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BEYOND FISCAL RETURNS: THE SOCIOECONOMIC IMPACT OF PUBLIC INVESTMENT IN BRAZIL

ALÉM DOS RETORNOS FISCAIS: O IMPACTO SOCIOECONÔMICO DO INVESTIMENTO PÚBLICO NO BRASIL

MÁS ALLÁ DE LOS RENDIMIENTOS FISCALES: EL IMPACTO SOCIOECONÓMICO DE LA INVERSIÓN PÚBLICA EN BRASIL

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ABSTRACT

Public investment is generally theorized to decrease inequality, yet its real-world influences hinge on governance. This study analyzes how Brazil's public budget allocation shapes socioeconomic vulnerability. The main finding reveals a paradox: while investments significantly reduce vulnerability in low-income areas, institutional failures undermine their potential, including misallocation and elite capture. Using subnational data on completed projects, we show that agricultural, urban, and social spending in disadvantaged regions achieves measurable welfare gains. However, these are systematically diluted by politicized resource distribution and weak oversight. Our findings highlight how political economy constraints transform public investment from a redistribution tool into a mechanism reinforcing spatial and class inequalities. By linking fiscal policy with vulnerability outcomes, we offer a framework for analyzing the institutional roots of inefficient spending in unequal democracies, with implications for reform in Brazil and beyond.

Keywords: Regional Inequality. Public Expenditures. Social Policy. Institutional Change. Development.

RESUMO

O investimento público é geralmente teorizado como um mecanismo de redução da desigualdade, porém seus efeitos no mundo real dependem da governança. Este estudo analisa como a alocação do orçamento público no Brasil molda a vulnerabilidade socioeconômica. O principal achado revela um paradoxo: embora os investimentos reduzam significativamente a vulnerabilidade em áreas de baixa renda, falhas institucionais comprometem seu potencial, incluindo má alocação e captura por elites. Usando dados subnacionais de projetos concluídos, mostramos que os gastos em agricultura, urbanismo e áreas sociais em regiões desfavorecidas promovem ganhos mensuráveis de bem-estar. No entanto, esses efeitos são sistematicamente diluídos pela distribuição politizada de recursos e pela supervisão fraca. Nossos achados destacam como as restrições da economia política transformam o investimento público de uma ferramenta de redistribuição em um mecanismo que reforça desigualdades espaciais e de classe. Ao vincular a política fiscal aos resultados de vulnerabilidade, oferecemos um quadro para analisar as raízes institucionais do gasto ineficiente em democracias desiguais, com implicações para reformas no Brasil e além.

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Revista ARACÊ
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Palavras-chave: Desigualdade Regional. Gastos Públicos. Política Social. Mudança Institucional. Desenvolvimento.

RESUMEN
Se teoriza generalmente que la inversión pública disminuye la desigualdad, pero sus efectos en el mundo real dependen de la gobernanza. Este estudio analiza cómo la asignación del presupuesto público en Brasil moldea la vulnerabilidad socioeconómica. El hallazgo principal revela una paradoja: aunque las inversiones reducen significativamente la vulnerabilidad en áreas de bajos ingresos, las fallas institucionales socavan su potencial, incluyendo la mala asignación y la captura por parte de las élites. Utilizando datos subnacionales de proyectos finalizados, mostramos que el gasto en agricultura, urbanismo y áreas sociales en regiones desfavorecidas genera ganancias de bienestar mensurables. Sin embargo, estos efectos se diluyen sistemáticamente debido a la distribución politizada de recursos y a la supervisión débil. Nuestros hallazgos destacan cómo las restricciones de la economía política transforman la inversión pública de una herramienta de redistribución en un mecanismo que refuerza las desigualdades espaciales y de clase. Al vincular la política fiscal con los resultados de vulnerabilidad, ofrecemos un marco para analizar las raíces institucionales del gasto ineficiente en democracias desiguales, con implicaciones para reformas en Brasil y más allá.

