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# THEMES IN MATHEMATICS STUDIED IN THE LIGHT OF PHENOMENOLOGY

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**ABSTRACT**

This study aims to understand which themes of Mathematics are researched in the light of the theoretical and methodological foundation of Husserlian Phenomenology. To this end, a bibliographic study was carried out in the works published by the research group Phenomenology in Mathematics Education (FEM), of the São Paulo State University of Mesquita Filho (UNESP), chosen under the criteria of consolidation, production and Husserlian focus. Phenomenology was used as a theoretical/methodological field, with which it was possible to constitute nuclear ideas, which allowed us to weave understandings about the guiding question. Among the views cast, it is noted that most of the works that apply phenomenology to mathematics tend to focus on Geometry. This emphasis can be justified by the visual and spatial character of Geometry, which naturally lends itself to a phenomenological approach. However, the concentration in this field leaves important gaps, which are the other areas of mathematics that could also benefit from a phenomenological analysis.

**Keywords:** Mathematics. Phenomenology. Geometry.

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## INTRODUCTION

Over the centuries, Mathematics has remained a fertile field for investigations, given its nature as a language that transcends disciplinary boundaries, providing an important conceptual and instrumental foundation for the investigation and analysis of phenomena. Whether in the sphere of Physics, Biology, Economics, Engineering or in other areas of knowledge, Mathematics presents itself as a formal system capable of providing models and techniques that enable the description, interpretation and prognosis of the behavior of both natural and artificial systems. Mathematics, as a philosophical discipline, invites us to reflect on the fundamental structure of the universe and the laws that govern it. The possibility of representing abstractions and formalizing relations between concrete and abstract concepts is essential for understanding the world.

In this work we focus on mathematics from the perspective of philosophy, more specifically the Phenomenology of Edmund Husserl (1859-1938), which emerged in the nineteenth century, gained robustness in the twentieth century and continued its trajectory advancing with studies of themes that branch out between schools and thinkers, as is the case of Martin Heidegger's existential ontology and Max Scheler's ethics of values. The influence of Husserlian phenomenology was such that today it has become difficult to evaluate it in all its extension and depth. In this philosophy, the focus is on the phenomenon that manifests itself to those who inquire about it (Husserl, 2006). This statement does not simplify a methodology, it opens a problem for Husserl, as he has to answer how a researcher can conduct his conceptions, his worldviews, his hypotheses and, at the same time, experience the genuine manifestations that are shown when targeting the phenomenon.

Given the relevance of Mathematics and Phenomenology, we want to understand here: *which themes of Mathematics are researched in the light of the theoretical and methodological foundation of Husserlian Phenomenology?* To this end, a qualitative study of bibliographic nature will be carried out, searching for studies by researchers who focus on Mathematics from a phenomenological perspective, which can contribute to the understanding of what is questioned. It is understood that such researchers may be those who make up the research group Phenomenology in Mathematics Education - FEM. The data from this survey will be analyzed in the light of the phenomenological methodology.

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## THE HUSSERLIAN PHENOMENOLOGY

The term "phenomenology", throughout the development of philosophical knowledge, is constituted by various semantic nuances. All of them corresponded to the investigative needs of those who employed him. During the modern period, phenomenology was understood as the field of study of appearances, that is, of illusions, as established by Lambert. With the advent of Hegelian philosophy, the term was marked in the title of "The phenomenology of spirit" (1807), in which the word has the role of defining the paths traced and likely to be followed by the individual consciousness in an attempt to identify itself with a consciousness, universal spirit (Abbagnano, 2007).

Later, it moved to the domain of descriptive psychology, in which it gained the sense of psychic appearance, a situation that is apprehensible and prior to psychic reality (Abbagnano, 2007). In this last sense, the word acquires, once again, a content of something that dwells on illusion, initial confusion in the face of an object to acquire greater depth later.

Currently, the use of phenomenology is intrinsically related to the philosophy of Edmund Husserl, who renewed the word based on his studies on the theory of knowledge. The question that leads to the development of Husserl's concept of phenomenology concerns the possibility of the individual, through his consciousness, being able to apprehend the surrounding reality with objectivity. At the beginning of the twentieth century, the question was under the domain of psychology, whose proposition was that logic offered dogmas applicable to all thought, so that the field of logic would be grounded in psychology. To this proposition, Husserl counterposes the arguments that logical formulations do not need physical referents, while psychology only reflects empirical experiences and generalizes them (Zilles, 2007).

To maintain the primacy of logic and reject psychologism, Husserl will pay attention to the very foundations that would make a theory of knowledge possible. In this sense, the philosopher will determine two categories: the "noeses" and "noemas". The first corresponds to the specific ways in which consciousness faces an object, while the second indicates the content and meanings of that object. These same categories can occur at two levels: a transcendental level, in which the relationship of consciousness with the object and its meanings are based on the ideal aspirations that the ego has for the object, and an objective level, in which the ego aims only at the meanings and contents already established in reality for the object (Husserl, 1986, apud Zilles, 2007; Mora, 2001).

