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APPLICATION OF THE ITEM RESPONSE THEORY IN IMAGE PROCESSING

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ABSTRACT
The Item Response Theory (IRT) was applied to image processing where the item's difficulty parameter was used to reconstruct a region of interest in an image contaminated with noise. Shannon's entropy was used for the dichotomization of the image pixels. To demonstrate the efficiency of the proposed method, it was used on simulated data that can be related to functional magnetic resonance imaging (fMRI). The results on simulated data showed that IRT achieved a high percentage of accuracy in identifying the region of interest in the image and that, on average, the estimation converges to the true region of interest, thus proving to be a promising method for the analysis of such data.

Keywords: IRT. Shannon Entropy. fMRI. Simulated Data.

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INTRODUCTION

Item Response Theory (IRT) is one of the main statistical techniques that aims to measure latent variables or traits such as an individual's ability in a specific discipline or consumer satisfaction levels, among others. Essentially, IRT proposes models that describe the directly proportional relationship between the latent variable and the probability associated with an event of interest.

Originally, IRT was developed for application in educational assessment (Batista et al. 2013), but in recent years this technique has been applied for various purposes such as psychometrics (Zanon et al. 2016), evaluation of speech synthesizers (Oliveira et al. 2020), in tests for functional magnetic resonance imaging (fMRI) applications (Thomas et al. 2013), in leadership measurement (Scherbaum et al. 2006), as an auxiliary tool in Artificial Intelligence (Martínez-Plumed et al. 2019), among others.

So far, no application of IRT in image processing has been found in the literature, which poses a significant challenge. The main difficulty in this application is that IRT uses dichotomized data, thus requiring a process for dichotomizing the image pixels.

In this work, we intend to use IRT to highlight or select a certain region of an image. For the pixels in the region of interest of the image, the dichotomization will assign the value 1, and for the remaining pixels, the value zero.

ITEM RESPONSE THEORY MODEL

For the analysis of dichotomized items (in our case, zeros and ones), three logistic models are typically used, differing by the number of parameters (Andrade et al. 2000): the one-parameter logistic model (item difficulty), the two-parameter model (item difficulty and discrimination), and the three-parameter model (item difficulty, discrimination, and guessing). Among the three mentioned logistic models, the most used is the three-parameter logistic model (3PL), given by:

$$P\left(U_{ij}=1\middle|\theta_j\right)=c_i+(1-c_i)\frac{1}{1+e^{-Da_i(\theta_j-b_i)}},$$

With $i=1,2,\cdots,I$ and $j=1,2,\cdots,N$, where $P\left(U_{ij}=1\middle|\theta_j\right)$ is the probability that individual j with latent trait θ_j answers item i correctly; b_i is the item difficulty parameter measured on the same scale is the item difficulty parameter measured on the same scale

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as θ_j; a_i is the item discrimination parameter i, proportional to the slope of the Item Characteristic Curve at point b_i (Andrade et al, 2000); c_i is the random hit parameter of item i; D is a scale factor, constant and equal to 1, however, the value 1.7 is used when the logistic function is desired to provide results similar to those of the normal ogive function.

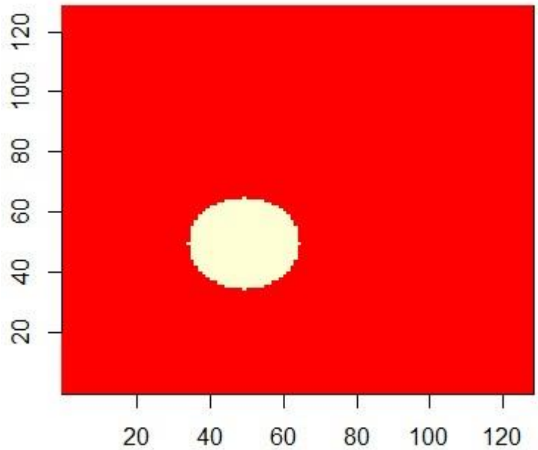
From ML3 it is possible to obtain the other models. To obtain the 2-parameter logistic model, simply use $c_i = 0$. To obtain the 1-parameter logistic model, it is necessary to use $c_i = 0$ and $a_i = 1$.

