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DETERMINATION OF FIBONACCI SEQUENCE TERMS THROUGH TWO COMPUTATIONAL ALGORITHMS

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ABSTRACT
The Fibonacci sequence arose from the following problem: A pair of newborn rabbits was placed in a fenced place. Determine how many rabbit pairs they will have after one year, assuming that each month one pair of rabbits produces another pair and that each pair begins to breed two months after their birth. From this problem the Fibonacci sequence we know was built. When we build the sequence, we see that the number of rabbits increases a lot over time. In this way, it is laborious to calculate the number of rabbits when increasing the number of months. Hence, the following question arises: how many rabbits will they have after n months? For this, two computational algorithms will be built in VisuAlg: one algorithm to determine the first n terms of the Fibonacci sequence and another to determine the nth term of the Fibonacci sequence using Binet's formula. The general objective of the work is to present a computational algorithm to determine the first n terms of the sequence. The specific objectives of the work are: to present the motivating problem of the Fibonacci sequence; display the Fibonacci sequence as a recurrence and establish a formula for determining the terms of such a sequence. First, a bibliographic research was carried out that had as its source the books of Hefez (2016), Alencar (1981) and Burton (2010) and the article by Silva (2020). Then, the computational algorithms were built in VisuAlg. Finally, some considerations were made, briefly showing the difficulties faced in the research and a reflection on the methodology used.

Keywords: Fibonacci sequence. Recurrences. Computational Algorithms.

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INTRODUCTION

Fibonacci sequence is a subject of number theory and number theory is a very beautiful topic of mathematics. In addition, the Fibonacci sequence has several applications. Finally, knowing a formula to determine the nth term of the sequence and having a computational algorithm to determine the first n terms of the sequence makes the study and application of the topic more practical.

Leonardo de Pisa, son of Bonacci, hence the nickname Fibonacci, was a very important Italian mathematician for the Western world. In 1202, he published the book Liber Abaci, which had all the knowledge about numbers and algebra of the time. This work was responsible for introducing the Indo-Arabic numeral system in Europe and consequently for the development of algebra and arithmetic in the West. (HEFEZ, 2016)

Fibonacci proposed and solved the following problem: A pair of newborn rabbits was placed in a fenced place. Determine how many pairs of rabbits they will have after one year, assuming that each month a pair of rabbits produces another pair and that a pair begins to breed two months after their birth.

From this problem the Fibonacci sequence we know was built. We observed that the number of rabbits increases a lot over time. Consequently, it is difficult to calculate the number of rabbits when increasing the number of months. Hence the need to know: how many rabbits will they have after n months?

For this, a formula will be presented to determine the number of rabbits after n months and a computational algorithm will be developed to determine the first n terms of the Fibonacci sequence.

The objectives of the present work are:

- General objective: To present a computational algorithm to determine the first n terms of the Fibonacci sequence.
- Specific objectives: To present the motivating problem of the Fibonacci sequence, to display the Fibonacci sequence as a recurrence, and to establish a formula for determining the terms of the Fibonacci sequence.

The research of this work was bibliographical. The bibliographic research was based on books and articles. The textbooks that were used are: Burton (2010), Hefez (2016), Iezzi (2013) and Mathias (2017). The article that was used is: Silva (2020).

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THE FIBONACCI SEQUENCE

We will initially deal with some fundamental results for the understanding and development of subjects addressed later.

Definition: A finite sequence is any function of the set , in which $fN_n \longrightarrow RN_n = \{1,2,3,\ldots,n\}$. (IEZZI, 2013)

Example: The sequence of positive integer divisors of 60.

$$D(60) = (1,2,3,4,5,6,10,12,15,20,30,60).$$

Definition: An infinite sequence is any function of the set , in which $fN \longrightarrow RN = \{1,2,3,\ldots\}$. (IEZZI, 2013)

Example: The sequence of positive prime numbers.

$$P = (2,3,5,7,11,13,17,19,\ldots).$$

Definition: A sequence is called recurrence in which two rules are given: the first to identify the first term (or more terms) and another to calculate each term from the previous one (or previous terms). (IEZZI, 2013)

Example: The sequence of odd numbers can be defined by recurrence:

$$\begin{cases} x_1 = 1 \\ x_{n+1} = x_n + 2 \end{cases}$$

In which .$n \geq 1$

$$I = (1,3,5,7,9,11,13,15,\ldots).$$

The Italian mathematician Fibonacci proposed the following problem: A pair of newborn rabbits was placed in a fenced place. Determine how many pairs of rabbits they will have after one year, assuming that each month a pair of rabbits produces another pair and that a pair begins to breed two months after their birth. (HEFEZ, 2016)

The following table shows the amount of rabbits in each month:

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Table 01: Number of pairs of rabbits in each month.

