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MAPPING OF THE TEACHING AND LEARNING PROCESS OF PLANE GEOMETRY UNVEILED IN SCIENTIFIC PRODUCTIONS IN BRAZIL

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ABSTRACT

This research focuses on the need to answer the question: how is the didactic-methodological process presented/developed in the initial training of the teacher and the teaching of Plane Geometry in graduate research in Brazil? In this context, the objective is to unveil and map the processes of teaching and learning geometry, with regard to the calculation of polygon areas and perimeters, based on the academic productions undertaken in the national graduate programs. The research was developed in a qualitative approach, of the bibliographic review type, through a methodology adopted in a mapping. The methodological procedures chosen were an advanced search of theses and dissertations in the Digital Library of Theses and Dissertations (BDTD), using filters with inclusion and exclusion characteristics, for the classification and organization of the selected scientific productions. For data analysis, we used categorization regarding the training of mathematics teachers; theories and methods for the teaching of Plane Geometry; and dialogue of pedagogical praxis between Higher Education Institution (HEI) and school. Among the results, it was noticed that, in the southern region of Brazil, the scenario of the productions found after the filters, there are HEIs with graduate programs actively producing research on the theme of teaching and learning of Plane Geometry. The study of area and perimeter calculation is a small part of what is researched in graduate programs with the line of research in Mathematics Education; with interlocutions between the training of the Mathematics teacher and the theories, methods and processes of teaching geometry. It is worth noting that the southern region of the country also fully included master's and doctoral programs with this line of research.

Keywords: Didactic-Mathematical Knowledge. Mathematics teacher. Plane Geometry. Theory. Pedagogical Practice. Teaching.

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INTRODUCTION

This study is based on the premise that mathematical thinking is closely linked to human development, in whose evolution and development, conceptions regarding mathematical making and teaching stood out, as Martins (2012) points out, because Mathematics originated from the relationship between man and nature. By the way, Mathematics Education is a recent study in Brazil, since in the 1990s alone, there was a significant number of scientific productions in Brazil. In view of the didactic evolution and the teaching and learning of Mathematics, it is proposed to answer: how is the methodological didactic process presented/developed in the initial training of the teacher and the teaching of Plane Geometry in graduate research in Brazil? In this sense, the research aims to unveil the processes of teaching and learning geometry, particularly with regard to the calculation of polygon areas and perimeters, from the state of the mapping of academic productions undertaken in national *stricto sensu graduate programs* in Brazil.

In the search for an answer to the problem-question, the research was developed in a qualitative approach, which, according to Denzin and Lincoln (2006), involves an interpretative approach to the world, which demonstrates that its researchers study things in their natural scenarios, trying to understand the phenomena by the meanings that people give them. Thus, for this research, a methodology based on a mapping of scientific productions was adopted. Mapping can:

> [...] to contribute effectively to the theoretical studies of a specific area of knowledge, allowing the identification of the conceptual aspects involved in the study, limitations and potentialities, in addition to enabling the categorization of information (Motta; Kalinke; Mocrosky, 2018, p. 69).

Systematic mapping can also lead to the construction of a literature review, which, according to Falbo et al. (2017, p. 1), is "a broad review of existing primary studies on a specific research topic that aims to identify the evidence available on that topic". Vosgerau and Romanowski (2014) argue that the works that follow this line have as their main focus to observe several works on the chosen theme and, subsequently, to list the possible gaps that exist for new interpretations and research. In this sense, research mapping is more concerned with the characterization of studies than with making conjectures and analyses about the information investigated. Therefore, it unveils the approximations between teacher training, theories and dialogue with the teaching of Plane Geometry.

At first, as a methodological procedure, a search was made in the repository of the

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Brazilian Digital Library of Theses and Dissertations (BDTD). From an advanced search, in all fields, for Plane Geometry, basic education, mathematics teacher training, teaching and learning, 37 (thirty-seven) scientific productions were found. After being organized and filtered, they were categorized for a descriptive and qualitative analysis, with 03 (three) dissertations and 03 (three) theses. This BDTD survey was conducted in January 2024.

These scientific works will be presented in tables, according to the categories of analysis: the training of the mathematics teacher, theories and methods for the teaching of Plane Geometry and the Interlocution of the pedagogical praxis between IES and school. The graduate programs in each federative unit and their historiography will also be mapped, as to the region of the country.

METHODOLOGY

METHODOLOGICAL PROCEDURES

This is a qualitative research, which, for Gaskell (2002, p. 65), "provides the basic data for the development and understanding of the relationships between social actors and their situation". It also defines that "the objective is a detailed understanding of beliefs, attitudes, values and motivation, in relation to people's behaviors in specific social contexts". It is a research that adopts a systematic mapping methodology. Fiorentini et al. (2016) define research mapping as follows:

> “[...] a systematic process of gathering and describing information about the research produced on a specific field of study, covering a certain space (place) and period of time. This information concerns the physical aspects of this production (describing where, when and how many studies were produced over the period and who were the authors and participants of this production), as well as its theoretical-methodological and thematic aspects" (FIORENTINI et al., 2016, p. 18).

Therefore, a methodological procedure was initially adopted that used the repository of the Brazilian Digital Library of Theses and Dissertations (BDTD). With an advanced search, according to the one available in the digital library itself, the following topics were consecutively searched in all fields and as inclusion criteria; *basic education*; *mathematics teacher training;* and *teaching and learning*. The result brought to light 37 (thirty-seven) scientific productions between theses and dissertations, which, a posteriori, were selected by filters with exclusion criteria: modality different from regular education, publication outside the period between 2012 and 2022 and different theme of plane geometry. Such exclusion criteria/filters are justified by the need to organize the data, as well as the best

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parameter for categorization and analysis according to the methodology of this mapping.

In view of the exclusion criteria, 11 (eleven) were outside the period determined from 2012 to 2022; 09 (nine) repeated productions were excluded; 11 (eleven) exclusively contemplated the teaching modality for Youth and Adults (EJA) and Professional and Technological Education (EPT), which were also excluded because they dealt with topics such as andragogy and heutagogy. Productions with themes other than Mathematics and Plane Geometry were also found – such as the study of diffraction in Physics and the study of Statistics – as well as productions with access unavailable for consultation and reading in the repository, which were also excluded. Under these conditions of appraisal, 31 (thirty-one) of the 37 (thirty-seven) initially selected were withdrawn. Thus, as an object of research, there was a selection of 06 (six) scientific productions, being 03 (three) dissertations and 03 (three) theses.