Palabras clave: Desigualdad Regional. Gasto Público. Política Social. Cambio Institucional. Desarrollo.

Revista ARACÊ
ISSN: 2358-2472

1 INTRODUCTION

In several instances around the world, emerging economies have fallen prey to misallocation of public funds, resulting in a cycle of unfavourable outcomes that hinder these countries from improving their standard of living (Acconcia et al., 2014), where institutional constraints and failure of critical socioeconomic governance often undermine the effectiveness of policy (Bremer et al., 2023). More public investments should stimulate economic development, leading to a decrease in poverty. Conventional growth models, derived mainly from (Solow, 1994) neoclassical growth model and (P. M. Romer, 1994) endogenous growth model, highlight the significance of capital accumulation, labour growth, and technological progress as critical growth drivers. These models indicate that internal aspects such as knowledge, human capital, and innovation, which can be fostered through investments in infrastructure, human capital, and research and development (R&D), have the potential to enhance productivity and sustain long-term economic growth. The significance of public investment lies in stimulating economic development and addressing social vulnerabilities, contributing to a more egalitarian and resilient economy.

Conventional growth models have significant drawbacks in effectively addressing the stratified distribution of public investment across socioeconomic strata, mainly the high- and low-income sectors of the economy. The peripheral economic sectors face structural barriers to initiating development, including lack of capital, institutional marginalization, and cross-cutting vulnerabilities. In the high-income sector, further investments in physical capital may result in declining returns due to existing substantial capital stock. This may impede the effectiveness of government spending in stimulating continued expansion. In contrast, the low-income sector of the economy often requires more initial capital, which poses a significant obstacle in initiating the development process. Notably, allocating public funds to achieve social reproduction benefits, such as reducing poverty, increasing lifespan, and reducing child mortality, becomes an economic challenge and a socio-political dilemma of redistributive justice. The critical question is whether the distribution of public investments between the high- and low-income segments of an economy effectively addresses social vulnerability to maximize social welfare and achieve sustainable development for the entire economy. In this regard, it is challenging to identify the impact of the use of public resources on social welfare. The challenge lies in quantifying these socioeconomic externalities of public investment, particularly their distributional consequences across stratified populations.

Revista ARACÊ
ISSN: 2358-2472

This study aims to understand the socioeconomic efficiency of public budget allocation in generating social benefits in Brazil between 2000 and 2010. We studied the distributional impact of completed projects funded through transfer contracts (parliamentary amendments) on multidimensional welfare indicators across Brazilian municipalities to understand the causal effects of effectively applying public resources. Our investigation focuses on the impact of these investments on variations in the Social Vulnerability Index (SVI), created by the Brazilian Institute of Geography and Statistics (IBGE). By analyzing these variations, our objective is to determine the efficacy of public investments in mitigating social vulnerability. Using, we offer insight how well-executed public projects in urban infrastructure, education, and healthcare can transform communities and improve the life. The significance of this study lies in its ability to provide empirical evidence of the social aftermath of poorly managed public investments. By focusing on tangible indicators, such as the SVI, we anchor our findings to measurable outcomes, painting a clear picture of the broader social fallout of such misallocations.

Public investments are crucial to developing a country's low- and high-income regions. Optimally-allocated and well-directed public investments can stimulate economicactivity, create jobs, and improve overall quality of life. In low-income areas, investments in infrastructure, such as roads and bridges, facilitate trade and mobility, which are essential for economic growth. Investments in education enhance human capital, leading to a more skilled workforce that can drive innovation and productivity in the high-income areas of the country. Similarly, investments in healthcare improve population health, which is a funda- mental component of human capital and economic stability. However, the effectiveness of these investments often depends on the efficiency of their execution and their optimal allo- cation. Properly allocated and managed public investments can significantly reduce poverty and inequality and foster sustainable development and social progress.