Month	Number of couples in the previous month	Number of newborn couples	total
1st	0	1	1
2nd	1	0	1
3th	1	1	2
4th	2	1	3
5th	3	2	5
6th	5	3	8
7th	8	5	13
8th	13	8	21
9th	21	13	34
10th	34	21	55
11th	55	34	89
12th	89	55	144

Source: Prepared by the authors.

This was the solution given by Fibonacci. From this problem the following definition emerged:

Definition: The sequence of positive integers where and , for all $F_1 = F_2 = 1 F_n = F_{n-1} + F_{n-2} n > 2$, $n \in N$, is called the Fibonacci sequence. (SILVA, 2020)

Therefore, we get the terms of the Fibonacci sequence using the recurrence above. Below we find the first 12 terms of the Fibonacci sequence using this definition.

$$
\begin{gathered}
F_1 = 1 \\
F_2 = 1 \\
F_3 = F_2 + F_1 = 1 + 1 = 2 \\
F_4 = F_3 + F_2 = 2 + 1 = 3 \\
F_5 = F_4 + F_3 = 3 + 2 = 5 \\
F_6 = F_5 + F_4 = 5 + 3 = 8 \\
F_7 = F_6 + F_5 = 8 + 5 = 13 \\
F_8 = F_7 + F_6 = 13 + 8 = 21 \\
F_9 = F_8 + F_7 = 21 + 13 = 34 \\
F_{10} = F_9 + F_8 = 34 + 21 = 55 \\
F_{11} = F_{10} + F_9 = 55 + 34 = 89 \\
F_{12} = F_{11} + F_{10} = 89 + 55 = 144
\end{gathered}
$$

Thus, the first 12 terms of the Fibonacci sequence are the numbers:

$$1,1,2,3,5,8,13,21,34,55,89,144$$

In 1843, a French mathematician Jacques-Philippe-Marie Binet discovered a formula for the Fibonacci sequence. The formula is as follows: (BURTON, 2010)

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

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Using this formula, we will determine the first term of the Fibonacci sequence.

$$F_1 = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^1 - \left(\frac{1-\sqrt{5}}{2}\right)^1\right]$$
$$F_1 = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)\right]$$
$$F_1 = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}-1+\sqrt{5}}{2}\right)$$
$$F_1 = \frac{1}{\sqrt{5}}\left(\frac{2\sqrt{5}}{2}\right)$$
$$F_1 = 1$$

As the exponent increases, it becomes unfeasible to use Binet's formula. Therefore, a computational algorithm will be built, in Visualg, to determine the nth term of the Fibonacci sequence.

INTRODUCTION TO VISUALG

There is a growing demand for mathematical and computational knowledge in today's society. Therefore, it is necessary for the bachelor in mathematics to be more prepared to use some computational tools in his work. Mainly, computational tools for algorithms that solve problems of pure and applied mathematics.

We must know that the machine (computer) is not intelligent. It needs instructions to execute what we want. The difference between a human and a machine is that the computer runs algorithms at a much faster rate than the human being.

But what would be an algorithm? According to MATHIAS (2017), an algorithm is the specification of an orderly logical sequence of steps that must be followed to perform a task, ensuring its repetitiveness. Therefore, an algorithm is a set of steps to solve a given problem.

Visualg is an application used by students of Algorithm and Programming Logic. The look is based on Portugol (a pseudocode written in Portuguese). It was developed by Antonio Carlos Nicolodi in partnership with Claúdio Morgado de Souza. It all started in 1996 when Claudio Morgado had created Visualg Algorithm for an undergraduate course. Claúdio Morgado passed the source code to Nicolodi of version 2.0 of the program, so that it could be improved. After a while, Morgado had to leave the project and asked Nicolodi to continue developing and improving the app.