The estimations of the parameters a_i, b_i and c_i of the items and of the latent trait θ_j of the individuals are one of the main objectives in IRT. For these estimations, the Marginal Maximum Likelihood method proposed by Bock & Lieberman (1970) will be used. The development of this estimation method is present in Andrade, Tavares & Valle (2000).

IRT APPLICATION IN IMAGE PROCESSING

To exemplify the use of TRI in image processing, the following situation will be considered: An image of size 128×128 pixels that contains a circle of radius 15 u.m and centered at the point (50,50), Figure 1 illustrates this image. Consider that a group with ten images in this format are interspersed with another group with ten images of the same size, but without the presence of the circle, for eight moments, which will give a total of 160 images. Furthermore, the set of 160 images is repeated 16 times.

Figure 1. image of size 128×128 pixels containing a circle with radius 15 μm and center at point (50,50).



This simulation may correspond to a functional magnetic resonance imaging experiment – fMRI (Campelo et al, 2014; Araujo et al, 2003), with a block paradigm with 8 stimulus section periods and interspersed 8 periods with rest and rest sections. 10 images

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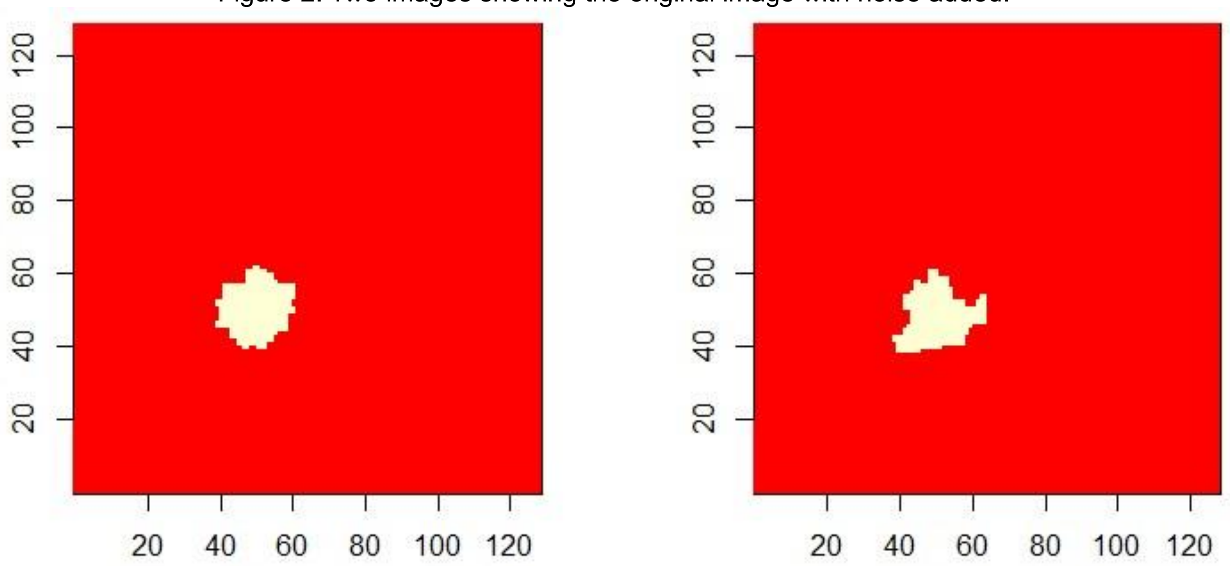
are taken in each section (totaling 160 images over time), each exam would be performed on 16 patients. Thus, each pixel (in the case of fMRI, these are the voxels) corresponding to the images, per patient, will have a time series. To make the simulations as close to reality as possible, an additive Gaussian noise with mean 0 (zero) and variance 1 (one) will be added to all time series. Figure 2 shows two images with added noise.

Figure 2. Two images showing the original image with noise added.



DICHOTOMIZATION

The simulation considered in this article is similar to fMRI data with a block paradigm, so to perform data dichotomization, Shannon entropy will be applied, as described in (Campelo et al, 2014; Araujo et al, 2003) and which will be briefly described below.

Shannon Entropy in Data Dichotomization

Shannon entropy is a way of measuring information based on probability theory, where the amount of information transmitted by a message is measured according to its predictability of occurrence. Thus, if this probability of occurrence is small, the message contains a lot of information; otherwise, if it is predictable, the message contains little information (Campelo et al, 2014).