These constraints frequently distort investment allocation, particularly in federal systems with asymmetric development trajectories where regional elites capture resources. Our analysis extends this insight by utilizing the SVI of Brazil; this study examines how different levels of public investment influence social outcomes across core-periphery economic regions in low- and high-income areas. We analyzed data from a prominent public bank responsible for managing the application of public resources to quantify the impact of well- executed versus poorly-executed projects. Although higher public investment generally leads to economic growth and poverty reduction, inefficiencies and mismanagement in low-

Revista ARACÊ
ISSN: 2358-2472

income areas can significantly diminish these benefits, resulting in incomplete or ineffective projects.

This study addresses the relationship between the level of public expenditure and the degree of development in Brazilian society. Brazil was selected because of its importance to the global and regional economy, the large volume of data available, and the possibility of extrapolating the results to other developing countries. Public investments have long been recognized as a fundamental driver of Brazil's economic growth and social progress.

Historically, Brazil has channelled significant resources into public investments to fuel its developmental aspirations, ranging from infrastructure projects to social programs. These investments promise to stimulate economic activities, foster job creation, and enhance the overall quality of life. However, Brazil faces challenges in managing its public investments. Bureaucratic hurdles, corruption, and a lack of strategic planning often impede realizing the full potential of these investments. (De Desenvolvimento & others, 2019; World Bank, 2023) highlight that while Brazil's commitment to infrastructure development is commendable, the quality of its infrastructure often remains sub-par due to such inefficiencies and corruption. This study contributes to the literature in numerous ways by enriching our under- standing of the interplay between public expenditure and social output in Brazil, which often finds itself in the global spotlight for its governance challenges. It also offers an opportunity for understanding these challenges in other developing countries and providing support for more effective development strategies that can stimulate economic growth in destressed regions. Based on the theoretical economic growth model, similar to (Ba r r o & Sa l a - i -Ma r t i n , 1 9 9 0 ; So l ow, 1 9 9 4) , we developed a simple theoretical model and hypothesis about how public investments might mitigate a country's socioeconomic vulnerability. The results of empirically testing this hypothesis are presented in Table 4. These findings reshape debates about public investment efficiency by integrating growth theory with sociological analyses of vulnerability (Noël, 2020).

We then bridged a crucial gap in the literature by shedding light on the tangible social repercussions of public investment allocation in high- and low-income areas of the economy. Consequently, we test a policy-relevant hypothesis, which states that public investment benefits the economy by decreasing the socioeconomic vulnerability of the low-income group, rather than that of the high-income group. The results on asymmetric socioeconomic returns are presented in Tables 7, 9 and in the Internet Appendix. The results reveal a lacking rational approach to policymaking, where many regions still suffer misallocation and mismanagement

Revista ARACÊ
ISSN: 2358-2472

of the public budget, leading to a spiral of poor results in human development indicators and preventing the population of these regions from achieving a higher quality of life. The related results of the regional analysis are presented in Table 8 and in the Inter- net Appendix. Our findings emphasize the potential for well-directed public investments in stimulating diverse economic sectors, leading to comprehensive growth that includes underdeveloped regions. The related results of the regional analysis are presented in Table 10 and in the Internet Appendix.

The rest of the paper is structured as follows. In Section 2 review the related literature and explain our contributions to current literature. We present a theoretical model and develop related hypotheses in Section 3. Then, in Section 4 we discuss our data and how we construct the main variables. Section 5 presents the empirical results. Finally, Section 6 concludes.