The development of this scientific work adopted a categorization procedure. This is because it helps to define a relevant role of organization for the mapping, description and analysis of Brazilian scientific productions already classified. Here is how the categorized organization took place: (i) the training of the Mathematics teacher; (ii) theories and methods for teaching Plane Geometry; and (iii) the interlocution of pedagogical praxis between HEI and school. The categories can lead us to the achievement of the research objective, unveiling the processes of teaching and learning geometry, with calculation of areas and perimeters of polygons, through a content analysis in the academic productions developed in the national *stricto sensu graduate programs*. Therefore, the analysis of the collected data focused not only on a description, but also on a qualitative, analytical and essay discussion, with possible significant inferences, focusing on the answer to the initial problem of this research. In addition to mapping, it brings to light the theories and methodologies developed in the initial training of the Mathematics teacher, analyzing the relationship between the institution of higher education and the contributions, direct or indirect, in basic education, presented in the scientific productions, classified as the object of research in this study. It should be noted that the research, when contemplating basic education, considered all teachers who teach Mathematics, whether in the early years of elementary school, or in the final years of elementary school, or in high school.

In this organization, the research aimed to analyze in scientific productions the initial and continuing training of the Mathematics teacher, his praxis and possible contributions to the teaching of Plane Geometry. It was also sought to know the theories, unveil the trends

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and relevant methodological processes in these researches published in graduate programs between 2012 and 2022, a moment that contemplated the phase of remote teaching throughout the national territory, when a state of pandemic was declared The period brought significant and updated data that can direct and foster new studies on this topic, post-pandemic, as well as a direction for the organization and adaptation of new practices after remote teaching.

RESULTS

PRESENTATION OF SCIENTIFIC PRODUCTIONS

The selected scientific productions were identified by letter and number. The letter "D" was used when it was of the dissertation type and "T" when it was the Thesis type. The number accompanied by a letter indicates the order of selection. Here, we opted for ascending order, according to the year of publication, after being classified, according to the advanced search of the BDTD itself, and filtered by the researcher by exclusion criteria, adopted, described and defined in the methodological procedures. Six (6) scientific productions were found that were adequate to the objective and methodology proposed in the research, as shown in Chart 01. The productions appear in terms of their type, identified with Thesis (T) and Dissertation (D), theme, their author and advisor, year of defense and higher institution.

Describing the research object adopted, there were 03 (three) dissertations and 03 (three) theses, all produced in Brazilian graduate programs between the years 2012 and 2022, presented in Table 01 below. The time period, consisting of an interval of 10 (years), considered a relevant period, encompassed scientific productions before and after the covid-19 pandemic and, mainly, research developed during remote teaching.

Table 01: Presentation of the selected research

D	Theme	Author	Advisor	Year/ Institution
1	The contribution of origami in geometry: developing skills and concepts in the training of mathematics teachers	Elisane Strelow Gonçalves	Prof. Dr. Josias Pereira da Silva	2018/UFPel
2	Sociocultural mathematics *versus* academic mathematics in the context of the future teacher: an ethnomathematic study	Paulo Policarpo Campos	Prof. Dr. Everton Lüdke	2018/UFRS
3	History of mathematics in geometry activities: a proposal for initial teacher training	Kamila Gonçalves Celestino	Prof. Dr. Márcio André Martins	2020/Unicentro/PR

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4	Learning to formulate in order to learn how to solve: registers of semiotic representation and creativity in the learning of geometry	José Luiz Rosas Pinho	Prof. Dr. Méricles Thadeu Moretti	2021/UFSC
5	Teacher training for the teaching of mathematics in the early years mediated by potentially significant teaching units (UEPS)	Graziela Ferreira de Souza	Prof. Dr. Nilcéia Aparecida Maciel Pinheiro	2021/FTFPR
6	Development of geometric thinking in the early years of elementary school based on Van Hiele's theory: contributions from a teacher training course	Marilda Delli Colli	Prof. Dr. Zenaide de Fátima Dante Correia Rocha	2022/UTFRS

Source: selection of theses and dissertations at BDTD.

These are the dissertations (D) produced in graduate programs, such as: Graduate Program in Teaching of Natural Sciences and Mathematics, area of concentration in Teaching and Learning of Natural Sciences and Mathematics, State University of the Midwest, in the city of Guarapuava, in the state of Paraná/PR; Graduate Program in Mathematics Education of the Institute of Physics and Mathematics of the Federal University of Pelotas/RS; Graduate Program in Mathematics Teaching at the Technology University of Londrina/PR. Theses (T) are from the Graduate Program in Scientific and Technological Education at the Federal University of Santa Catarina, in Florianópolis/SC; Graduate Program in Science and Technology Teaching in Ponta Grossa/PR; and Graduate Program in Science Education at the Federal University of Rio Grande do Sul, in Porto Alegre/RS. Coincidentally, the selection of theses and dissertations exclusively contemplated a single region of Brazil. The South region was contemplated with public institutions of higher education in its 3 (three) states: Paraná, Santa Catarina and Rio Grande do Sul.

For the presentation of dissertations and theses, the selection involved scientific productions exclusive to the southern region of Brazil. The Sucupira platform (2024), in the field *of evaluated* and *recognized courses*, informs a *ranking* of graduate programs, and the region is in 3rd (third) place, after the Southeast and Northeast regions. This platform reported a number of 1559 master's and doctoral courses - academic and professional - in the three federative units - Paraná, Santa Catarina and Rio Grande do Sul - that make up the southern region of Brazil. It is believed to be a significant factor for this mapping in the selection of scientific productions for research.

In view of this, the research portrayed here leads us to analyze the initial and

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continuing training of the Mathematics teacher, his pedagogical praxis and possible contributions to the teaching of Plane Geometry. It was also sought to know the theories, trends and relevant methodological processes through the selected researches. Therefore, we resort to a theoretical basis that grounds and illuminates the textual, descriptive and interpretative construction of this work.

SUBSTANTIATING THE CATEGORIES OF ANALYSIS

In the search for the answer to the research question, a systematized literature review was organized, called the Theoretical Framework. This denomination occurred to exclude any and all aspects of staticness. Severino (2004) warns that the theoretical framework serves as a guideline and orientation for paths of reflection, and not as a model and form of thought. The author highlights the dynamic role of the theoretical framework and completes by stating "creative thought cannot formally enslave itself to it" (Severino, 2004, p. 162). And it is by recognizing the function, its limits and dynamism that the theoretical framework was built, believing in the possibility of having it as "a filter through which one sees reality, suggesting questions and indicating possibilities", as defined by Luna (2002, p. 32). In this sense, it was intended to organize a theoretical framework that would support the discussion and reflection on the theses and dissertations already selected. The categories adopted for content analysis in this research will guide the theoretical framework.