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In this process, especially from the transcendental sphere, consciousness, equipped with intentionality, that is, in the act of directing itself to the object, fills it with meanings, the individual, in his natural or objective attitude, takes the objects with meanings and contents already given by the surroundings, since he is not detached from his relations with the world. Only from the phenomenological reduction, which puts the world in parentheses, is it possible for consciousness to perceive the ideal relations that remain with the object, which is free from the pre-definitions that the world imposes on consciousness. In this way, it would be possible to reach the essence of things, since the subject-object relationship is overcome in order to reach the constituent consciousness of the object itself, the consciousness of something (Husserl, 1986, apud Zilles, 2007; Mora, 2001).

That said, phenomenology comes to constitute itself as both a theory and a methodology (Mora, 2001), since it also enables a look free from interference from the environment to the object of scientific investigation, in addition to offering a process through which it is possible to seek the conceptual purity of what the research aims at as a purpose: the essence of the entities that surround the existence of the individual.

In summary, Husserlian phenomenology is a philosophical approach that aims to apprehend the structure of human experience and consciousness. Husserl proposed "phenomenological reduction," a method that suspends prejudices and judgments concerning external reality in order to focus on the way in which objects are perceived and experienced in consciousness. Such a methodology enables the exploration of the essences of subjective experiences, revealing how the world manifests itself to individuals, regardless of its objective existence. Phenomenology, therefore, seeks to describe experiences from the point of view of those who experience them, emphasizing the intentionality of consciousness, that is, its characteristic of always being directed to something.

## THE CONSTITUTION OF MATHEMATICAL KNOWLEDGE IN THE LIGHT OF HUSSERLIAN PHENOMENOLOGY

Phenomenology, in its Husserlian sense, comprises an attempt to reestablish the objectivity of the relations between the subject and the object, the focus of investigation, supplanting the radicalism of psychology. In this sense, it is constituted as a questioning of the production of human knowledge in the face of a complex reality, focusing, beyond reality itself, on the reality that is established in the individual's experience.

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That said, the notion of perception generates interfaces of contact with phenomenology. It is worth considering, as Bicudo (2012) states, that the natural sciences already establish, at their core, a preconceived view of Nature, which prioritizes a theoretical function, which presupposes a language that supports the observation of what was objectively inserted into the language itself. To this theoretical view, Husserl will oppose a more comprehensive view, capable of apprehending more freely the subject's contact with the "life-world"; this being the scenario in which the experience unfolds and which is capable of offering concrete bases to the individual's deductive process.

This non-theoretical posture advocated by Husserl is intimately linked to the apprehension not only of the external reality – as in the case of seeing a beautiful landscape – but also of the internal effects aroused by it – the pleasure and serenity that is transmitted in the observation, in the act itself, and also in the correlations sustained in the reflection on the act itself that develops from the experience. There is, therefore, in this posture, a dynamic that constitutes the concomitant realities of feeling, perceiving and reflecting. Such a notion expands the possibilities of subject-object dynamics, as is the case of aesthetic pleasure. On the other hand, the theoretical-practical posture, in the face of perception, is a process of abstraction, which is no longer based on the real presence (Bicudo, 2012).

It is also necessary to emphasize that such a phenomenological posture encompasses, beyond the simple subject-object relationship, the entire surrounding reality in its potentialities and specificities. Thus, in the very act of perceiving, the entire social, historical and cultural context is implicated. This fact prevents the act of perception and reflection of perception in the phenomenological process from falling into a radical subjectivism, whose nature would end up clouding the view of reality (Bicudo, 2012).

In this sense, the non-theoretical posture frees the individual from initial theoretical presuppositions in the face of reality, enabling a purer contact with the life-world, which does not consider only the subject's pretensions, but everything else that is expressed in the surrounding nature and that cannot be relegated to the background, at the risk of adulterating and profoundly restricting the desired perception of reality.

The constitution of mathematical knowledge, according to Edmund Husserl's phenomenology, involves the analysis of how mathematical objects appear in consciousness. For Husserl, mathematics is not just a set of formulas and theorems, but an intellectual construct deeply rooted in intuitive experience and the intentionality of

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consciousness. Phenomenology seeks to understand how mathematical concepts emerge from lived experiences and how the human mind apprehends them intentionally, that is, directed to an object. In this sense, mathematical objects are considered *ideal objects*, existing independently of time and space, but whose understanding depends on the conscious activity of the subject who conceives them. Using Geometry as an example, it is observed that it "has its thematic sphere in ideal products, in idealities from which more and more idealities at various higher levels are produced" (Husserl, 2006, p. 15).

Ideality in the Husserlian perspective is linked to the constitution of the intentionality of subjectivity, the foundation where experiences, including geometric ones, gain meaning both for the individual and for the community of subjects. An ideal object, once formed, acquires a temporal duration that extends throughout history, crossing time and space. This is achieved through records, such as writing, which preserve in culture and in the history of knowledge the ideas that have been conceived, discussed, revisited and objectified over time.