2 LITERATURE REVIEW

Our study focuses on research that evaluates the impact of public investment on Brazil's regional economies. These studies show differing results across distinct sectors and regions. (L i p s c omb e t a l . , 2 0 1 3) highlighted the substantial beneficial effects of transporta- tion infrastructure expenditures on local economic growth and employment. (Bu s t o s e t a l . , 2 0 1 6) studied the impact of agricultural investments and concluded that allocating public funds to this sector may positively affect productivity and reduce poverty, especially in rural regions. (Ca s e l l i & Mi c h a e l s , 2 0 1 3) analyzed the effects of oil discoveries and associated public investments in Brazil, emphasizing the presence of both positive and negative im- pacts. Their results showed that while the original oil discoveries brought about economic prosperity, later public investments and governance issues resulted in negative consequences, such as misallocation of resources and environmental concerns. (Go l d f a j n e t a l . , 2 0 2 1) investigated the influence of government spending on the Brazilian economy, specifically in regions with different degrees of development, highlighting the need for targeted public investment to drive economic development, especially in underdeveloped areas with infrastructural deficiencies. (S p e n c e , 2 0 2 1) emphasized the significance of public investment in stimulating economic growth, particularly in highly developed areas. Targeted expenditure in these areas may promote innovation and increase productivity. (Ak h t a r i e t a l . , 2 0 2 2) explored the political aspects of public investment, providing insights into how political variables in- fluence investment choices and their

Revista ARACÊ
ISSN: 2358-2472

resulting economic consequences in various Brazilian regions. (Arma n d e t a l . , 2 0 2 0 ; As h e r & No v o s a d , 2 0 2 0) examined the efficacy of public investment in rural regions, highlighting the significance of customized policies in tackling regional inequalities. (De Mello Jr, 2002) offers a historical analysis of previous public investment strategies and their enduring effects on Brazil's regional economies. Our study shares similarities with these studies as it emphasizes the significance of considering sector-specific intricacies when assessing the impact of public investment in Brazil. Our study emphasizes both the promising prospects for development and possible challenges of public investment, which are directly linked to a region's life expectancy and per-capita income levels.

Vulnerability is a multifaceted concept that includes the susceptibility of individuals, communities, or systems to potential harm or adverse effects. The nature and scope of vulnerability can vary, encompassing the economic, social, environmental, and even psychological realms (Cutter et al., 2003). This broader understanding of vulnerability underscores its intrinsic links with societal structures, economic systems, and environmental changes. Recognizing the pervasive nature of vulnerability, countries and organizations have developed indices and measures to capture its intricacies, aiming to inform related policies and interventions (Cutter et al., 2012). (Bobonis et al., 2022) analyzed the causal relationship be- tween vulnerability and clientelism in Brazil, concluding that reducing vulnerability among the local population substantially reduces politicians' demands for private goods, largely among residents who have the possibility of being in clientelist relationships.

Our study focuses on analyzing the effects of public investment on the growing vulnerability rate across several domains: economic, social, environmental, and psychological. Prior research emphasizes the varied effects of public investment on important aspects of the economy, including productivity, human capital development, employment, and macroeconomic stability. (Hs i e h , 2 0 0 2) discussed the relationship between public investment in education and human capital accumulation, arguing that such investments may lead to a more competent workforce and contribute to long-term economic development. (Bu c h h e im & Wa t z i n g e r , 2 0 2 3) examined the immediate influence of public investment on employment, emphasizing its capacity to stimulate job creation, particularly in construction and allied industries. (Bouakez et al., 2023) examined the impact of government expenditure on economic stability, showing that targeted public investments help reduce volatility in the business cycle and encourage long-term growth. Our research contributes to the existing body of literature examining the impact of public investment allocation on the socioeconomic

Revista ARACÊ
ISSN: 2358-2472

aspects of the economy (Bremer et al., 2023; Noël, 2020; Solga, 2014). (Az z imo n t i , 2 0 1 1) emphasized that political instability and electoral competitiveness obstruct effective public investment, often resulting in inefficient allocation. (B a t t a g l i n i & C o a t e , 2 0 0 7) explores the inefficiencies that result from political bargaining, where resource allocation is determined more by political influence than by eco- nomic need. (B a r d h a n , 2 0 0 2) contends that decentralization can improve public investment efficiency by customizing spending on local needs. However, this strategy requires a robust institutional structure to prevent corruption and mismanagement. (Esteban & Ray, 2006) investigated the correlation between inequality and public investment allocations and found that increased inequality might result in less public investment in crucial services, prolonging poverty and underdevelopment.

