_N}{w_V}\right) > 0,$$

assuming that w_V is penalized exponentially due to T_d, while w_N is affected only indirectly.

Proof:

The wages of non-vulnerable and vulnerable workers are given by:

$$w_N = \frac{P_N}{\eta}, \quad w_V = \frac{P_V}{\eta},$$

where:

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- P_N and P_V are the marginal productivities of non-vulnerable and vulnerable workers, respectively,
- $\eta > 1$ represents the firm's degree of market power.

The productivity of vulnerable workers is affected directly by T_d, as:

$$P_V = A_V \cdot X_V^{\phi V} \cdot e^{-\beta T_d^2}.$$

Thus, the wage of vulnerable workers depends exponentially on T_d:

$$w_V = \frac{A_V \cdot X_V^{\phi V} \cdot e^{-\beta T_d^2}}{\eta}.$$

The productivity of non-vulnerable workers (P_N) is independent of T_d directly but may be affected indirectly through changes in aggregate labor input (L) or total production (Q). For simplicity, assume P_N remains approximately constant or changes marginally relative to P_V.

The ratio of wages is:

$$\frac{w_N}{w_V} = \frac{P_N}{P_V}.$$

Substituting P_V:

$$\frac{w_N}{w_V} = \frac{P_N}{A_V \cdot X_V^{\phi V} \cdot e^{-\beta T_d^2}}.$$

Taking the derivative with respect to T_d:

$$\frac{\partial}{\partial T_d}\left(\frac{w_N}{w_V}\right) = \frac{\partial}{\partial T_d}\left(\frac{P_N}{A_V \cdot X_V^{\phi V} \cdot e^{-\beta T_d^2}}\right).$$

Since P_N, A_V, and X_V are constants with respect to T_d, the derivative becomes:

$$\frac{\partial}{\partial T_d}\left(\frac{w_N}{w_V}\right) = \frac{P_N}{A_V \cdot X_V^{\phi V}} \cdot \frac{\partial}{\partial T_d}\left(e^{\beta T_d^2}\right).$$

The derivative of $e^{\beta T_d^2}$ is:

$$\frac{\partial}{\partial T_d}\left(e^{\beta T_d^2}\right) = 2\beta T_d \cdot e^{\beta T_d^2}.$$

Substituting back:

$$\frac{\partial}{\partial T_d}\left(\frac{w_N}{w_V}\right) = \frac{P_N}{A_V \cdot X_V^{\phi V}} \cdot 2\beta T_d \cdot e^{\beta T_d^2}.$$

Since all terms (P_N, A_V, $X_V^{\phi V}$, β, and $e^{\beta T_d^2}$) are positive, and $T_d > 0$, it follows that:

$$\frac{\partial}{\partial T_d}\left(\frac{w_N}{w_V}\right) > 0.$$

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Thus, the ratio between w_N and w_V increases with T_d, because w_V decreases exponentially with T_d due to the penalty on P_V, while w_N is affected less directly.

Theorem 3: The Ratio of Firm's Profits to Total Labor Costs Decreases as T_d Increases

Statement: The ratio between the firm's profits (Π) and its total labor costs (C_L) decreases as the temperature deviation (T_d) increases. Formally:

$$\frac{\partial}{\partial T_d}\left(\frac{\Pi}{C_L}\right) < 0,$$

considering that the firm's total output (Q) decreases with T_d, while the total labor costs (C_L) increase due to a higher demand for non-vulnerable workers (L_N).

Proof:

1. *Firm's Profits (Π):* The firm's profit is defined as:

$$\Pi = P \cdot Q - C_L,$$

where:

- P is the price of the product (assumed constant),
- Q is the total production, and
- C_L is the total cost of labor.

The total production (Q) is given by:

$$Q = A \cdot K^{\alpha} \cdot L^{1-\alpha},$$

where L is the aggregate labor input:

$$L = \left[\theta L_N^{\rho} + (1-\theta) L_V^{\rho}\right]^{\frac{1}{\rho}}.$$

As shown in Lemma 3, Q decreases as T_d increases because L_V is penalized exponentially:

$$L_V^{\text{effective}} = L_V \cdot e^{-\beta T_d^2}.$$

Thus, Q decreases with T_d, reducing the firm's overall profits (Π).