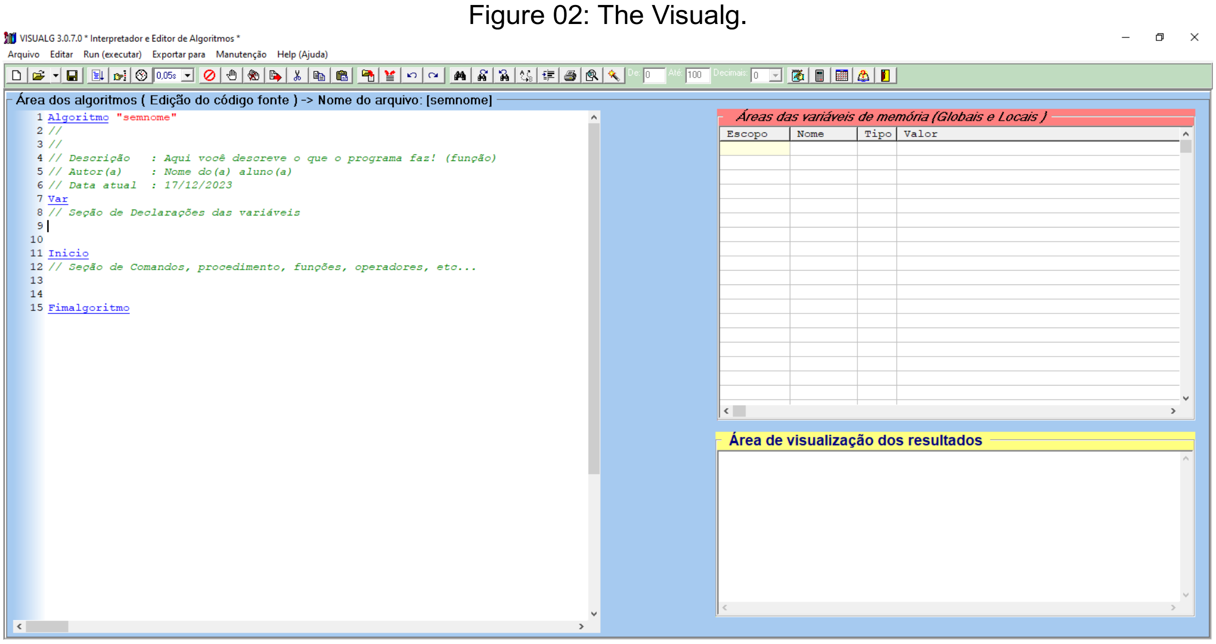
In 2014, version 3.0 of the app was released. The success was so great that version 3.0 of the Visualg application already has more than 30 million downloads in more than 95 countries and is used by more than 50 thousand schools as a teaching/learning tool.

Figure 01: Antonio Carlos Nicolodi

Source: https://www.infoescola.com/noticias/professor-brasileiro-desenvolve-metodo-e-aplicativo-que-facilitam-o-aprendizado-de-programacao/

The following figure shows the main Visualg screen. On this screen we have a menu with the following options: File, Edit, Run, Export to, Maintenance and Help. We also have a toolbar, an algorithm area, a memory variables area, and a results visualization area.

Figure 02: The Visualg.



Source: Prepared by the authors.

The decision structure has instructions that must be executed only when the condition is satisfied, that is, when the condition is true. Syntax:

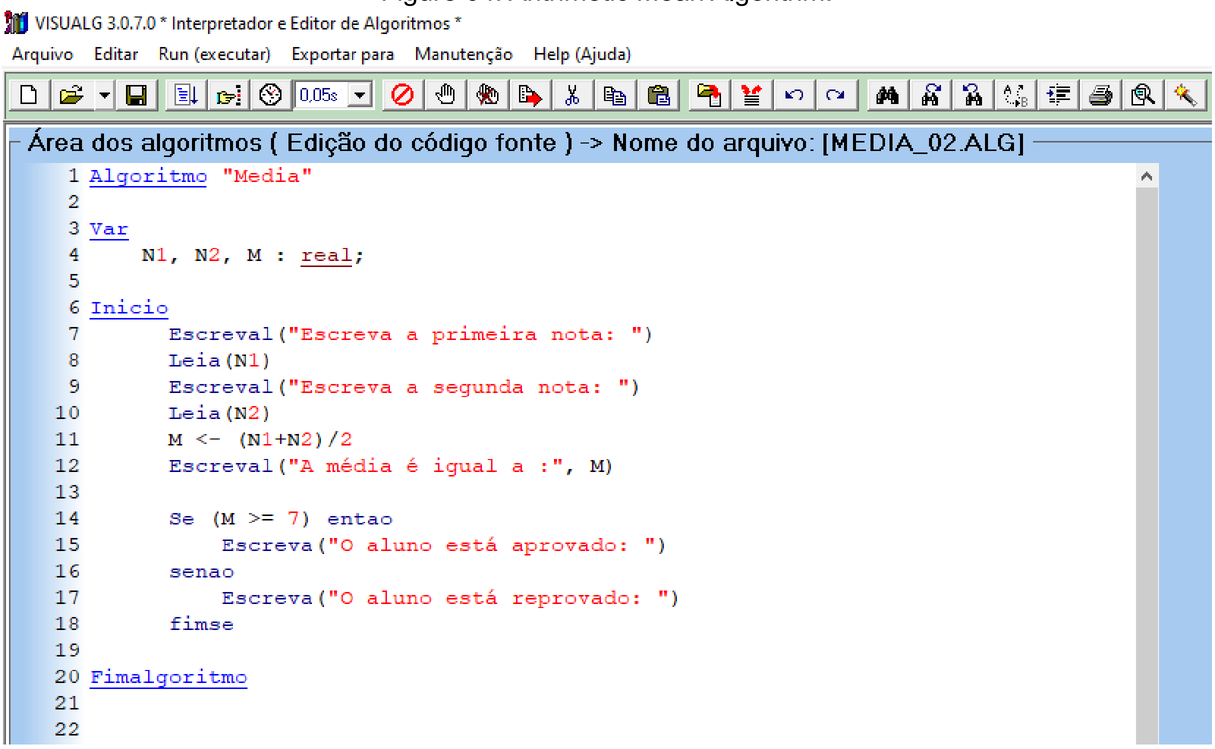
Figure 03: Syntax of the decision structure.

```
Se (expressão condicional) entao
(bloco de instruções)
Fimse
```