Theoretical analysis and hypothesis development

This section presents our theoretical model for analyzing the relationship between public investment, economic development, and social vulnerability. Initially, we develop a theoretical economic growth model similar to those of (ADB et al., 2016; Barro & Sala-i-Martin, 1990; T. Romer, 1988; Solow, 1994) to optimize total social welfare by considering the distribution of public investment.

The economic model examines the impact of government expenditure and public investment on the performance of various economic sectors and the general welfare of its population. Government investment in public goods such as infrastructure, education, and healthcare directly influences the productivity and efficiency of these sectors. Enhancing road infrastructure and transportation networks may facilitate corporate operations and the delivery of commodities, thus improving productivity in both high- and low-income sections of the economy.

We assume the economy consists of two sections with high- and low-income. The production function is defined as:

$$Y = F(K, L, I_H, I_L, A) \qquad (1)$$

Where:

Revista ARACÊ
ISSN: 2358-2472

Y is the economic output, K is the capital, L is a representative of the labor factor of the economy and A is the technology factor. The public investment allocated to the high-income section and the low-income section are I_H and I_L, respectively.

The social vulnerability function G is function of optimal public investments and it can be defined as:

$$V = G(I_H, I_L), \qquad (2)$$

Where:

V represents social vulnerability. We define the social welfare function as:

$$W = H(Y, V), \qquad (3)$$

which increasing with economic output and decreasing with social vulnerability.

The model reduces the complexity of the economy to two primary sectors: one characterized by high earnings and the other by low income. Policymakers should strive to maximize social well-being by optimally allocating public investments for both the economy's high- and low-income sections.

Given the total public investment I, the optimal allocation is characterized by following the (Barro & Sala-i-Martin, 1990) approach. The Production Function is:

$$Y = AK^{\alpha}L^{\beta}I_H^{\gamma}I_L^{\delta}, \qquad (4)$$

Where:

$\alpha + \beta + \gamma + \delta = 1.$

Inspired by (Castel, 1995), social vulnerability V is defined as:

$$V = \phi(I_H) + \psi(I_L), \qquad (5)$$

Where:

Revista ARACÊ
ISSN: 2358-2472

$\phi(I_H)$ and $\psi(I_L)$ are functions where investments reduce vulnerability. Functions $\phi(I_H)$ and $\psi(I_L)$ are concave and decreasing functions of I_H and I_L, respectively:

$$\phi'(I_H) < 0 \quad \text{and} \quad \psi'(I_L) < 0, \qquad (6)$$

$$\phi''(I_H) \leq 0 \quad \text{and} \quad \psi''(I_L) \leq 0. \qquad (7)$$

We define social welfare function W as a combination of the above components:

$$W = U(Y) - V, \qquad (8)$$

Where:

$U(Y)$ is a utility function of economic output, assuming it is a concave function.

The central planner aims to maximize social welfare W by choosing the optimal allocation of I_H and I_L subject to budget constraints: $I_H + I_L = I$, where I is the total public investment available. To study how the government (central planner) might optimally allocate public investments to its high- and low-income sectors, we develop the following proposition:

Proposition 1. The optimal allocation of public investments for high-income and low-income sections of an economy depend on the marginal utilities and marginal impacts of public investment on social vulnerability. The proof is provided in the appendix.

The results of Proposition 1 show that the optimal allocation of public investment for the high- and low-income sectors depends on the marginal impact of public investment on social vulnerability. Next, we extend the model to study the effect of the allocation of public investment across high- and low-income areas on the socioeconomic vulnerability of the country.

Proposition 2. *Public investment might decrease the country's socioeconomic vulnerability. The proof is provided in the appendix*.