By way of clarification, the theoretical framework was constructed and organized with regard to its theme, defined by categories. The categories in this research are (i) the training of the mathematics teacher; (ii) theories and methods for teaching Plane Geometry; and (iii) the interlocution of pedagogical praxis between HEI and school. Together, they bring close relationships in the productions classified for this study. In this sense, the construction of the theoretical framework presents and discusses theories, in their definition, in their characteristics and in the collaborative inferences to the process of teaching and learning geometry, correlated to each category.

It should be noted that the systematic review enriched the analysis of the data and the organization by categories was based on the theme of each selected scientific production. Thus, the category *of Mathematics teacher training* (author's emphasis) came as a guide in the understanding of the D1, T2, D3 and T4 researches, which were developed in the field of initial and continuing education, of the teacher who teaches

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Mathematics and of the teacher trainer of the Mathematics degree. The category *theories and methods for the teaching of Plane Geometry* (emphasis added) guided the understanding of relevant theories in the construction of mathematical knowledge in T5 and D6. The category *Interlocution of pedagogical praxis between HEI and the school* (emphasis added) contemplated and guided the analysis regarding the methods and trends of teaching Mathematics, the relations with the official documents in force and the contributions to the teaching of Plane Geometry in basic education in all the selected scientific productions.

To discuss Mathematics teacher training, teacher training was initially used, according to Nóvoa (2022):

> despite all the difficulties and problems faced with teacher training, any change in education and pedagogy can only come from within the teaching profession/professional, always with strong external support, namely from academics and universities (Nóvoa, 2022, p. 3).

Indeed, teacher training is fundamental for the necessary and urgent transformations in education. It can be highlighted that:

> Teacher training is a decisive field for change if it is possible to avoid the subordination of teachers and to contribute to the enhancement and strengthening of the teaching profession. It is this connection between training and profession [...] that has as its central point the professional knowledge of teachers (Nóvoa, 2022, p. 3).

It is known about the historical struggles faced for changes and transformations in favor of good teacher training and the quality of education. For Nóvoa (2022), since the second half of the nineteenth century, training has a purely instrumental appearance, with practice being a reference for teacher training. This is a traditionalist view, rooted to the present day in the field of teacher education. Modern and innovative trends have come over many years to challenge the traditionalist view of teacher education. The transformation becomes more evident in the first two decades of the twentieth century, when "the disciplines of pedagogy, psychology and sociology of education, history of education, organization of teaching systems, education or curriculum development" emerge (Nóvoa, 2022, p. 5). They start to occupy more and more space in teacher training programs in the implementation of the New School. This, according to Nóvoa (2022), is the most important educational movement of pedagogical modernity. With this movement, educational sciences emerged - in France in 1960 - which "gave rise to a true explosion of innovative practices"

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(Houssaye, 1984, p. 47).

Public policies aimed at teacher training in Brazil have caused real changes in education throughout the national territory: there is the valorization of children – from the New School, dating from 1930 onwards – with the valorization of autonomy and respect for freedom; the concern with educational technology in 1964; and, from 1980 onwards, the spontaneity of the New School movement stands out, which minimized the effects of technicism. It is an effect arising from industrialization in Brazil, which brought new contours to the training of Mathematics teachers.

THE TRAINING OF MATHEMATICS TEACHERS IN BRAZIL

In the middle of the twentieth century, an economic strategy aimed at Brazil's late industrial development stimulated a significant growth in primary education. This resulted in a growing need for Mathematics teachers, generating a situation of great demand and importance.

Currently, the initial training of mathematics teachers is the responsibility of higher education institutions (HEIs), and they, by themselves, are not able to ensure adequate professional training of teachers. We agree with Nóvoa when he states that universities:

> they are indispensable as spaces of knowledge and science, but they need the collaboration of schools and teachers of basic education, and other actors. This collaboration cannot be based on hierarchies of power and unbalanced relationships, namely between university and school professors (Nóvoa, 2022, p.13).

The transformation of the university, with regard to teacher training, can be identified in its current organization, in the search for adjustments in the Political Projects of Courses, followed by the Opinions of CNE/CP No. 09/2001 and CNE/CES No. 1,302/2001. Basically, these guiding documents outlined and formatted a new profile of graduates from higher education courses, notably those from teaching degrees.

In the light of these perspectives proposed in the opinions, official documents used to support the advances in the initial training of the Mathematics teacher, the figure of the *specialist teacher emerges*, who, according to Nacarato (2006, p.132), is the Mathematics teacher who works in the final years of elementary school and high school. He is the professor who graduated from the degrees in Mathematics.

As advances, Nacarato (2006, p. 133) emphasizes "the collective work of the training institutions"; "the role played by the teaching practice and the supervised internship" and the respect, considerations and innovations brought by the opinions of CNE/CP 09/2001 and CNE/CES 1.302/2001. Consequently, there is a process of engagement between teacher training institutions regarding their institutional organization for a Course Political Project (PPC) aligned with the proposal of a teaching degree with equity, commitment and focus on teacher training. The training institution can and should assume a role of link between the student of the Mathematics degree course with the school and other spaces through teaching practice and supervised internship. It is believed that, using practice and internship, it is possible to provide students with a moment of discussion and reflection on the construction of their own teaching practice. Undoubtedly, it is of great relevance, in the initial training of the Mathematics teacher, this moment of interaction, after all it provides "the constitution of teaching knowledge and the resignification of school knowledge during the Teaching Practice and the Internship, having as its training axis the research and/or the systematic reflection of practice" (Fiorentini et al., 2002, p.145).

The responsibility of the training institution is great, especially when one takes into account the profile of the graduate of the degree in Mathematics, in the future teacher who teaches Mathematics, destined to teach in the final years of elementary school and high school. In addition to the teaching practice, content knowledge and pedagogical didactic knowledge built in initial training are considered. The link between the knowledge of specific content of Mathematics and the theories and methods for teaching must be contemplated in the curricular plan of the Teaching Degree in Mathematics, in order to achieve "advancement" (Nacarato, 2006) around the construction of didactic-mathematical knowledge.

TEACHER'S DIDACTIC-MATHEMATICAL KNOWLEDGE: THEORIES AND METHODS FOR TEACHING PLANE GEOMETRY

In order to conceptualize the didactic-mathematical knowledge of the teacher who teaches Mathematics, as well as to know the theories that study and develop this type of knowledge, a theoretical framework was created, based on the content knowledge of Shulman (1996) and Ball et al. (2008). The theoretical concepts described here were identified in the mapped scientific productions. In addition to the concept, theoretical characteristics and the pedagogical and methodological relations of didactic-mathematical knowledge were presented, defined by Godino and Batanero (1994) and Godino, Batanero and Font (2007). In the context of the construction of knowledge of Plane Geometry, a

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characterization of the Logical-Mathematical Thinking of Piaget (1964) and Kamii (1982; 1985) was made, in order to reflect in the research, in an epistemological way, the knowledge of the teacher, as proposed in this research. Regarding the theoretical foundation of geometric thinking, he used Fischbein (1993), Duval (1995) and Pais (1996). And to understand the method, we turned to Van Hielle (1957). This category is justified to contemplate, mainly, the analyses of T5 and D6 productions.