How does this process of objectification occur? In Husserl (2006), we understand that, along with perception, an articulating thought emerges that builds understandings, which are manifested through language. This expression reveals what is presented in the act of perceiving and understanding. Language, when shared in intersubjectivity between co-subjects, allows these individuals to turn to what is said, understanding what is expressed and participating in the discussion, bringing contributions, agreeing, disagreeing and exploring new articulations. Objectification, or the formation of objective knowledge, is concretized in the repetition and practices of these understandings expressed by the subjects, regardless of whether they belong to the same time, culture or society.

Husserl proposes that mathematics, like other forms of knowledge, should be grounded in a pre-reflective and intuitive experience, which he calls "meaning-giving." This phenomenological approach implies that mathematical knowledge is constituted through acts of synthesis that occur in consciousness, where scattered elements of intuitions are unified into coherent mathematical concepts. Husserlian phenomenology, therefore, offers a unique perspective on the nature of mathematical knowledge, emphasizing the importance of intuition and subjective experience in the formation of mathematical concepts. This is in contrast to the formalist view that sees mathematics as a mere symbolic manipulation, highlighting instead the role of consciousness in the construction and validation of mathematical knowledge.

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## RESEARCH METHODOLOGY AND PROCEDURES

A bibliographic research is carried out, from a phenomenological perspective and posture (Bicudo, 2011), to understand which themes in Mathematics are investigated from the theoretical and methodological perspective of Husserlian Phenomenology. The intention is to promote a movement of understanding free of assumptions that can anticipate results. The interpretations developed throughout this research emerge from the investigative process, based on the works published by the research group Phenomenology in Mathematics Education FEM, of the São Paulo State University of Mesquita Filho (UNESP), chosen under the criterion of consolidation, production and Husserlian focus. The WEF is a 40-year-old group (1984 – 2024), which at UNESP began together with the first Graduate Program in Mathematics Education (PPGEM) in the country, and is coordinated by Professor Dr. Maria Aparecida Viggiani Bicudo, the main reference in Husserlian phenomenology in Brazil and one of the main in the world.

The database, initially, was the composition of the members of the Group, available in the Directory of Research Groups of the Lattes Platform. Subsequently, the analysis of the Group's researchers was carried out, curriculum by curriculum, exploring the published articles, seeking in them themes of Mathematics approached from the perspective of phenomenology. The process of searching for the analyzed material resulted in a vast number of productions, given the long trajectory and intense activity of the group. In view of this scenario, as an inclusion criterion, a time frame of the last 10 years (2014 - 2024) was determined and, as an exclusion criterion, texts that do not deal with mathematical themes in the light of phenomenology and those that, although they are phenomenological texts, do not present the specificity of a mathematical theme in the discussion, such as,  the article "Brazilian indigenous peoples in the Mathematics textbooks of the Early Years of Elementary School," by Liana Cézar Barros, Ana Paula Purcina Baumann and José Pedro Machado Ribeiro, does not deal with a specific mathematical theme, but rather with how indigenous peoples are represented in these teaching materials.

## DESCRIPTION AND ANALYSIS OF TEXTS

In all, 104 articles were highlighted, all published in indexed journals. To facilitate the understanding of the analysis process, it is confirmed to the reader that a phenomenological approach was adopted in all texts. However, due to the large amount of

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materials, here is only a synthesis of how this movement occurred. Considering the importance of providing the reader with a comprehensive overview, four texts were selected that represent different approaches.

Text 1 "A phenomenological look at Dynamic Geometry", investigates the relationship between movement, perception and knowledge in the context of Dynamic Geometry, using a phenomenological approach based on the works of Husserl and Merleau-Ponty. The authors present *Dynamic Geometry* as a new look at Geometry, a geometry that *is* only in motion; the objects that are evidenced in the software interface cannot be affirmed as such before the subject throws himself at it in motor acts, provoking configurations and/or deconfigurations, exposing, or not, invariants from which it can be defined. The computer interface is discussed as a space where the body and technology intertwine, allowing users to express and understand their thoughts through gestures and interactions. The research highlights the importance of the subject's intentionality in the learning process, suggesting that Dynamic Geometry is not only a tool, but a means of constituting knowledge, where movement and perception are fundamental to the educational experience (Pinheiro, Bicudo and Detoni, 2019).

Text 2 "A work on digital technologies in the discipline of Calculus in a Mathematics Degree course" presents the *Fundamental Theorem of Calculus as the* main theme. The article discusses the importance of thinking in Mathematics classes, emphasizing that problem solving begins with reflection and the construction of meaning even before practical execution. Qualitative research, focusing on the phenomenological approach, is used to understand the students' experiences during the activities, highlighting the interaction and dialogue between them when using digital technologies, such as the GeoGebra software, to understand the Theorem. The analyses of the group discussions reveal how students question, validate hypotheses, and elaborate ideas, promoting an active and reflective learning environment, where the understanding of mathematical concepts is deepened through collaboration and exploration (Pavanelo, 2022).