Therefore, we formulate the following hypothesis to transform the theoretical model into an empirical one and study the impact of public investment on social vulnerability:

Hypothesis 1: *Public investment might mitigate the country's socioeconomic vulnerability*

Optimizing the allocation of resources between high- and low-income sectors is crucial for maximizing economic development and social welfare through public investment. Although both sectors see advantages in heightened investment, their requirements and

Revista ARACÊ
ISSN: 2358-2472

preferences vary. For example, affluent regions require more sophisticated infrastructure development and technological investments to maintain and support economic expansion. Conversely, low-income areas may need more assistance in vital sectors, such as education and healthcare, to combat socioeconomic disparities. Hence, it is imperative to allocate public funds towards mitigating social vulnerability, particularly in economically underdeveloped regions. The government may enhance overall social resilience by expanding access to healthcare, social assistance programs, and disaster relief infrastructure, thereby mitigating economic shocks and the impact of natural catastrophes. Reducing vulnerability in low-income communities safeguards people from descending into poverty and fosters long-term financial stability.

The central planner attempts to find a balanced solution and suggests policy implications while considering diverse investment strategies in high- and low-income areas. High-income areas often yield prompt returns due to better infrastructure and are less risky than those in low-income areas. Therefore, the central planner allocates a higher level of investment by ensuring that low-income areas receive sufficient investment, so that general economic growth can become more inclusive and sustainable. This strategy reflects the status quo of numerous economies. However, sustainable allocation policies should consider increasing investments in low-income areas to reduce social vulnerability. Based on this observation, we propose the following proposition:

Proposition 3 *Considering the allocation of more or equal public investments in highincome areas, increasing public investment in low-income areas reduces social vulnerability more significantly than in the high-income areas. The proof is provided in the appendix*.

The results of Proposition 3 indicate that a sustainable investment strategy could involve increasing investment in low-income areas by a substantial margin, while maintaining or slightly expanding investment in high-income areas. Based on these theoretical results, we establish our hypothesis and use regression analysis to examine the influence of public investment on social vulnerability:

Hypothesis 2 *Public investment helps mitigate the socioeconomic vulnerability of the low-income economic sector with greater impact than investment in the high-income sector.*

Data and Variables

In this section, we present the data sources, how the variables were constructed, and the related statistics. The data used in this research was collected from two primary public

Revista ARACÊ
ISSN: 2358-2472

sources. First, the SVI is a metric first developed by the IBGE and produced by the Institute of Applied Economic Research (IPEA), a federal public foundation in Brazil linked to the Ministry of Economy. Its research activities technically and institutionally support the government in formulating public policy and development programs in Brazil. The database was built using 2000 and 2010 demographic census data.

To address and understand the nuances of social disparities, Brazil introduced the SVI. This metric offers an insightful perspective on socio-economic disparities, taking inspiration from broader concepts like “unsatisfied basic needs,” “multidimensional poverty,” and “human development.” Unlike metrics that focus solely on monetary dimensions, the SVI provides a comprehensive view addressing numerous facets of societal vulnerability.

At their core, vulnerability and social exclusion are predominantly political notions, providing a fresh interpretative lens for the processes of societal development. These inherently political concepts provide a comprehensive understanding of societal dynamics beyond purely economic metrics and, as such, can influence public policy, magnifying the state’s responsibility to ensure citizens’ well-being.

The SVI is a testament to Brazil’s commitment to this goal. It seeks to identify and highlight areas in the Brazilian territory that lack certain “assets” which, ideally, should be available to every citizen. These assets, facilitated by state actions, are fundamental to a population’s well-being and overall quality of life.

The SVI is structured around three pivotal dimensions of social vulnerability: (1) urban infrastructure, which represents the availability and quality of basic amenities and infrastructure in urban locales; (2) human capital, which measures educational and health status, capturing the potential and existing capabilities of the population; and (3) income and work, which assesses the economic well-being, employment status, and overall economic security of the population.

These subindices are derived from a collection of 16 indicators obtained from the demographic variables of the IBGE census for 2000 and 2010. The SVI is computed by assigning each sub-index an equal weight, ensuring a balanced representation of all aspects. The indicators within the sub-indices are normalized on a scale between 0 (the ideal scenario) and 1 (the worst possible scenario).