Regarding the definition and epistemological characteristics of the knowledge of the Mathematics teacher, Piaget (1985) states that knowledge emerges from man. Overcoming the dichotomous idea between content knowledge and methodology, it is believed in the interaction and diversity of ideas, through which the teacher must build his own knowledge to teach. The teaching process needs to start with the understanding of teacher knowledge, arriving at knowledge to learn and to be taught, as stated by Shulman (1986, 1987). It is crucial, then, that the teacher's work should be planned basically in a series of activities that provide specific instruction, among new learning opportunities for both the student and the teacher.

One of the pioneers in the studies of teacher knowledge (Shulman, 1986) proposed a model in 3 (three) categories: *content knowledge*, *pedagogical content knowledge* (PCK) and *curricular knowledge* (emphasis added). Subsequently, (Shulman, 1987) expands to 7 (seven) categories, which he calls *categories of basic knowledge*. Of the seven categories, *pedagogical content knowledge* (PCK) is of special interest, since it identifies the *distinctive body* of knowledge for the teaching process (Shulman, 1987 apud Pino-Fan and Godino, 2015, p. 89). *Pedagogical content knowledge* (PCK) represents the combination of content and pedagogy in understanding "how a specific topic, problem, or theme is organized, represented, and adapted" (Pino-Fan and Godino, 2015, p. 90) to meet the diversity of interests as well as the abilities of students.

Still based on the ideas of Shulman (1987), regarding the notions of content knowledge and pedagogical content knowledge, Debora Ball (2008) et al. advocate the concept of *mathematical knowledge for teaching* (MKT), defined as "the mathematical knowledge that the teacher uses in the classroom to produce instruction and growth in the student" (Hill, Ball and Schilling, 2008, p. 374 apud Pino-Fan and Godino, 2015, p. 89). And so other researchers followed, building a relevant process in the field of research on the knowledge of the teacher, in their training. For Silverman and Thompson (2008), "Although mathematical knowledge for teaching has begun to gain attention as an important

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concept [...], there is limited understanding [...] how it can be developed in the minds of teachers" (p. 499).

In this context, the research community still asks about teacher training: what is it? How can mathematical knowledge be recognized and how can it be developed for teaching?

In the search for answers with new research and, mainly, with the diversity of dimensions for understanding the knowledge of the Mathematics teacher in the context of the issues of recognition and development of knowledge, Pino-Fan and Godino (2015) assert that "there is no universal agreement on a theoretical framework to describe the knowledge of mathematics teachers" (p. 95) and add:

> The models of mathematical knowledge for teaching elaborated [...] in mathematics education include very broad categories, [...] that allow a more detailed analysis of each type of knowledge involved in an effective teaching of mathematics. (Godino, 2009, p. 19)

From the observations of Godino (2009) and characteristics of the studies that were already being discussed among researchers in the field of Mathematics Education, such as the discipline of Didactics of Mathematics[3], throughout Europe, the theory defined by Godino (2009) as Didactic-Mathematical Knowledge of the Teacher, in the Ontosemiotic approach, emerges.

Supported by a *broader refinement* for the understanding of the knowledge of the Mathematics teacher, Godino (2009) proposes the model of didactic-mathematical knowledge (CDM), which interprets and characterizes the teacher's knowledge in three dimensions: *mathematical dimension, didactic dimension* and *metadidactic-mathematical dimension* (Pino-Fan & Godino, 2015, p. 96). The *mathematical dimension* includes two subcategories of knowledge, defined by *common knowledge of the content* and *expanded knowledge of the content*, which are related to the specific *mathematical knowledge* that is considered necessary, but not sufficient for the practice of teaching. Therefore, in addition to the mathematical content, the teacher must have knowledge about the various factors that influence the planning and implementation of teaching. From the *Didactic Dimension* of the CDM, there are subcategories, also defined by facets and with characteristics of analysis for the knowledge built and developed by the Mathematics teacher.

[3] Didactics of Mathematics is a term currently in disuse in Brazil, however, out of respect for European authors, such as Godino (Spanish) and his collaborators, cited, the original spelling was maintained.

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The subcategories or facets are the *epistemic facet*, a specialized knowledge in the *mathematical dimension*; the *cognitive facet* (emphasis added), which derives from a "knowledge about the cognitive aspects of students"; the *affective facet* is the "knowledge about the affective, emotional and attitudinal aspects of students"; the *interactional facet*, defined as "knowledge about the interactions that occur in the classroom". Next, there is also the "knowledge about the resources and means that can improve student learning", established as the *mediational facet* and the *ecological facet*, which defines the "knowledge about the curricular, contextual, social, political, and economic aspects that influence the management of student learning" (p. 98).

According to Pino-Fan & Godino (2015, p. 99), the epistemic facet of CDM, in summary:

> it will allow teachers or future teachers to answer questions such as: (i) In addition to its solution, is there another way to solve the task? (ii) How would you explain the solution of the task to a student who was unable to solve it through the procedures developed in the classroom? (iii) What knowledge is mobilized to solve the task? (Our translation).

In this context, it is essential that the teacher who teaches mathematics and, in particular, Plane Geometry mobilizes the basic, expanded and didactic knowledge of the specific content that will be taught in class.

In order to analyze the research selected in this study concerning the theories and methods for teaching Plane Geometry, according to the categories, it is necessary to discuss and reflect on theorists who have researched from the theory of Logical-Mathematical Thinking (Piaget, 1964) and Geometric Thinking (Duval, 1995). As for the teaching method, the family (Hielly, 1957) conceptualizes and characterizes a teaching method.

3.5 REMEMBERING THE THEORIES

By resorting to theoretical concepts, one starts, initially, from the concept of mathematical logical thinking, considered from a discourse of Piaget's Genetic Epistemology (1976), which emphasizes the relationship between the child and the environment, from which a cognitive structure is built.