Consider X a discrete random variable associated with the probability vector $(P_1, \dots, P_n, \dots, P_N)$. According to Shannon, entropy is obtained as follows:

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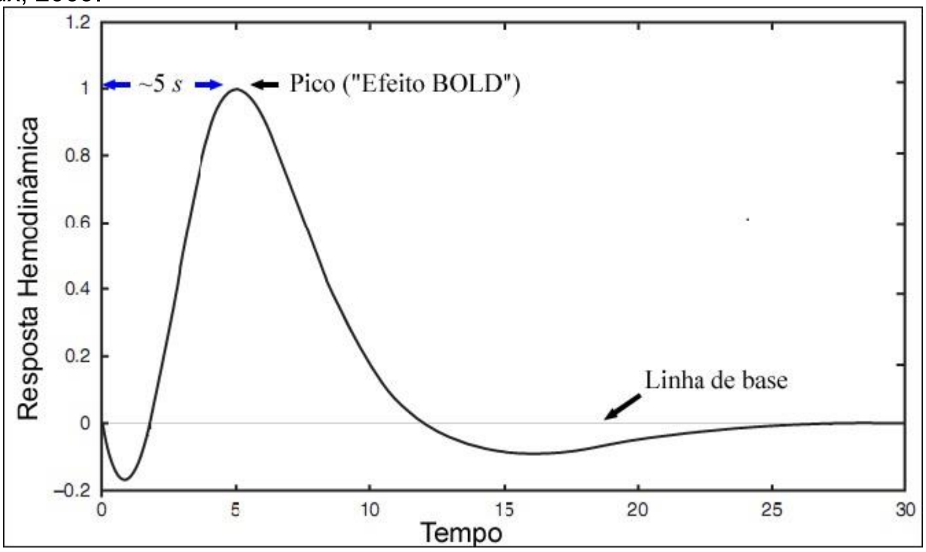
$$H(X) = -\sum_{i=1}^{n} p_i \log_2 p_i. \quad (1)$$

Commonly, entropy concepts allow a comparison of the characteristics of a system in numerical terms, verifying what the probability distribution of that system is like. The method adopted in this work consists of computing the time series of a voxel.

Let h(t) be the time series of a pixel (which corresponds to the behavior of the hemodynamic response over a period of time in fMRI). It is also assumed that the hemodynamic response of a voxel located in an active region follows the same temporal evolution shown in Figure 3.

Figure 3. Hemodynamic Response Function for a short-duration stimulus. Image taken and adapted from Mulert & Lemieux, 2009.



The objective, therefore, is to calculate the entropy of this signal (Araújo et al, 2003). First, the signal will be divided into two windows W1 and W2, where W1 corresponds to the hypersignal period (stimulus) and W2 corresponds to the baseline values (rest). Considering W as an entire epoch (Campelo et al, 2014; Araujo et al, 2003).

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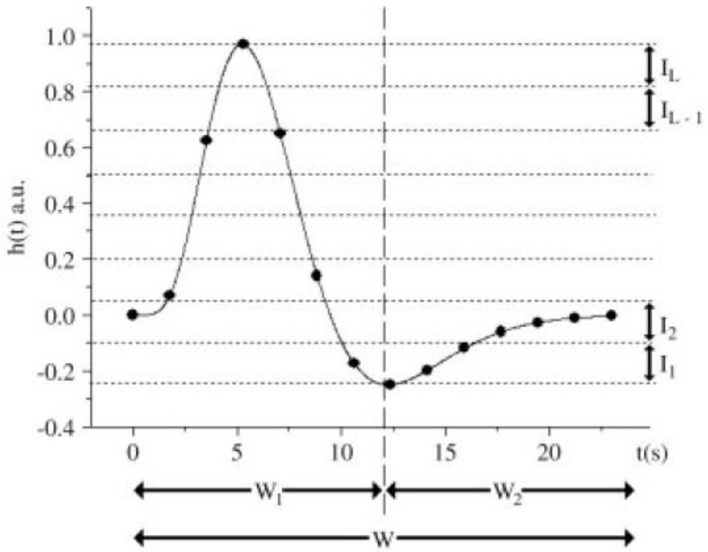
Figure 4. Time course of a BOLD signal in an active voxel in one epoch. Image taken from Sturzbecher (2011).