Text 3 "Mathematics teachers' understandings of the presence of Algebra in Elementary School II" exposes an investigation on the understanding of *Algebra* by Elementary School teachers, revealing that the discipline is seen as abstract and complex, generating discomfort among educators. The research highlights the difficulties of students in moving between Arithmetic and Algebra, especially in the use of letters as variables and in the translation of information from natural language to algebraic language. Teachers emphasize the importance of developing algebraic thinking and the need for pedagogical strategies that connect Algebra to students' daily lives, recognizing that teaching this subject is challenging and requires the assimilation of many concepts in a short period. The research suggests that it is essential for educators to show the relevance of Algebra, beyond simply complying with the curriculum, to facilitate students' understanding of the subject (Silva, *et. al.* 2021)

Text 4 "The relevance of knowing and understanding the incompleteness of Mathematics: a phenomenological look at Gödel's Incompleteness Theorem" discusses the importance of understanding the incompleteness of Mathematics, especially in the light of *Gödel's Incompleteness Theorem*, and its implications for mathematicians and non-mathematicians, including educators and students. The qualitative research, of a bibliographic nature, explores how incompleteness challenges the traditional view of Mathematics as a complete science and accessible only to a few, proposing a phenomenological approach that connects Mathematics to the lived world. The article argues that understanding incompleteness not only transforms the perception of the limits of the axiomatic method, but also opens space for new possibilities for knowledge and learning (Pinheiro, Batistelat, 2021).

The abstracts presented above expose different mathematical themes and ways in which they are studied in the light of phenomenology. In the same way, a synthesis of all the texts taken for analysis was made. To facilitate the reader's understanding, it is considered crucial to show the detailed process of this analysis, as well as the convergences between the texts. However, due to the need to synthesize the article, it is also presented, in synthesis, the movement carried out to explain and analyze the mathematical themes and their phenomenological environment.

In conducting the phenomenological analysis, we searched for the themes in Mathematics addressed in the works, which are identified as Significant Units (US). These units are organized in such a way as to facilitate the identification of subsequent convergences and are named in a systematic manner. In the first text, for example, the units were designated as US1T1 (Significant Unit 1, Text 1), US2T1, US3T1, and so on. To preserve the count, even in the analysis of Text 2, the sequence is maintained, changing only the number of the text.

Chart 1, which will be presented below, illustrates some of the HUs identified in the texts, providing a preliminary view of the analytical units used in the phenomenological study.

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Table 1: Significant Units found.

| o. | Text | Significant Units |
|---|---|---|
| | A phenomenological look at Dynamic Geometry - José Milton Lopes Pinheiro, Maria Aparecida Viggiani Bicudo, Adlai Ralph Detoni | US1Q1 - Dynamic Geometry |
| | A work on digital technologies in the discipline of Calculus in a Mathematics Degree course - Elisangela Pavanelo, Fabiane Mondini, Luciane Ferreira Mocrosky, Anderson Luis Pereira | US2Q2 - Differential Calculus<br>US3Q2 - Integral<br>US4T2 - Fundamental Theorem of Calculus |
| | Mathematics teachers' understandings of the presence of Algebra in Elementary School II - Lais Cristina Pereira da Silva | US5S3 - Algebraic Expressions<br>US6S3 - Features |
| | The relevance of knowing and understanding the incompleteness of Mathematics: a phenomenological look at Gödel's Incompleteness Theorem - José Milton Lopes Pinheiro, Rosemeire de Fatima Batistela | US7Y4 - Gödel's Incompleteness Theorem<br>US8S4 - Mathematical Logic<br>US9S4 - Fundamentals of Mathematics |
| | Investigating and exploring with technology: a possibility to teach trigonometry - Carolina Cordeiro Batista, Rosa Monteiro Paulo | US10T5 - Trigonometry |
| | The game Slice Fractions as a space to teach fractions and the opening to the constitution of mathematical knowledge - Cristiano Natal Toneis, Rosa Monteiro Paulo | US11T6 - Fractions |
| | Demonstration with technology: a study on the number of diagonals of a prism - Elisangela Pavanelo, Ana Carla de Paula Leite Almeida | US12T7 - Raw<br>US13S7 - Mathematical Demonstrations<br>US14T7 - Diagonals of a Prism |
| | The Teaching of Differential and Integral Calculus I: Possibilities of investigation. - Fabiane Mondini, Luciane Ferreira Mocrosky, Rosa Monteiro Paulo | US15T8 - Features<br>US16T8 - Differential Calculus<br>US17T8 - Graphics<br>US18T8 - Integral |
| | A study on the formal treatment of rational numbers. - Joel Gonçalves dos Santos, Fabiane Mondini | US19S9 - Mathematical Demonstrations<br>US20T9 - Rational numbers |
| 0 | Collaborative Learning in Dynamic Geometry Environments - José Milton Lopes Pinheiro | US21T10 - Dynamic Geometry |
| 1 | Mobile devices in the teaching of Spatial Geometry from the perspective of learning mobility - Érika Cruz Silva, Marli Regina dos Santos | US22T11 - Polygons<br>US23T11- Polyhedra<br>US24T11 - Prisms<br>US25T11- Cylinders |
| 2 | Paving the plan: a study with Mathematics and Art teachers. - Marli Regina dos Santos | US26T12 - BisectorUS27Q12 - Midpoint<br>US28T12 - Angle |