Source: Prepared by the authors.

To understand a little about the programming structure in Visualg we will show an algorithm. This algorithm calculates the arithmetic average of two grades of a student. If the average is greater than or equal to 7 then the command that shows the phrase 'The student is approved' is executed, otherwise another command that shows the phrase 'The student has failed' is executed.

Figure 04: Arithmetic Mean Algorithm.



Source: Prepared by the authors.

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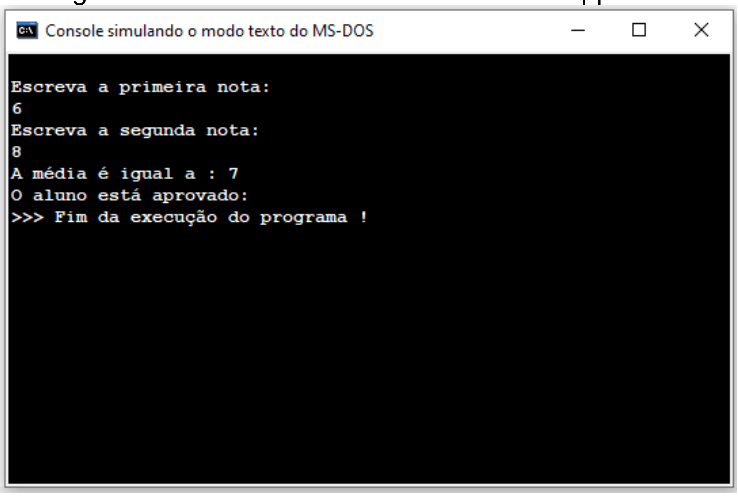
First the algorithm is given a name. Then the variables that will be used are declared. In this case, the variables N1, N2 and M (all of the real type) were used. After declaring the variables, two grades are requested from the student and then the student's average is shown. Finally, using the conditional structure, it is written on the screen whether the student has passed or failed.

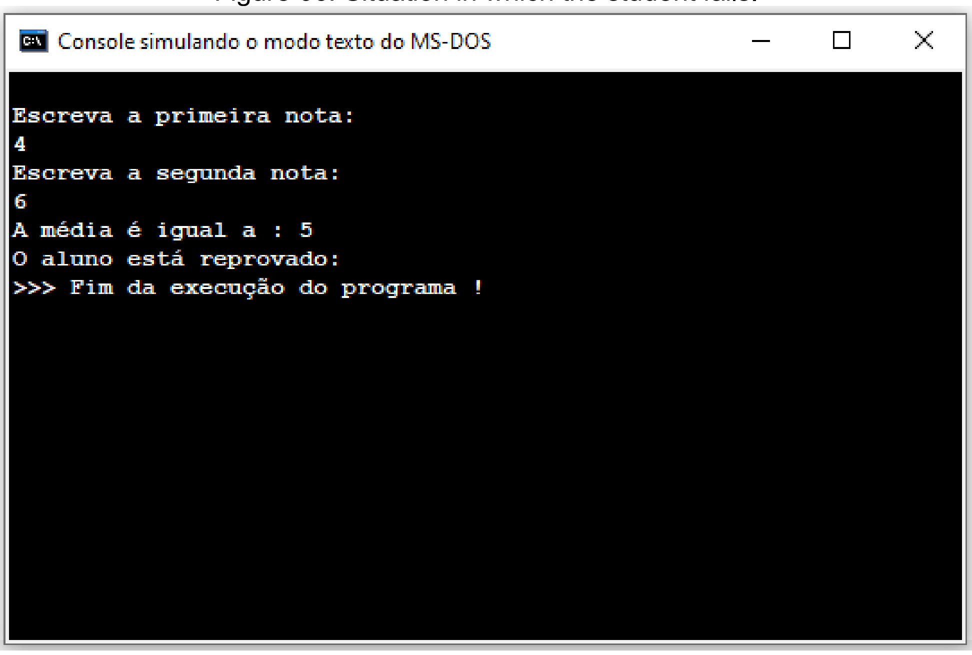
Next, we have the execution of this algorithm. Showing a situation in which the student is approved and a situation in which the student is failing.4