The SVI plays a pivotal role within the Atlas of Social Vulnerability (Atlas da Vulnerabilities Social), a dedicated government portal designed to make vulnerability data accessible and interpretable. This platform provides a panoramic view of vulnerability and

Revista ARACÊ
ISSN: 2358-2472

social exclusion across various administrative levels, from municipalities to states and even broader metropolitan regions. With its detailed insights, the Atlas underscores the importance of understanding vulnerabilities at the most localized level (municipal), revealing stark intra-municipal disparities. Each of its three dimensions consolidates various variables from the IBGE demographic census, painting a holistic picture of the living conditions and vulnerabilities of the population.

Delving deeper into the SVI, it encompasses 16 selected indicators from the ADH Platform. These are meticulously organized into three primary dimensions of social vulnerability: the Urban Infrastructure of the specific territory, the Human Capital contained within the households of that region, and the Income and work conditions of its residents. Each of these dimensions consolidates various variables from the IBGE demographic censuses, painting a holistic picture of the living conditions and vulnerabilities the populace faces.

The SVI developed in Brazil is an indicator that allows governments to detail the living conditions in all socioeconomic layers of the country, identifying those who are vulnerable and at social risk. Some of the main characteristics that label the state of social vulnerability are the precarious conditions of housing and sanitation, the means of subsistence, and the lack of a familiar environment. The people considered “socially vulnerable” are losing their representatives in society, generally depending on third parties to help ensure their survival. SVI is a concept that characterizes the condition of a group of individuals at the margin of society. Therefore, people and families are in the process of social exclusion, mainly by economic factors. The concept used by this research intends to indicate the access, lack, or insufficiency of some “assets” in some areas of the Brazilian territory that should be at the disposal of the whole population.

In this study, we attempt to understand the social costs and benefits of public resource usage. To do so, this study used data from transfer contracts from one decade and compared them with data from the SVI. To understand the public investment variables used in this study, it is important to explain the transfer contract system. We try to provide details about what is the stages of the transfer contracts system, its resource funds, and the method of extinction of contracts. Additionally, we explain how to control the execution of the related public budget and how to inspect the good application of public resources.

After the 1988 Constitution, and owing to the growing demand for public construction around the country, the federal government created two methods to distribute public

Revista ARACÊ
ISSN: 2358-2472

resources to states and cities: by agreement or through transfer contracts. The transfer contract is the main format used by the federal government to provide cities with the resources necessary for public construction. Transfer contracts are administrative instruments that facilitate the transfer of financial resources from federal coffers. This process involves a federal public financial institution or an agent overseeing the transactions. The agent is responsible for ensuring the effective operation of the project's goal and facilitating payments to contractors.

As an agent, the Brazilian state-owned bank CAIXA is responsible for overseeing and inspecting contractual execution and processing the related payments. As an agent of the Federal Government of Brazil, CAIXA operates in the contracting stage, performing technical analysis and physical and financial monitoring, assesses the accountability of the transfer contracts, and operationalizing payments to contractors using Service Contracts. The process begins with a ministry selecting the project (called "Manager of Resources"). Subsequently, using the SICONV government platform, the project is sent to CAIXA for approval. The aspects covered in the project analysis are environmental impact, engineering, document regularity, proof of legal instruments, bidding legality, and budget adequacy. After approval is communicated to the Manager of Resources and the city's legal representatives, the next step is implementation and using the resources. This study focuses on transfer contracts.

The second data source is an open public database known as the Brazilian Portal of Open Data, an initiative of the federal government that serves as a management system for the entire life cycle of agreements, transfer contracts, and partnership terms. Here, acts are registered from the formalization of the proposal to the final rendering of accounts.