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| | | |
|---|---|---|
| | | US29T12- Symmetry |
| 3 | Contributions to Instructional Design and Cybertraining through student feedback on interactive mathematical comics. - Maurício Rosa, Vinícius Pazuch | US30T13 - Functions |
| 4 | History of Mathematics in elementary school: a qualitative research related to the multiplication operation. - Ivan Álvaro dos Santos, Tânia Baier | US31T14 - Natural Numbers<br>US32T14 - Multiplication |
| 5 | Of the sense of beauty in Mathematics and of what proved beautiful to us in the proof of Gödel's incompleteness theorems. - Rosemeire de Fátima Batistela | US33T4 - Gödel's Incompleteness TheoremUS34T15 - Fundamental Arithmetic Theorem |
| 6 | A proposal for teaching the structures of algebra inspired by a phenomenological conception of the construction of its knowledge - Verilda Speridião Kluth, Paola Andrea Gaviria Kassama, Tiago Nunes Castilho, Carlos Alberto Tavares Dias Filho | US35T16 - Fundamentals of Mathematics<br>US36T16 - Multiplication |

Source: the author

Chart 1 presents 16 of the 104 texts analyzed, highlighting 36 Significant Units that highlight some of the themes in Mathematics discussed in phenomenological approaches. Considering all the texts, *62 US$ stands out*, and there are few themes in Mathematics other than those presented above, they are: US37T4 – *Mathematical Modeling*, US38T7 – *Circumference*, US49T9 – *Differential Equations*, and US62T20 – *Quadrilaterals*.

By revisiting the Significant Units (US) and exploring their particularities, we understand that some of them converge to broader areas of mathematics, here understood as convergence groups and which we name as Nuclear Ideas - IN, understood as structuring the investigated phenomenon (mathematical themes in the light of phenomenology). This process, in the phenomenological approach, is known as Nomothetic Analysis, which I understand "is the moment to transcend the analysis of individual data [...], paying attention to convergences and divergences that, once articulated, point to "great convergences" (Pinheiro, 2018, p. 97).

In Chart 2, which follows, we show the convergence movement, which constituted 4 Nuclear Ideas.

Table 2: Movement for the constitution of the Nuclear Ideas

| *Significant Units* | *Nuclear Idea* |
|---|---|
| US1T1, US2T5, US10T5, US4T15, US12T7, US14T7, US1T10, US10T8, US17T11, US18T11, US15T33, US6T25, US27T14, US5T20, US12T11, US30T22, US19T11, US20T12, US21T1, US5T21, US25T27, US12T20, US22T12, US23T12, US27T14, US2Q34, US2Q34, US67T12 | IN1 - IN1 - The study of Euclidean Geometry |

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| | |
|---|---|
| US2T2, US3T2, US3T10, US4T2, US6T3, US6T8, US6T8, US2T8, US15T8, US3T8, US6T13, US78T25, US24T36, US81T5, US62T7, US18T19, US28T45 | IN2 - Differential and Integral Calculus |
| US5T3, US8T4, US9T4, US11T6, US17T13, US13T7, US28T19, US13T9, US4T11, US3T11, US16T9, US24T14, US25T14, US81T60 | IN3 – Algebra and Fundamentals of Mathematics |
| US7S4, US7S4 | IN4 - Philosophy of Mathematics |

Source: The author.

US, as well as NI, are understandings that are manifested in the investigative process, that is, they are revealed to the researcher who, intentionally, places himself in the act of questioning, positioning himself as someone who seeks to know, without assuming the theoretical posture that previously establishes hypotheses or preconceptions about the questioned.

Thus, the Nuclear Ideas show themselves in the flow of convergences, constituted by the rigor of phenomenological research. Therefore, we understand that articulating these ideas is a way to present the reader with understandings about the research question: *which themes of Mathematics are researched in the light of the theoretical and methodological foundation of Husserlian Phenomenology?* We begin this articulation with the Nuclear Idea 1 (IN1), in which we bring the meanings that are shown, highlighted in italics and bold.

## IN1 - THE STUDY OF EUCLIDEAN GEOMETRY

Husserlian Phenomenology, based on the ideas of the philosopher Edmund Husserl, seeks to understand the essence of human experiences from the rigorous description of phenomena as they are experienced and perceived. By applying this approach to research in Mathematics, especially in the field of Geometry, a horizon of possibilities opens up to explore how geometric concepts manifest themselves in consciousness and how they are constituted through intuition and lived experience.

*Dynamic Geometry (DG),* for example, emerges as a fertile field for the application of Phenomenology, as it deals with the interaction of the subject with geometric representations that change in real time. In text 1 "A phenomenological look at Dynamic Geometry", the authors approach DG in the light of phenomenology, emphasizing the importance of the intentionality of the movement of the self-body in the interaction with DG software. The analysis reveals that, in the experiences of the subject-mover, the technological devices and their physical and logical interfaces become extensions of their body when manipulating geometric objects, which results in the creation of a unique space of experiences, where movement, perception and knowledge are intertwined, constituting the unity of movement-perception-knowledge.