It is relevant for this study to investigate the teaching and learning process, as well as to know the treatment of theories and methods for the construction of mathematical knowledge regarding the teaching of Plane Geometry, in basic education, via selected research. This demonstrates the need to consider that logical-mathematical knowledge, according to Piaget (1978), is a gradual construction, resulting from a child's mental action on the world. Piaget (1978) adds that the evolution of the logic and morality of the human being can be summarized in four stages of mental development: sensorimotor – develops from birth through the *ability to perceive the world*, or rather, in the evolutionary ability to coordinate sensations and their movements; *intuitive or symbolic* – children's logic develops (from the age of 2) and derives from the discovery of the symbol, turned to itself in the affective aspect and knowledge; *concrete operative* – "logic ceases to be intuitive and becomes operative, thinking becomes more coherent, approximately at the age of 12" (Silva, 2014, p. 3); *Formal operative* – it is the stage of adolescence, in which logical thinking reaches thinking by hypothesis, based on previous experiences, becoming capable of reflecting and autonomously organizing the rules and carrying out deliberations.

It is important to highlight that, for the construction of logical-mathematical knowledge, considering the psychogenesis of Piaget (1978), it is crucial to discuss the process of training the Mathematics teacher.

About the theoretical concept of the Construction of Geometric Thinking, it is discussed from Duval (1995), who analyzes the difficulties using a cognitive approach. For the author, in cognitive functioning, the student can develop the understanding, operation and control of the variety of mathematical processes in the classroom. The French philosopher and psychologist Duval (1995) developed the theory of Semiotic Representation Registers, which "analyzes the functioning of cognition linked, above all, to mathematical activity and the problem related to its learning" (Silva and Costa, 2020, p. 3). From this perspective, the understanding is based on the difficulties of students in basic education.

Such representations are essential for the cognitive activity of thought and can show the same object in different ways. According to Duval (2003), semiotic representations are elaborations produced by the application of signs belonging to a system of representations that has its own interventions of signs and functioning. Thus, Silva and Costa (2020, p. 3 apud Duval, 1995) consider that an object of a mathematical nature can be represented by several registers of representation, because "contact and experience with various registers favor the learning process, understanding and production of knowledge in Mathematics" (p. 4). And in view of the diversity of representations, the author introduces two important operations related to Mathematical learning. These are the treatment operations, defined as

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a transformation that occurs within the same register of semiotic representation, and the conversion operation, which is the change that occurs when a mathematical object changes from one semiotic representation to another. An example is the change from algebraic representation to graphical representation.

With regard to geometry, Duval (1995) proposes a model for the development of geometric thinking, also defined through the notion of apprehensions. The model is organized into four groups of geometric apprehensions: perceptual, discursive, operative and sequential.

Each geometric seizure is characterized, according to Costa and Santos (2020, p. 5-6). Perceptual Apprehension is that "which allows the immediate identification or recognition of a shape or object in the plane and in space". This model of apprehension "has the epistemological function of identifying objects in two or three dimensions and in space". The identification of the object is carried out through "cognitive treatments carried out automatically and unconsciously". For Duval (1995), this perceptive apprehension is related to the observer's first look and the interpretation of the geometric shapes of the object.

The characteristics of Duval's Discursive Apprehension (1995, p.6) are related to "[...] a denomination, a legend or a hypothesis". It comprises a justification (of a deductive nature) and the mathematical properties of the observed object/figure (Costa and Santos, 2020, apud Duval, 1995, p. 6).

Regarding Operative Apprehension, Costa and Santos (2020, p. 6) understand it as the "ability to operate on figures", composing, transforming, reconfiguring, comparing geometric objects in order to solve a geometry problem. This apprehension is also "centered on the possible modifications of an initial figure and on the possible reorganizations that these modifications make possible" (p. 6). It is focused on the possibilities of constructing and deconstructing geometric objects.

Duval's Sequential Apprehension (1995) arises when the desire to construct a figure or from the need to describe its construction. It concerns the order of construction of the figure, which does not depend only on mathematical properties, since the technique of using tools is also necessary. For the construction of geometric figures in the plane, the use of rulers, protractor, compass or some software can be exemplified, as exemplified by Costa and Santos (2020).

Duval's Geometric Apprehensions (1995) are fundamentally the first fruits for the Construction of Geometric Thinking and permeate the observation, manipulation,

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construction and deconstruction of a geometric object. They do not demonstrate the hierarchical necessity for the Construction of Geometric Thinking and, therefore, differ from other authors who discuss the need for the hierarchical organization of geometric thinking.

In order to better differentiate the concepts and theories that emphasize in their construction of geometric thinking the need for hierarchical organization, we highlight Van Hielle's model, which brings characteristics opposite to Duval's, but with the same objective of construction of geometric thinking.

The theory created by the Dutch couple Pierre Marie Van-Hiele and Gina Van-Hiele Geodolf (1957) arises from the experiences of the classroom itself, where they identified the problem: "Why did many students, who mastered most mathematical concepts, have difficulties in learning Geometry?" (Costa and Santos, 2020, p. 6).

In this reality, Van Hiele (1957), after analyzing, perceived a diversity of levels of understanding in the process of Construction of Geometric Thinking. He highlights that these levels should be gradual, in a hierarchical way, like the cognitive process. He also states that "the progress of a subject's geometric thinking does not depend on his age, nor on his biological maturity" (Costa and Santos, 2020, p. 7).

Van Hiele (1957), who based his research on Piaget and on cognitive development based on psychogenesis, goes further and defines the pedagogical function as primordial, since "what makes this development possible is pedagogical action" (Costa and Santos, 2020, p. 7). He concludes that the construction of geometric thinking is not only linked to ontogenetic maturation, but also to educational practices and justifies: "what promotes the development of geometric thinking is contact with appropriate activities" (p. 7). As for the contribution to the construction of geometric thinking, the psychogenesis of Piaget (1957) was respected and the adequacy of the pedagogical practice, in a gradual and hierarchical way, to the levels of Construction of Geometric Thinking was considered.

Van Hiele's (1957) model is defined and characterized in 5 (five) levels, which explain the structure of geometric thinking.

In level 1, students perceive geometric objects based on their physical appearance and justify their production/construction considering visual aspects, without explicitly using the properties of the observed object. One can exemplify the representation of a quadrilateral by cutouts "in groups of squares, rectangles, parallelograms, rhombuses, and trapezoids" (Van Hiele, 1957, p. 7).

At level 2, students are able to recognize geometric objects through their properties, adjusting to the more specific knowledge of geometric objects. However, they use a set of properties that are necessary for the identification and description of these objects. They still use a range of properties for object identification. It can be exemplified by the description of a square, in which he uses the properties of "4 sides, 4 right angles, equal sides, parallel opposite sides" (p. 7).

At level 3 or third level, the student, able to order the properties of geometric objects, constructs abstract definitions, differentiates between the necessary properties and the sufficient properties in the definition of a concept. You must also be able to understand simple deductions without the use of demonstrations. An example of this level can be taken when the student "describes a square by the minimum properties: 4 congruent sides and 4 right angles" (p. 7), also defining it as a special case of rectangle and rhombus. In the same way, when, in the case of a square, it knows that it is a parallelogram, because it has parallel opposite sides.