Figure 05: Situation in which the student is approved.



Source: Prepared by the authors.

Figure 06: Situation in which the student fails.



Source: Prepared by the authors.

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Repetition structures are also called loops and correspond to a programming modality in which the objective is to repeat, iterate, a certain part of a program, a certain number of times. Syntax:

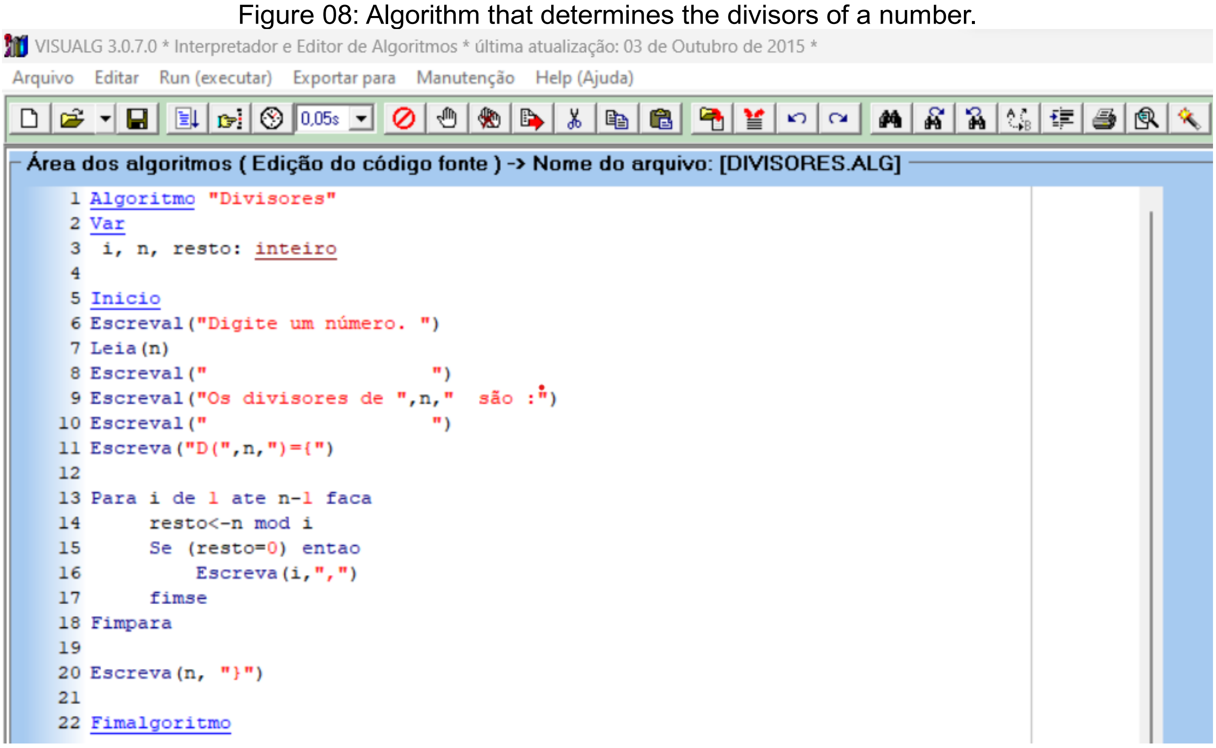
Figure 07: Syntax of the repetition structure.

```
Para i de 1 ate n faca
(bloco de instruções)
Fimpara
```

Source: Prepared by the authors.

Next, we have an algorithm that determines all the positive divisors of a number.

Figure 08: Algorithm that determines the divisors of a number.



Source: Prepared by the authors.

First the algorithm is given a name. Then the variables that will be used are declared. In this case, the variables **i**, **n** and **remainder** (all of the integer type) were used. After declaring the variables, the number you want to know what the divisors are.