Subsequently, the signal h(t) is divided into intensity levels IL (Figure 4). This division is performed by calculating the maximum and minimum amplitude of the signal within each epoch. Then, we calculate the probability that a given IL interval contains a portion of the signal $h(t_L)$, that is:

$$P^n(I_L) = \frac{n^\circ\ of\ valores\ of\ h(t_L) \in W_n\ in\ I_L\ interval}{n^\circ\ of\ values\ of\ h(t_L) \in W_n}.$$

With this information, the entropy for each epoch is calculated separately by (1). Each set of $h(t_L)$ values belonging to W_i will have an entropy value. Intuitively, it follows that the first portion of the signal, contained in W_1, should provide higher entropy values than that contained in W_2, since it is a more disorganized system.

At the end of this process, a cross-correlation is performed between the values of the temporal evolution of the entropies and a simulated sawtooth function (Campelo et al, 2014; Araujo et al, 2003). Therefore, significant correlations refer to the area's most likely to be active.

ITEM RESPONSE THEORY APPLIED TO DATA

For the application of IRT, each pixel (voxel) was considered as an item and each experiment (patients undergoing fMRI examination) as respondents, and the criterion for dichotomizing the response is the cross-correlation between the values of the temporal evolution of the entropies and a simulated sawtooth function. It was considered that correlations equal to or above 0.7 receive 1 (correct/active), otherwise they receive 0 (incorrect/not active). The one-parameter Rasch Model (2) was used to estimate the true activation region.

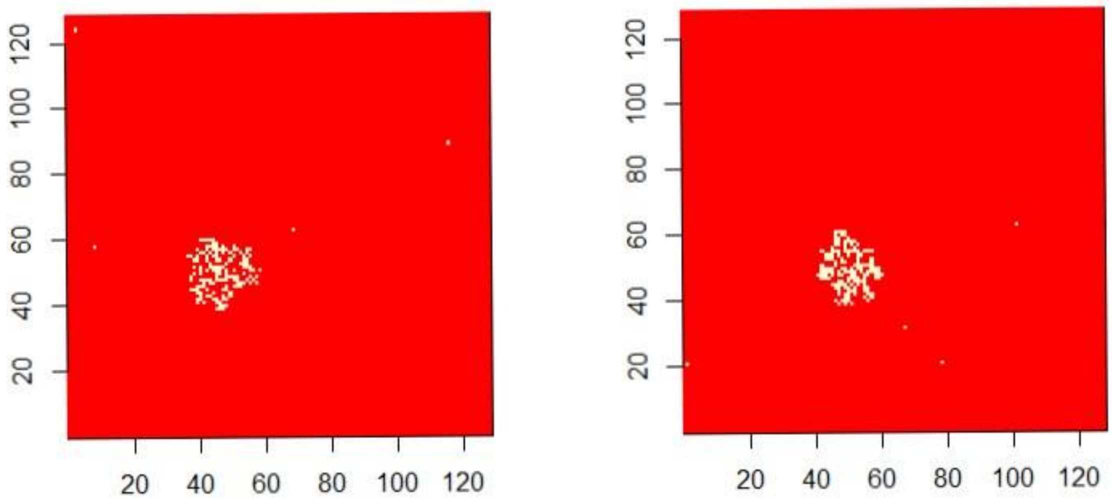
$$P\left(U_{ij}=1\middle|\theta_j\right)=\frac{1}{1+e^{-D(\theta_j-b_i)}}. \qquad (2)$$

Therefore, active pixels (voxels) will have many 1's and consequently result in low b's, and inactive pixels (voxels) (noise) will have many zeros, resulting in high b's, thus a matrix plot of the b's would indicate the true active region. However, the problem for estimation is that the number of respondents/patients is small (16 patients) and the number of items/voxels is large (16,384 voxels). To solve this problem, it was considered that the voxels belonging to the same regions in the images obtained from the 16 patients will be a response set, that is, there are now 16 items and consequently 262,144 respondents.