At level 4, students "are able to understand the role of the different elements of a deductive structure and develop original demonstrations or at least understand them" (p. 7). A capacity for structural understanding of the geometric form is built, identifying and qualifying the parts. As an example of this ability to understand, "The student demonstrates the properties of remarkable quadrilaterals by means of the congruence of triangles" (p. 7). It is a form of autonomy in the construction of geometric knowledge.

At level 5, "Students are able to work on different axiomatic systems and study various geometries in the absence of concrete models" (p. 7). We see a structured autonomy, consolidated in a hypothetical and deductive way and with security. An example can be given in the following situation: "the student establishes and proves theorems in a finite geometry" (p. 7). The ability that the student acquires to infer, explain, construct and deconstruct, in a safe way, mathematically, is perceived, which can also lead to an inductive, intuitive, safe and autonomous understanding.

Van Hiele's (1957) hierarchical levels of construction of geometric knowledge are based on Piaget's Logical-Mathematical Thinking. The authors Costa and Santos (2020) discussed Van-Hiele's levels of geometric thinking according to the source of Jehin and Chenu (2000), in the work "How to evaluate geometric reasoning?", in which he presents the levels in a table, ensuring hierarchical linearity.

The development of geometric thinking, in the perceptions of Duval and Van Hiele, occurs in a procedural way. This construction occurs more broadly in the theory of registers of semiotic representation, which analyzes the functioning of cognition linked, above all, to mathematical activity and to the problem related to its learning (Duval, 1995). There is also, adding the perceptions, the need for an adequate practice. Van Hiele (1957) argues that the development of geometric thinking, respecting a cognitive hierarchy, goes through levels that begin "with the recognition of geometric objects by their global aspect, ending with the analysis of different axiomatic systems" (Costa and Santos, 2020, p. 7).

Certainly, the aforementioned theories define and characterize the process of construction of geometric thinking and with possibilities of good results in the process of teaching and learning Mathematics in basic education. The study of these theories, as reference assumptions for the initial training of the Mathematics teacher, guides him to the student's teaching process.

DIALOGUE OF PEDAGOGICAL PRAXIS BETWEEN HEI AND SCHOOL

As a reference for official documents for education in Brazil, the Federal Constitution of 1988, the Law of Guidelines and Bases of Education (LDB), No. 9394/96, as well as Resolution No. 02/2017 and Resolution No. 04/2018 can be cited. These legislative frameworks have characteristics in common, since they are education standards at the federal level and define a range of rights and duties.

In this topic, for the theoretical foundation, the focus is given to Resolution No. 02/2017 and Resolution No. 04/2018, which, consecutively, defines the National Common Curricular Base (BNCC) of Early Childhood Education to the final years of Elementary School and establishes the National Common Curricular Base of High School, also known and described as New High School (NEM).

Thus, for the study to which this research is focused, notably in the attempt to understand the interlocution of pedagogical praxis between IES and school, aspects of this interlocution will be sought, based, above all, on the BNCC. It will also be examined, if presented in the selected national researches, the aspects arising from the curricular organization of the initial training courses based on the CNE/CES Opinion No. 1.302/2001. This opinion defines the national curriculum guidelines for Mathematics Degree courses.

DISCUSSION

The analysis of the data of this research occurred by mapping, which will be described below. Its description will be obeyed according to the basis of the specificity of

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the research methodology. In this sense, it was understood and adopted that the term research mapping differs from the state of the art of research, because mapping is more concerned with the descriptive aspects of a field of study than with its results. The state of the art, in turn, refers to the identification, location and description of research carried out in a given time, space and field of knowledge.

Research mapping was understood as a systematic process of gathering and describing information about research produced on a specific field of study, covering a certain space (place) and period. Such information concerns the physical aspects of this production (describing where, when and how many studies were produced over the period and who were the authors and participants of this production), as well as its theoretical, methodological and thematic aspects.

The mapping of the survey highlights the southern region of Brazil, composed of the states of Paraná, Santa Catarina and Rio Grande do Sul. His important participation in the history of Mathematics Education in Brazil is recognized, in particular in the creation of the Brazilian Society of Mathematics Education (Sbem), in 1988, at the II National Meeting of Mathematics Education (Enem, Maringá/PR). Since then, the South has become a hotbed of studies and research in the area, driven by the expansion of graduate programs, with research that has exceeded a thousand defenses since the 1970s, distributed in more than 35 Higher Education Institutions (HEIs) and 30 (thirty) PPGs with studies in the area of Mathematics Education, as Melo (2012) states.

The analysis of the selected data will take place according to the categories already presented in the methodological procedures. The categories, even presented in separate topics, will maintain, in this analysis, a greater interlocution of proximities, so not all of them will be presented in tables, maintaining the analytical descriptive textual form.

However, there are categories in this work that manage to stand out by production, which elucidates the reflection and analysis between each scientific production. The focus comprises the years of publications, between 2012 and 2022, and the methodological procedures in light of the covid-19 pandemic. The scientific productions prior to 2020 were developed in person in the various methodological proposals, while the research developed after 2020 assumed methodological procedures with the use of techniques and instruments suitable for a virtual and/or remote system.

In the category called initial training of the Mathematics teacher, it is possible to point out separately the productions that work on this theme, as presented in Chart 2.

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All scientific productions are associated with teacher training, whether in initial or continuing education. Due to this panorama, in the field of research, it was decided to highlight, in the following table, the distinction, or not, between the typology of teacher training, especially between the initial training in the Mathematics Degree and in the Pedagogy Degree.

Table 2: Initial and continuing teacher training: relevant aspects

Kind	Research	Type of training	Academic aspects of the target audience	Mode
D1	The contribution of origami in geometry: developing skills and concepts in the training of mathematics teachers	Initial	Degree in Mathematics	EAD
D3	History of mathematics in geometry activities: a proposal for initial teacher training	Initial	Degree in Mathematics	Regular/face-to-face
T4	Learning to formulate in order to learn how to solve: registers of semiotic representation and creativity in the learning of geometry	Initial	Bachelor's Degree in Mathematics and other university students	Regular/Face-to-face
D6	Development of geometric thinking in the early years of elementary school based on Van Hiele's theory: contributions from a teacher training course	Continued	Teacher trainer in Higher Education, teacher of early childhood education and early years	Virtual (via Meet)

Source: Author's organization.

The research presented in Chart 2 has in common the theme of teacher training. However, in the table, there are different factors that should be highlighted and reflected on, such as: initial teacher training in the Mathematics Degree and in the initial teacher training in other teaching degrees. There is also a difference in the modalities of the undergraduate course and the continuing education of teachers.