The phenomenological perspective allows us to understand that the act of moving is not limited to a physical action, but also involves a cognitive process, emphasizing the intersubjectivity in the construction of mathematical knowledge. This approach enriches the shared experience between different subjects, showing how DG can be an effective means for the exploration and understanding of geometric concepts.

A tool to explore Dynamic Geometry is the GeoGebra software, which allows the interactive manipulation of geometric figures. Students can create and alter figures such as *triangles*, *quadrilaterals,* and *circles*, observing how their properties change or remain invariant. This direct manipulation helps to reveal the essences of the geometric figures, allowing students to perceive, for example, how the sum of the angles of a triangle remains constant regardless of the changes made to the sides. The phenomenological focus is on the experience lived during the interaction, on the perception of transformations and on the intuitive construction of concepts.

Regarding the study of *trigonometry*, which deals with the relationships between the angles *and* sides of triangles, Text 5 "Investigating and exploring with technology: a possibility for teaching trigonometry" argues that phenomenology can help explore how students experience and interpret trigonometry research activities, allowing educators to better understand the perceptions and meanings that students attribute to the learning. This understanding can, in turn, highlight more effective and student-centered pedagogical practices, promoting a more meaningful and collaborative learning environment.

Phenomenology can be used to analyze how the concept of angle is intuited and how trigonometric relations are understood from sensible experience. By exploring the essences of *triangles* and their properties, it is possible to investigate how the concept of *sine*, *cosine,* and *tangent* present themselves to consciousness and how they are used in solving practical problems. An example of a best practice present in the analyzed texts is the use of instruments such as the protractor and the ruler to measure angles in real triangles, constructed on paper or in three-dimensional models.

Students can explore how trigonometric relations arise naturally from these measurements, understanding how sine, cosine, and tangent are more than simple formulas, but relations directly intuited from sensible experience. This phenomenological approach allows students to develop a deeper understanding of these *functions*, rooted in practical experiences and observation.

*Polyhedra*, geometric solids formed by *polygonal faces*, can also be studied under phenomenological conception, exploring how their shapes and properties are intuited and perceived directly by sensory experience. By manipulating or building physical models, students can experience and understand the spatial structure of polyhedra, revealing their geometric essences through practical and visual interaction.

Text 7, "Demonstration with technology: a study on the number of diagonals of a prism" brings a study on *prisms*, which are three-dimensional figures that have flat and parallel faces and offer an opportunity to explore spatial perception and geometric intuition from a phenomenological perspective. Phenomenology, in this case, can help to understand how the notion of prism is constituted by the student and how this constitution is related to formal mathematical properties, such as the calculation of areas and volumes and the study of the *diagonals of a prism*. A hands-on activity for the study of prisms involves building physical models using materials such as paper, cardstock, or even building blocks.

Students can assemble different types of prisms and explore their properties by manipulating the models, counting the *faces*, *edges,* and *vertices*, and calculating the base areas and *volume*. The experience of assembling and manipulating the prisms allows students to intuit the spatial structure and understand the geometric relationships in a phenomenological way, building the concept of prism from the concrete interaction with the object.

Another theme discussed from the phenomenological conception is the *mathematical demonstration*, where students, when constructing and justifying each step, perceive the mathematical truth emerging gradually and consistently. For example, when exploring the *bisector*, hands-on activities that involve constructing split angles allow students to experience the *symmetry* and equality of the parts, connecting formal knowledge to sensible experience. Similarly, the concept of *midpoint* can be worked through manual measurements and constructions, where students directly perceive the equitable division of segments, revealing the midpoint as an intuitive geometric phenomenon that emerges from direct interaction with space.

Phenomenology offers a powerful tool to investigate perception and reflection in the constitution of mathematical concepts, revealing the structuring factors behind these constructions and offering insights on how to improve mathematics education. By relating these geometric concepts to Husserlian Phenomenology, a field of investigation is opened that seeks to understand not only mathematical formality, but also the lived experience of the subject who learns, explores and understands the concepts. The phenomenological approach allows for a deeper and more intuitive understanding of mathematics, revealing the fundamental structures that underpin mathematical knowledge and offering new perspectives for research and teaching. Thus, in the work with Phenomenology, the spaces of Geometry and other areas of Mathematics are experienced spaces, and mathematical knowledge is that which manifests itself in this experience.

IN2 - DIFFERENTIAL AND INTEGRAL CALCULUS

Calculus, often seen as an abstract and challenging discipline, can be re-signified through phenomenology, which promotes meaningful learning by connecting mathematical concepts with students' daily experiences, as evidenced in text 8, entitled "The Teaching of Differential and Integral Calculus I: Possibilities for Investigation," examines the connection between the teaching of *Differential Calculus* and *Integral Calculus* and phenomenology. The study highlights the relevance of an investigative approach, which allows students to explore and understand mathematical concepts. The phenomenological investigative process allows students to understand Calculus, taking it as a phenomenon, to which they intentionally turn, through perception and reflection, apprehending the implications of these acts, and giving them the mathematical structure.