2. *Total Labor Costs (C_L):* The total labor cost is given by:

$$C_L = w_N \cdot L_N + w_V \cdot L_V.$$

- w_V is directly penalized by T_d, as P_V decreases exponentially:

$$w_V = \frac{P_V}{\eta}, \quad P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}.$$

- As T_d increases, w_V decreases, but L_N increases due to substitution of non-vulnerable workers for vulnerable workers. This substitution leads to a higher cost contribution from $w_N \cdot L_N$.

Overall, C_L increases with T_d, as the demand for L_N rises and w_N is generally higher than w_V.

3. *Ratio of Profits to Costs* ($\frac{\Pi}{C_L}$)*:* The ratio of profits to costs is:

$$\frac{\Pi}{C_L} = \frac{P \cdot Q - C_L}{C_L}.$$

Taking the derivative with respect to T_d:

$$\frac{\partial}{\partial T_d}\left(\frac{\Pi}{C_L}\right) = \frac{\frac{\partial}{\partial T_d}(P \cdot Q - C_L) \cdot C_L - (P \cdot Q - C_L) \cdot \frac{\partial C_L}{\partial T_d}}{C_L^2}.$$

- As T_d increases:
  - Q decreases, so $\frac{\partial Q}{\partial T_d} < 0$, reducing $P \cdot Q$,
  - C_L increases due to a higher demand for L_N and increasing costs.

The numerator is dominated by the reduction in $P \cdot Q$ and the increase in C_L, both of which contribute negatively to the ratio. Since $C_L^2 > 0$, we conclude:

$$\frac{\partial}{\partial T_d}\left(\frac{\Pi}{C_L}\right) < 0.$$

Thus, the ratio of profits to labor costs decreases as T_d increases. This happens because Q declines due to the reduced productivity of vulnerable workers (P_V), while C_L rises due to the greater reliance on non-vulnerable workers (L_N) and their higher costs.

Theorem 4: The Impact of T_d on Profits (Π) is Amplified for Firms with a Higher Proportion of Vulnerable Workers

Statement: The impact of the temperature deviation (T_d) on the firm's profits (Π) is amplified for firms that have a higher proportion of vulnerable workers (i.e., firms with θ closer to 0). Formally, it must be shown that:

$$\frac{\partial^2 \Pi}{\partial T_d \, \partial \theta} > 0,$$

considering that firms with $\theta \to 0$ are more sensitive to the impact of T_d.

Proof:

1. *Firm's Profits (Π):* The profit function is:

$$\Pi = P \cdot Q - C_L,$$

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where:

- P is the price of the product (assumed constant),
- Q is the firm's total production,
- C_L is the firm's total labor cost.

The total production (Q) is given by:

$$Q = A \cdot K^{\alpha} \cdot L^{1-\alpha},$$

where L is the aggregate labor input:

$$L = \left[\theta L_N^{\rho} + (1-\theta) L_V^{\rho}\right]^{\frac{1}{\rho}}.$$

As T_d increases, the productivity of vulnerable workers (P_V) is penalized exponentially:

$$P_V = A_V \cdot X_V^{\phi_V} \cdot e^{-\beta T_d^2}.$$

This reduces the effective contribution of L_V to L, impacting Q more severely for firms with smaller θ, as their production relies more heavily on vulnerable workers.

2. *Total Labor Costs (C_L):* The total labor cost is:

$$C_L = w_N \cdot L_N + w_V \cdot L_V.$$

- w_V decreases with T_d, but its impact on C_L is smaller for firms with high θ (as they rely less on L_V). Conversely, for firms with $\theta \to 0$, w_V's decline has a more significant effect on C_L because L_V dominates the labor allocation.