Dissertations D1 and D6 are two researches that were developed within the scope of the initial training of Mathematics teachers, and D1 was carried out in a Mathematics Degree course in the regular and face-to-face modality and D6 was built in the initial training of teachers in the distance modality (EAD). In Brazil, the legal bases for the Distance Education (EAD) modality were regulated by the Law of Guidelines and Bases of National Education, Law No. 9,394, of December 20, 1996, which was regulated by Decree No. 5,622, published on 12/20/05. The initial training of teachers − more precisely the degrees in Mathematics in the regular modality − is established by Opinion CNE/CES No. 1,302/2001. By the way, the curricular organization of the degrees in Mathematics is common for both teaching modalities, as they are guided by the same opinion of the

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CNE/CES.

The research presented in Chart 3 is the one that contemplates the category that explicitly leaves the theory, the teaching or research tendency, as well as the weaving with mathematical languages, within a thematic unit, specifically geometry. It is also possible to analyze the methodological procedures of each scientific production of these dissertations.

Research D6 and T4, also in the same scope, were based on teacher training, but research D6 was developed in the continuing education of teachers of early childhood education and the early years of elementary school. The T4 research, on the other hand, undertook processes and methods to stimulate creativity in solving and formulating problems, during the initial training of Mathematics teachers and in other undergraduate courses.

The initial and continuing education of Mathematics teachers do not appear in the selected research as the central theme of these scientific productions or even as a research problem. But they appear as a field of research, or as a context of research, which corroborates the great relevance of the theme still present in current research of graduate programs in Brazil.

Teacher education is a relatively new field of study in the Western world (Zeichner, 2005). It is a field in which one has the opportunity to know and unveil numerous factors and aspects related to the teaching and learning process. Both initial and continuing education of teachers as a field of research have great relevance, but they still need new research, as it is a field in full dynamism. The two types of training manage to connect the teacher (graduate) and the academic (in training) in a process of knowledge construction and adaptation and improvement of pedagogical practice. For this reason, Nacarato et al. (2006) highlight the potential of this mixed profile and state that in them not only academic teachers approach school contexts, but future teachers learn collaboratively from teachers in practice. It is a necessary interaction for the initial training of teachers and can produce moments of experimentation and consolidation in the continuing education of teachers in practice.

Chart 3: Theories and methods for teaching Plane Geometry

ID	Theory and/or Methods and Teaching Trend	Level of discussion of the content	Theme of study in geometry
D1	Origami	Final years of elementary school and high school	Basic concepts of plane and spatial geometry

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D3	History of Mathematics Trend	Final years of elementary school and high school	Squaring the circle; height of the pyramid; equivalence of areas; Triangle Resemblance
D8	Van Hiele's Theory of Geometric Thought	Early years of elementary school	Fundamental concepts of plane geometry

Source: Prepared and organized by the author.

The three dissertations selected in Chart 3 are within what this research proposed. It is noted that the level of discussion of the content on the theme of the teaching of Plane Geometry in basic education includes from the initial and final years of elementary school to high school.

The D8 research approaches Van Hiele's Geometric Thinking model as a theory. This model or theory is developed from the cognitive development presented by Piaget (1964).

The couple Pierre Marie van Hiele and Dina van Hiele-Geldof, Dutch professors and researchers, started from the experience in their classrooms, where they observed that students were certainly learning arithmetic and not learning geometry. This theory, considered a learning model, "is a possible strategy for reversing the problem in the teaching of geometry, because, because it originated in the classroom, the theory combined the cognitive and pedagogical aspects of the teaching of geometry" (Nasser, 1998 apud Silva, p. 2). The D6 research proceeded, methodologically, with a pedagogical training for teachers who teach Mathematics at different levels of schooling. The profile of the participating teachers goes through the experience with teaching Mathematics in early childhood education until the final years of elementary school. In its results, the D8 research discusses a satisfaction regarding the achievement of its objectives and recognizes the contributions to a new meaning of the teaching of geometry. In addition, it emphasizes the valorization of prior knowledge and the use of manipulative materials, as well as emphasizes the need for actions aimed at the continuing education of teachers who teach Mathematics. For Fiorentini et al. (2002), in addition to initial training in the adequacy of teaching degrees, continuing education is also a transforming agent in teacher training.

The D1 research identified that the research embarked on other areas of knowledge for the construction of the basic concepts of Plane Geometry. The research discusses the construction of knowledge of geometry seeking in the arts the pedagogical practice of folding and origami. In addition, it uses the construction of videos with a demonstration of folds in the construction of some geometric solids. The methodological procedure of research D1 is in agreement with Brasil (2017), for whom the teaching of Geometry "involves the study of a broad set of concepts and procedures necessary to solve problems in the physical world and in different areas of knowledge" (p. 226).

The teaching of Mathematics has been adapting to changes in society. The emergence of new trends in teaching is already noticeable and Mathematics Education has not "been left out" (Cavalcanti, 2010). In this area, the D1 research reveals one of the new trends for the teaching of Mathematics, which is the use of the History of Mathematics as a pedagogical resource. History of Mathematics can be a powerful aid in the teaching and learning process in order to manifest, in a peculiar way, mathematical ideas, situate temporal and spatial great ideas and problems, along with their motivations and historical precedents and also see the problems of the past, as well as find solutions to open problems. In the words of Fonseca, "the study of history is fundamental to perceive movement and diversity, enabling comparisons between groups and societies in different times and spaces" (2003, p. 40). It is so fundamental that the research underlines the use of the History of Mathematics as a pedagogical resource since the PCN of 1997. Fonseca (2003) corroborates the results of the D1 research that the History of Mathematics "allows social experiences to be seen as a constant process of transformation; a process that takes on very different forms and that is the product of the actions of men themselves" (p. 40).

The three researches selected in Chart 3 emphasize theories, trends in the teaching of Mathematics as potentiators of the construction of knowledge of geometry in basic education. The themes addressed are present, such as mathematical skills for basic education, as described in Brasil (2017). And as skills to be built in mathematics classes, in the period of elementary and high school years, there are those highlighted in the dissertations as fundamental concepts of Plane Geometry.

The research has been demonstrating, based on the analyses, the reading of the abstracts, the methodological processes − especially the research objects − and the problems that guide these scientific productions, an interlocution between the theories and the skills standardized in the BNCC (Brasil, 2017).

For this purpose, these researches were analyzed and mapped, identifying, through theories and the methodological process, what becomes relevant in the interlocution and in the possible transforming inferences of the teaching practice – for the participating teacher – and of the student practice – of the student participating in each research. We believe in a considerably important dialogue in the introduction, construction, fixation and consolidation of skills by year of schooling.