RESULTS

Shannon entropy (Campelo et al, 2014; Araújo et al, 2003) was applied to dichotomize the noisy data of each respondent (patients in the case of fMRI), as shown in Figure 5 (corresponding to the data presented in Figure 2). Blank points are considered active when the correlation of the temporal evolution of entropy with a sawtooth function is greater than 0.7.

Figure 5. Region dichotomized using Shannon entropy.



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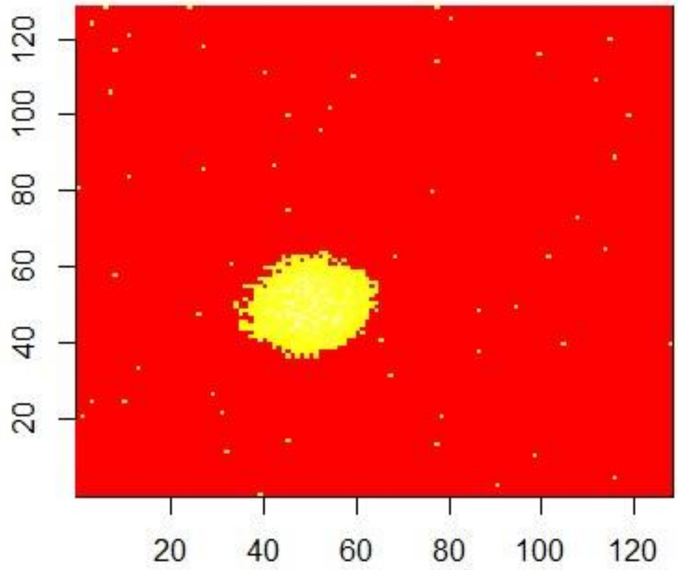
Then, IRT was applied to the inversion mentioned in section 3.2. Figure 6 shows the estimation of the region of interest (in the case of fMRI, it is the activation region) from estimates of the difficulty parameter of item b, as described in section 3.2. The method presented a 98% accuracy percentage; however, it is observed that some false positive points occurred in the image recovery.

Figure 6. Region of interest estimated by IRT through the item difficulty parameter.



To better evaluate the estimation using IRT, the results of 100 estimates of the same estimation region were verified. Figure 7 represents the histogram of the percentage of correct answers of 100 estimates using IRT. It can be seen that the percentage of correct answers is in the range of 97.80 to 98.89%.

Figure 7. Distribution of the proportion of correct answers in 100 estimates using IRT.

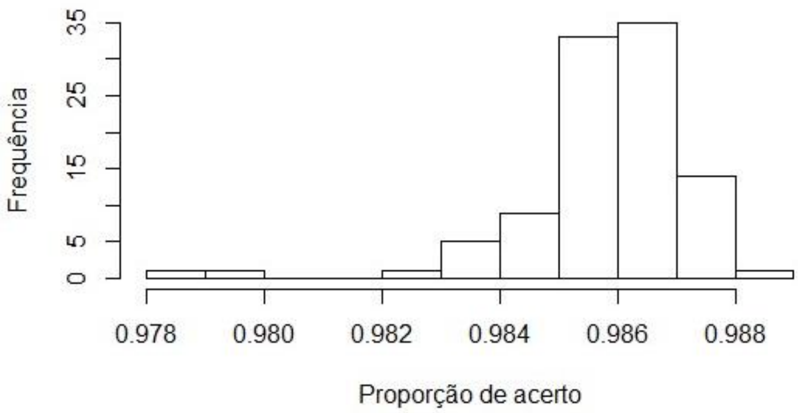


Figure 8 represents the average image of the 100 estimates, it can be seen that on average the estimate is close to the true region of interest.

FINAL CONSIDERATIONS

The contribution of this work was to present a new method for image processing based on Item Response Theory. To apply IRT, Shannon's entropy was used to analyze the data in a dichotomized way. The results in simulated data showed that IRT was able to bring a high percentage of accuracy in the region of interest of the image and that, on average, the estimation converges to the true region of interest, thus showing itself to be a promising method for the analysis of these types of data.

It is important to note that the study only used simulated data, as these data were representative of an fMRI situation. Therefore, a study involving the analysis of several slices using some type of anatomical cut is necessary in order to be able to analyse the levels of brain activity in a volumetric way. In addition, the aim is to verify the suitability of IRT models for polytomous and subsequently continuous responses, since the activity levels will be better represented by these types of models.

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