3. *Mixed Partial Derivative ($\frac{\partial^2 \Pi}{\partial T_d \, \partial \theta}$):* Taking the first derivative of profits with respect to T_d:

$$\frac{\partial \Pi}{\partial T_d} = \frac{\partial (P \cdot Q)}{\partial T_d} - \frac{\partial C_L}{\partial T_d}.$$

The term $\frac{\partial Q}{\partial T_d}$ is negative, as Q decreases with T_d due to the decline in L_V:

$$\frac{\partial Q}{\partial T_d} = A \cdot K^{\alpha} \cdot (1-\alpha) \cdot L^{-\alpha} \cdot \frac{\partial L}{\partial T_d}.$$

The labor input L depends on θ:

$$\frac{\partial L}{\partial T_d} = \frac{1}{\rho} \cdot \left[\theta L_N^{\rho} + (1-\theta) L_V^{\rho}\right]^{\frac{1}{\rho}-1} \cdot (1-\theta) \cdot \rho \cdot (L_V \cdot P_V)^{\rho-1} \cdot \frac{\partial (L_V \cdot P_V)}{\partial T_d}.$$

Now consider the second derivative of profits with respect to T_d and θ:

$$\frac{\partial^2 \Pi}{\partial T_d \, \partial \theta} = \frac{\partial^2 (P \cdot Q)}{\partial T_d \, \partial \theta} - \frac{\partial^2 C_L}{\partial T_d \, \partial \theta}.$$

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For $\frac{\partial^2 (P \cdot Q)}{\partial T_d \, \partial \theta}$, as $\theta \to 0$, the dependence of Q on L_V becomes stronger, amplifying the impact of T_d on profits. Specifically:

- For low θ, a small change in T_d causes a large reduction in Q, because the firm relies heavily on L_V, which is sensitive to T_d.

For $\frac{\partial^2 C_L}{\partial T_d \, \partial \theta}$, as $\theta \to 0$, the firm's total labor cost is increasingly dominated by L_V, and changes in w_V (which decreases with T_d) amplify the changes in C_L.

Combining these effects:

$$\frac{\partial^2 \Pi}{\partial T_d \, \partial \theta} > 0,$$

because the reduction in Q and the increase in C_L due to T_d are more pronounced for firms with smaller θ.

Thus, the impact of T_d on profits (Π) is amplified for firms with a higher proportion of vulnerable workers ($\theta \to 0$).

CONCLUSION

This study investigates the impact of climate variability on the productivity of vulnerable and non-vulnerable workers. It addresses how deviations in optimal temperature conditions influence labor markets, particularly emphasizing the disparity between these two worker groups.

The theoretical foundation draws upon heat stress models, psychosocial theories of work, and climate justice principles. These frameworks elucidate the physiological, psychological, and economic effects of climate stressors on labor productivity, emphasizing their disproportionate impact on vulnerable workers.

Our research utilizes a Cobb-Douglas production model with a CES function to differentiate the productivity dynamics between vulnerable (L_V) and non-vulnerable workers (L_N). The results demonstrate that productivity for vulnerable workers decreases exponentially as temperature deviates from the optimal range ($T_d = |T - T_0|$), captured by the penalty term $e^{-\beta T_d^2}$, where β represents sensitivity to temperature changes. Firms, facing reduced productivity in vulnerable workers, shift their reliance towards non-vulnerable labor. This transition intensifies existing income and employment inequalities. The elasticity of substitution (σ) further influences this dynamic, dictating the ease with

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ISSN: 2358-2472

which firms replace vulnerable labor. Our findings highlight significant productivity declines and wage disparities, particularly in climate-sensitive industries.

Integrating these results with the theoretical frameworks reveals that climate-induced productivity losses exacerbate social inequalities, aligning with the concept of climate justice. Vulnerable workers, often employed in lower-income, outdoor-intensive roles, face amplified job insecurity and reduced earning potential. Psychosocial stressors, such as increased mental strain and health risks, compound these challenges, underscoring the need for policies promoting equitable adaptation strategies. Community resilience frameworks also suggest that robust social networks can mitigate some adverse effects, offering pathways to enhance worker adaptability.

Given these findings, future research should explore policy interventions targeting structural inequalities. These could include investments in technology to shield vulnerable workers from environmental risks or initiatives aimed at enhancing community support systems. Expanding the model to incorporate longitudinal data on labor mobility and firm behavior in response to climate adaptation strategies would provide further understanding of sustainable labor practices amidst a changing climate.

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ISSN: 2358-2472

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