In the first year of schooling, considering the mathematical objects around location and recognition, the individual should be situated in space, with their own references or associated with the other. The T4 and D1 surveys can contribute to their participants and, consequently, to a better basic education, since they provide opportunities for discussion and experiences through the continuing education carried out in the surveys. T4 is based on the theory of Semiotic Registration and Representation (Duval, 1988) in a different context of object representation through a process of apprehension. For Duval (1983, p. 40):

> There is a cognitive paradox of mathematical thinking: on the one hand, the apprehension of mathematical objects can be only a conceptual apprehension and, on the other hand, it is only through semiotic representations that an activity on mathematical objects is possible.

It is a necessary apprehension for the student who proposes to build a representation of objectives for the construction of geometric knowledge. In research D1, the pedagogical practice with the use of origami is an important methodological resource beyond representation, since the construction of geometric solids, by means of folding, makes the object a manipulative material.

In the 2nd year of schooling, there is a proposal to study the mathematical object from the location and precise representation of geometric planes and solids, in order to build skills that make the student capable of identifying, naming and comparing structural aspects in a systematic way. The research that most associates his theories with the possible contribution to an adequate teaching practice is D3, which uses the History of Mathematics to give greater proximity between mathematical skills and the history of civilization. The D6 research gives visibility to Van Hiele's theory (1957), which values the cognitive development and previous knowledge of the student as relevant in a process of construction of geometric thinking through 5 (five) levels of learning. Similarly, T4 can contribute based on the understanding of Duval's (1995) levels of apprehension, given that semiotic representations are fundamental for the communication of mental representations and also for the cognitive activity of human thought. Now, the opportunity that the teacher participating in the T4 research has to know and understand the RRS and improve his teaching practice can generate fruitful results for the teaching of geometry in basic education.

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In the 3rd year of schooling, in view of the EF03MA12 (Brasil, 2017, p. 287) and EF03MA16 (Brasil, 2017, p. 289) skills, it is clear and unequivocal the need to understand Duval's (1988) levels of apprehension, so that the representation of plane figures and geometric solids are diversified, or not, but in a safe and autonomous way by the student. This is what the RRS can provide during the process of building geometric knowledge, as presented in the T4 and D6 research. They enable the teacher to develop pedagogical practices appropriate to the level of knowledge in a hierarchical way, according to Van Hiele (1957).

In the 4th and 5th grade, along with the skills, it is relevant to systematize, deepen and consolidate the mathematical object. Semiotic Representation and the theory of Geometric Thinking, according to Van Hiele, can contribute to the systematization and consolidation of these skills. The use of orality and art as pedagogical resources can provide students with the opportunity to construct geometric knowledge in an integral and safe way.

There are many associations and interlocution of the selected researches, which are similar because of the theory, methods and pedagogical practices adopted in the methodological procedures. In addition to those presented here, it is possible to raise the level of studies regarding the years of schooling up to the final years of elementary school and high school and perceive greater contributions to the process of teaching Plane Geometry in basic education.

CONCLUSION

This was a work based on the mapping methodology, in which it was objectively sought to unveil the teaching and learning processes of Plane Geometry, with regard to the calculation of areas and perimeters of polygons, from the academic productions developed in the national graduate programs. In this path, we sought to answer the guiding question, or the problem of this research: how is the methodological didactic process presented/developed in the initial training of the teacher and the teaching of Plane Geometry in graduate research in Brazil?

In the entire process of construction of this mapping, it was considered, from the first search for scientific productions in graduate programs in Brazil to the analysis of the selected productions, the construction of the systematic review of literature and its analysis. All this set of information outlined the answer to the problem of this research.

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Regarding the location, the southern region of the country was mapped, with the presence of graduate programs, namely: Graduate Program in Teaching Natural Sciences and Mathematics at the State University of the Midwest of Paraná in Guarapuava (PR); Graduate Program in Mathematics Education at the Federal University of Pelotas (RS); Graduate Program in Mathematics Teaching at the Federal Technological University of Paraná (PR); Graduate Program in Scientific and Technological Education at the Federal University of Santa Catarina, Florianópolis campus (SC); and the Graduate Program in Science and Technology Teaching at the Federal Technological University of Paraná, Ponta Grossa campus.

Having achieved the objective of the research in this mapping, the results were of great relevance, given that, despite the small number of selected and analyzed researches, it was possible to visualize the main higher institutions with graduate programs actively producing research, especially in the theme of teaching and learning of Plane Geometry in basic education.

Throughout this study, it was confirmed that the southern region of Brazil is a pioneer in research in Mathematics Education, remaining active, active even in the adversities of the covid-2019 pandemic period.

Among the researches, the present concern with the guarantee of the rights to initial and continuing education of teachers who teach Mathematics in the initial years, in the final years of elementary school and high school was revealed. This is because, in order to leverage the mathematical literacy of students in Brazilian schools, it is first necessary to invest in the continuous training of teachers.

In addition, it was noticed that the theme of area and perimeter calculation is only a small part of what is researched in Brazil, in graduate programs with the line of research in Mathematics Education. It was possible to perceive that the scientific productions had a broader look at the process of teaching and learning Mathematics, with regard to the construction of the teacher's knowledge regarding theories. With this, the theories *Logical-Mathematical Knowledge*, by Piaget (1964); *Construction of Geometric Thought*, by Van Hiele (1957); *Registro e Representação Semiótica*, by Duval (1988); and *Ethnomathematics*, by D'Ambrósio (1970).

Regarding the methods and teaching trends, the theoretical-methodological trend History *of Mathematics*, the use of *origami folding* and the *use of technologies* were mapped. This mapping is justified not only by the number of studies produced in the South region, but by the contributions that graduate programs in master's and doctoral programs bring to basic education.

It is important to emphasize that the same higher institutions, their graduate programs, have been maintaining a dialogue, through research and extension, in the school space, making use of the extracurricular workload. On the occasion, the increase in these mapped researches was highlighted not only for initial training, but also for the continuing education of teachers and participating academics. In other words, in higher institutions, the interaction of the tripod research, extension and teaching, based on a Mathematics Education focused on the quality of basic education, has favored the construction of mathematical knowledge.

In this sense, the theories for the construction of logical-mathematical knowledge and the trends for the teaching of Mathematics presented are of great relevance, especially for the construction of the knowledge of the Mathematics teacher and for the student, who needs to build geometric knowledge and become literate and autonomous.

Finally, the realization of this research was partially concluded in terms of the achievement of its objectives and with great possibilities of contributions to the academic community, especially because it works as a contribution to new research and discussions beyond the southern region of the country.

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