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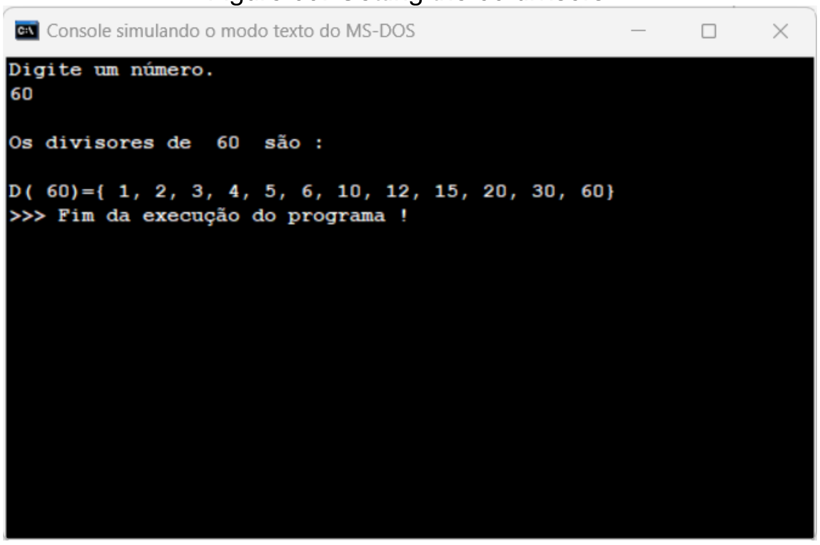
Finally, using the repetition structure, the divisors of the requested number are written on the screen.

Next, we have the execution of this algorithm. Showing the divisors of 60.

Figure 09: Getting the 60 divisors.



Source: Prepared by the authors.

COMPUTATIONAL ALGORITHM FOR DETERMINING THE TERMS OF THE FIBONACCI SEQUENCE

The following computational algorithm determines the first n terms of the Fibonacci sequence.

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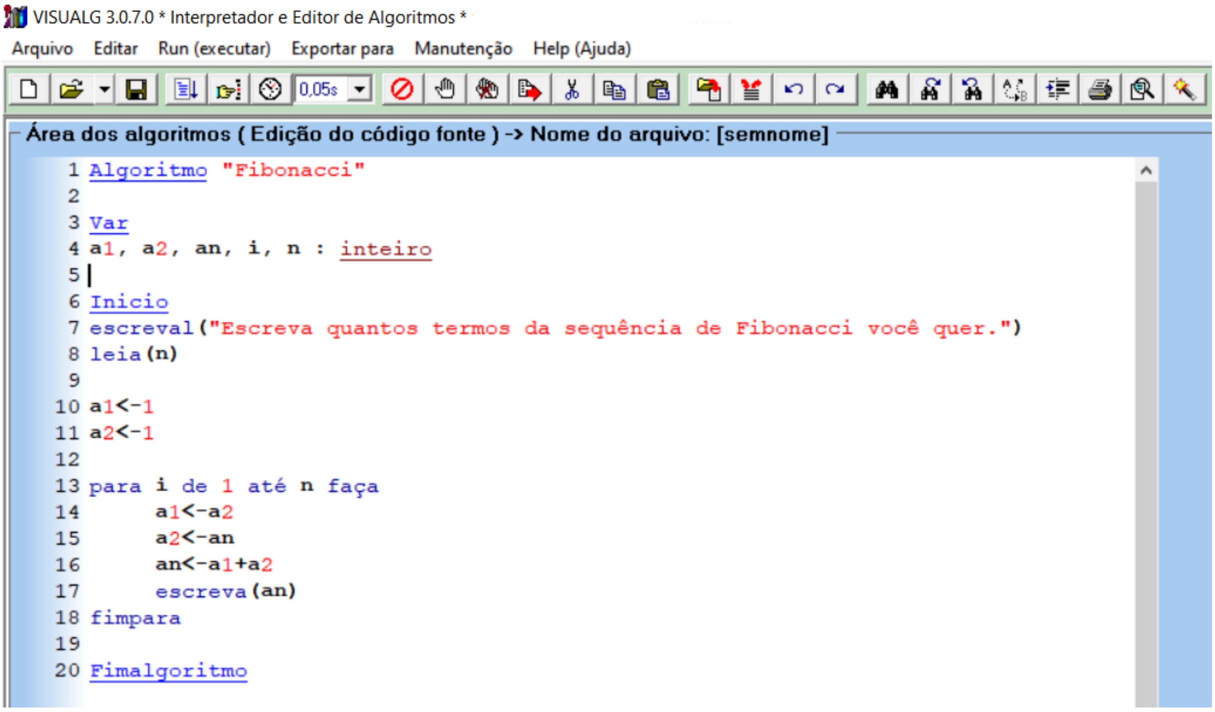
Figure 10: Algorithm for determining the terms of the Fibonacci sequence.