In the texts analyzed, some specific themes of calculus, *functions*, stand out, such as in Text 13 "Contributions to Instructional Design and Cybertraining through Student Feedback on Interactive Mathematical Comics", which highlights the aesthetic experience and the experience of students when interacting with Interactive Mathematical Comics, which mix graphic and narrative elements. Such experiences give the correlates of perception, which are interpreted articulated by the subjects, expanding the understanding of functions and how they are exposed in the life-world. In this context, the aesthetic experience involves the articulation between educational practice and the experience of beauty, where students become emotionally and cognitively involved with the content, such as when analyzing the situation of the taxi driver and the mathematical function that determines the cost of the ride. This approach leads students to a reflection that goes beyond the mere application of formulas, promoting a deeper and more contextualized understanding of the concept of function.

When it comes to *Differential Calculus*, phenomenology offers an approach that emphasizes the construction of knowledge from the student's experience. In the study of *derivatives*, for example, it is not treated only as a mathematical tool, but as a concept that must be lived and understood in its essence. Through the analysis of how functions behave, students are invited to explore the ideas of variation and change, observing how these changes are reflected in the first and second order derivatives. This approach allows for a more intuitive understanding of the concepts of Differential Calculus.

*Graphs* also play a fundamental role in the teaching of calculus, and Husserlian phenomenology recognizes their importance in the constitution of mathematical knowledge. When studying charts, students are not just passive spectators; They are encouraged to actively interact with graphical representations of roles. This interaction allows students to notice how changes in roles are reflected in their graphs, making it easier for them to understand concepts such as maximums, minimums, and inflection points. Phenomenology, in the experiences of perception and reflection, which bring mathematics as aesthetics and beauty, transforms graphs into powerful learning tools, which help students to better visualize and understand the abstract concepts of Calculus.

Integral, another central concept in the teaching of Calculus, is worked under a phenomenological context in the methodology that allows the student to experience, for example, the concept of area under the curve. This approach allows students to learn the idea of integral in a more meaningful way, connecting it with the notion of accumulation and continuous summation. The connection between the defined integral and the primitive function is explored so that the student can experience and understand the relationship between these concepts.

Digital technologies also play an important role in working with Husserlian phenomenology in the teaching of calculus. Tools such as *GeoGebra* allow students to visualize and manipulate functions and graphs, realizing that what is shown in the software interface as change is correlated to their act of moving, and understanding that the knowledge that is realized is also a corporeal knowledge, because the student can look at itself, and understand how its movement implies the constitution of knowledge. Technologies, when worked with a phenomenological perspective, are not seen only as an auxiliary resource, but as an extension of the student's intentionality. When the theme is

Calculus, it is exposed in digital interfaces and is explored in students' intentional actions, which provoke configurations and misconfigurations, graphic and conceptual, which, when observed, can reveal invariants, also graphic and conceptual, with which students can mathematically define what the interface shows.

Finally, Husserlian phenomenology proposes a teaching of Calculus that is centered on the student and that promotes the perceptual experience as a soil from which to develop theory, concept, and mathematical knowledge. This approach is not limited to imparting knowledge, but seeks to create a learning environment where students can explore, question, and build their own understanding of mathematical concepts. By integrating the themes of functions, differential calculus, graphs, and integrals into an investigation process, phenomenology offers a pedagogical approach that promotes deeper, contextualized, and meaningful learning.

IN3 - ALGEBRA AND FUNDAMENTALS OF MATHEMATICS

The phenomenological approach suggests that knowledge is not just a static definition, but a continuous process that unfolds in understandings and interpretations, mediated by language and social interaction. This can be seen in Text 6, "The game Slice Fractions as a space to teach fractions and the opening to the constitution of mathematical knowledge", in which the teaching of fractions is related to phenomenology by emphasizing that the constitution of mathematical knowledge occurs through the experience and sensitive experience of the student, who intentionally turns to the object of study, in this case, the *fractions*. Thus, when playing the Slice Fractions game, students not only interact with fractions as mathematical concepts, but also experience and express their findings, allowing knowledge to become intersubjective, reflecting the dynamics between the subject and the mathematical object in a context of active and collaborative learning.

In text 16 "A proposal for teaching the structures of algebra inspired by a phenomenological conception of the construction of its knowledge", the relationship between the teaching of *algebra* and phenomenology offers a rich and innovative perspective for the understanding and construction of mathematical knowledge. Phenomenology, as a philosophical approach, emphasizes the lived experience and the perception of the subject in relation to the world. By applying this perspective to the teaching of algebra, we can foster a learning environment that values students' active construction of knowledge rather than a mere transmission of information.

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In this context, phenomenology suggests that students should be encouraged to explore algebraic structures in ways that make sense to them. This implies a process of discovery, where students engage with expressions, their definitions and properties, such as rings and bodies, from their own experiences and intuitions. Thus, not only are axioms and theorems memorized, students are invited to experience the relationships and operations that define these structures, allowing for a deeper understanding.

Concepts such as *natural numbers* and the multiplication operation  are also addressed in the analyzed texts. They are proposed in order to connect the students' daily experience with algebraic abstraction. For example, when working with *algebraic expressions* that involve multiplication, students can relate these operations to practical situations, such as calculating areas or distributing objects, facilitating the transition from arithmetic to algebraic thinking. This approach not only enriches learning but also helps students see mathematics as a dynamic and ever-evolving discipline that is connected to diverse areas of knowledge.