Source: Prepared by the authors.

The structure of the algorithm is quite simple. First the algorithm is given a name. Then the variables that will be used are declared. In this case, the variables a1, a2, an, i and n (all of the integer type) were used. After declaring the variables, the number of terms that you want from the Fibonacci sequence is requested , in which case it will be n. Finally, using the repetition structure, the first n terms of the sequence are written.

By running the algorithm written in Visualg, we were able to determine the first 12 terms of the Fibonacci sequence. The following figure shows the first 12 terms of the Fibonacci sequence.

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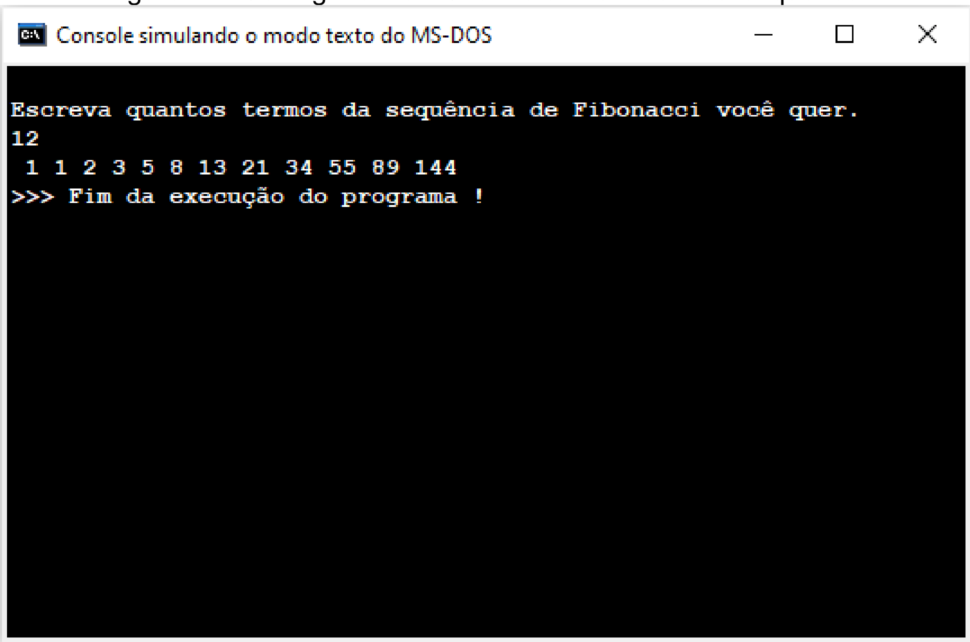
Figure 11: Getting the first 10 terms of the Fibonacci sequence.

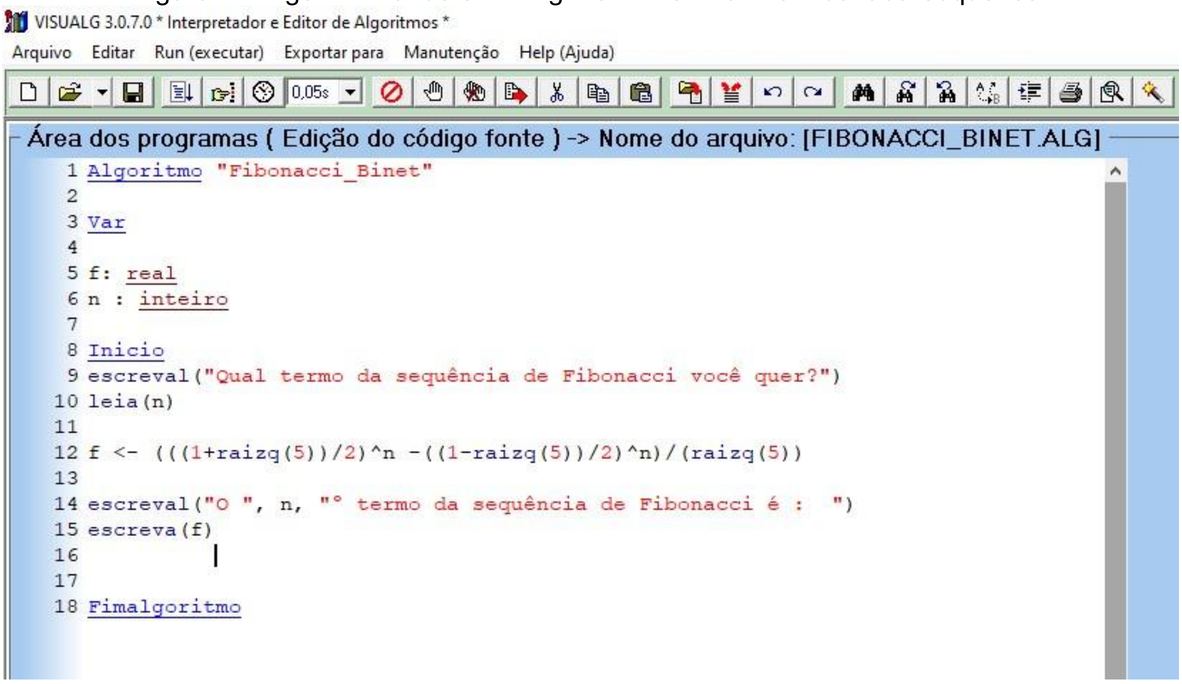


Source: Prepared by the authors.

The following computational algorithm determines the nth term of the Fibonacci sequence. For this, Binet's formula below is used.

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Figure 12: Algorithm for determining the nth term of the Fibonacci sequence.



Source: Prepared by the authors.

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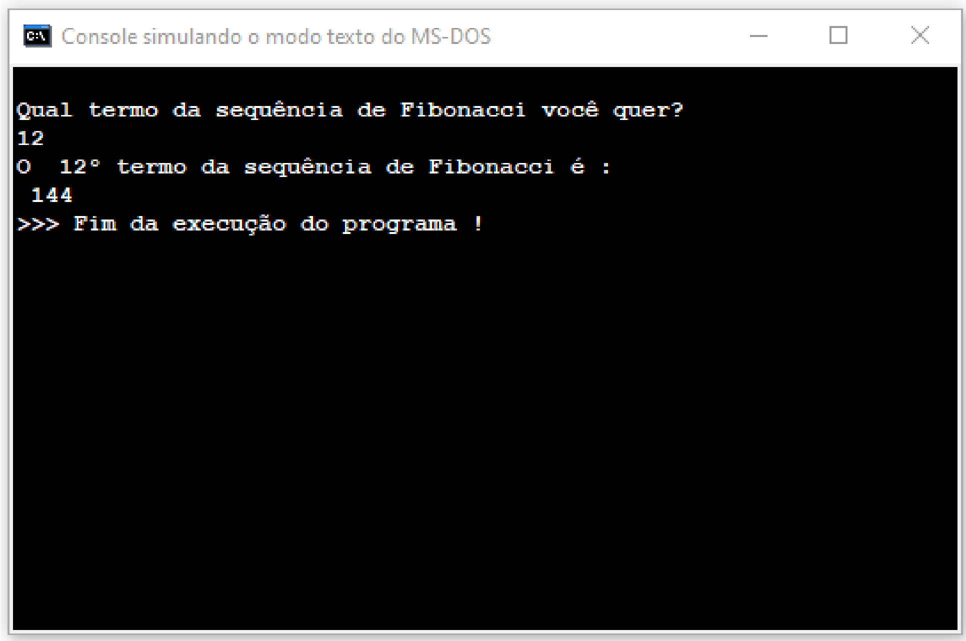
Figure 13: Getting the 12th term of the Fibonacci sequence.



Source: Prepared by the authors.

Therefore, the number of rabbits after one year (12 months) will be 144. The number of rabbits after n months will be given by recurrence or by Binet's formula. Finally, we obtain these values using the computational algorithms implemented previously.

FINAL CONSIDERATIONS

In the present work, two computational algorithms were constructed in Visualg: one to determine the first n terms of the Fibonacci sequence and the other to determine the nth term of the Fibonacci sequence. The first algorithm was run and the first 12 terms of the Fibonacci sequence found were the numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 and 144. The second algorithm (using Binet's formula) was executed and the number found was 144.

There was a bit of difficulty in making the algorithm using Binet's formula, because the amount of parentheses needed to write the formula in Visualg was higher than that of the formula written in mathematical language. It was necessary to replace the brackets of Binet's formula with parentheses in Visualg.

Finally, we learn that knowing a formula to determine the nth term of the Fibonacci sequence makes the job of finding this term easier. In addition, having a computational algorithm to determine the first n terms of the sequence and another to find the nth term makes the study and application of the topic more practical.

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