In addition, Phenomenology highlights the importance of the historical and cultural context in the formation of knowledge. In the teaching of algebra, this can be translated into the exploration of the origins of algebraic structures and how they have developed over time. This perspective can include *mathematical logic*, which serves as the basis for the formalization of the demonstrations and the construction of rigorous arguments. By understanding how logic articulates with algebraic operations, students can develop a clearer view of how algebraic expressions are manipulated and interpreted.

Finally, by integrating phenomenology into algebra teaching, educators can create a space where  classical *logic* and *mathematical demonstrations* are presented as tools for knowledge construction. The articulation between algebraic expressions, mathematical logic, natural numbers and multiplication, in the phenomenological perspective, occurs in the opening of the possibility of a knowledge that values the lived experience and its correlates, which, once understood by the student and shared with others, for reflection and analysis, can contribute to the structuring and validation of objective knowledge in the flow of the learning movement.

## IN4 - PHILOSOPHY OF MATHEMATICS

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For Bicudo and Garnica (2011, p. 39), the Philosophy of Mathematics "is defined by proceeding according to philosophical thinking, that is, through critical, reflective, systematic and universal analysis, when dealing with themes concerning the region of inquiry of mathematics". It differs from philosophy in that the classic questions of this science, such as "what exists?" or "what is knowledge?", focus on mathematical objects.

A theme on which these questions are raised, in the texts studied, concerns *Gödel's incompleteness theorems*, as addressed in text 15, "On the meaning of beauty in mathematics and what was shown to be beautiful for us in the proof of Gödel's incompleteness theorem". Theorems establish the existence of true propositions that cannot be demonstrated within a consistent formal system, and provoke a deep reflection on the nature of mathematical knowledge. Phenomenology allows one to investigate not only the logical structure of the theorem, but also the experiences and intuitions that led Gödel to his conclusions, revealing the intrinsic beauty that permeates mathematics.

The relationship between Phenomenology and Gödel's theorem can be observed in the way mathematical intuition manifests itself during the process of demonstration. Gödel, in developing his argument, used the arithmetization of metamathematics, a method that transforms statements about formal systems into arithmetic expressions. This transformation is not merely technical; it reflects an aesthetic experience that can be understood phenomenologically. The beauty of mathematics, in this context, emerges from Gödel's ability to articulate complex concepts in ways that resonate with mathematical intuition, allowing other mathematicians and philosophers to realize the depth of his discoveries.

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In addition, Phenomenology can illuminate the experience of incompleteness that Gödel's theorem imposes on the field of mathematics. Incompleteness is not just a technical limitation, but rather a revelation about the limits of human knowledge. Through a phenomenological lens, it can be argued that this incompleteness provokes a reassessment of the search for absolute truths in mathematics. The experience of confronting incompleteness can be seen as an invitation to reflect on the nature of mathematical knowledge, leading to a deeper appreciation of the nuances and complexities that characterize the discipline. Finally, phenomenology also allows us to consider the cultural and philosophical implications of the incompleteness theorem. Gödel's work not only challenges traditional notions of consistency and completeness but also influenced twentieth-century philosophical thought, leading to new interpretations of the nature of truth and knowledge.

Phenomenological analysis of the theorem reveals how aesthetic experience and mathematical intuition intertwine, providing a richer and more multifaceted understanding of the impact of Gödel's theorem on mathematics and philosophy. Thus, phenomenology not only enriches the appreciation of the theorem, but also broadens the horizon of possibilities for the investigation of mathematical knowledge.

## WEAVING A SYNTHESIS AND ARTICULATING FINAL UNDERSTANDINGS

Phenomenology invites us to look at mathematics not only as a set of abstract rules and formulas, but as a discipline that emerges from the relationship between the subject and the world, where mathematical concepts are meanings that reveal themselves through direct experience. Throughout the research, it is noted that most of the works that apply phenomenology to mathematics tend to focus on Geometry. This emphasis can be justified by the visual and spatial character of Geometry, which naturally lends itself to a phenomenological approach. However, the concentration in this field leaves important gaps, which are the other areas of mathematics that could also benefit from a phenomenological analysis.

Areas such as algebra, analysis, and statistics, while perhaps less intuitive, also contain rich potential for phenomenological exploration. For example, the concept of function in algebra can be investigated from the way students experience the relationship between variables and results. Similarly, in analysis, the notion of limit can be studied through the experience of approximation and convergence, and in statistics, the

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understanding of probabilities can be analyzed by the way the student perceives and interprets uncertainty and variation.

Phenomenology, as the soil with which Mathematics is studied, has the potential to humanize and deepen teaching, by recognizing that mathematical meanings, as well as all scientific knowledge, are born from perceptive experiences in the life-world, in which they are subjects and co-subjects of these experiences. Encouraging the application of this perspective in areas beyond geometrics can enrich the teaching of mathematics as a whole and offer students a deeper understanding of the discipline. Therefore, there is a demand to broaden the horizons of research in this field, seeking to work on other mathematical themes from a phenomenological perspective.

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