Unfinished construction is a problem in many transfer contracts and has been discussed in many forums in Brazil because of its impact on public accounts and society. When construction stops, the direct consequence is delayed project completion and thus delayed benefits to citizens. Such construction can be classified as abandoned construction" or simply "paralyzed construction."

Paralyzed constructions have been stopped for any number of reasons but can be resumed again. For example, constructions with abnormal speed can become paralyzed and must be inspected by the proper organs. Abandoned constructions are those for which public management entities are no longer interested in resuming, or if there is no condition for resumption. Constructions with no conditions for resumption because of the high costs of

Revista ARACÊ
ISSN: 2358-2472

doing so due to bad weather, for example, are also classified as abandoned. Inadequate project planning, which requires the use of transparent methodology for programming the construction schedule, is considered one of the main reasons for unfinished construction. The Court of Union Accounts is a public court that supervises Brazil's proper application of public resources. They are responsible for ensuring that public managers are accountable for improper allocation of public accounts in contracts with suspicions of mismanagement. To do that, all stakeholders must clearly understand the concept of "paralyzed construction." For this study, completed constructions are those that are not considered abandoned. Our study considers public investments linked to projects completed up to 2008.

Data from both databases were cleaned, integrated. This process involves handling missing values, normalizing variables, and aligning them for cross-analysis. Special attention was paid to ensure that the variables were compatible, thus providing a robust framework for interpreting the findings.

We provide an overview of how investments are distributed across various categories and states in Brazil. Although crucial, this distribution only scratches the surface of the complexities involved in public investment patterns in Brazil, which we will delve deeper into. Investments are categorized into nine distinct sectors: urbanization, agrarian develop- ment, sports, tourism, sanitary sewage, water supply, education, fighting hunger, and public security. Each of these sectors has unique challenges and requirements, and the allocation of funds should ideally be based on a careful analysis of each's specific needs and impacts. The priorities set in these categories often reflect government policy directions and societal values. For example, focusing on education and public security can signal a government's long-term development strategy.

When examining the distribution of investments in these investment categories and comparing the amounts distributed to different states, a heterogeneous distribution pattern emerges throughout the different investment types. For example, the total sum of investments in S˜a o Paulo was the highest, whereas there was little investment in the Federal District. This type of geographic disparity could indicate various issues, ranging from regional political influences to economic factors. An interesting question is whether richer states are becoming richer and thereby exacerbating regional inequalities.

We briefly discuss the total sum of investments, noting that Sa˜ o Paulo received the most significant portion. It may be beneficial to compare these numbers with the population size, GDP, and social needs of each state. The greatest investments in infrastructure were

Revista ARACÊ
ISSN: 2358-2472

made in the states of São Paulo, Minas Gerais, Bahia, and Santa Catarina. Investments in agrarian development were mostly in Minas Gerais, São Paulo, Paraná, and Rio Grande do Sul. Investments in sports followed the same distribution as agrarian development. The states with the greatest investment in tourism were Ceara˜, Minas Gerais, Paraná, and Rio Grande do Sul.

Investment in sanitary sewage is unusual, with very small investments for most states. In this category, the greatest amounts were invested in Rio Grande do Sul and Minas Gerais; however, these were still very small. Direct investments in water supply were also very small, with only some significance in Minas Gerais. Investments in fighting hunger were also insignificant, and investments in public security were significant only in Mato Grosso do Sul. Aligning these investment patterns with indicators, such as the SVI, could offer a comprehensive view of whether investments effectively target areas of need. Some studies suggest a mismatch between public investment and social necessities. In summary, although the initial description provides a snapshot of the investment distribution, a more nuanced analysis that considers various factors— categoric focus, regional disparities, volume relative to need, and alignment with social indicators—can offer a richer understanding of public investment patterns in Brazil.

This table describes the variables used in the regression analysis

Table 1

Definitions of variables

Variable	Definition
Public_Invest	The natural logarithm of total completed public investments (2000-2008).
Sport_Invest	The natural logarithm of total completed public investments in sport-related projects (2000-2008).
Tour_Invest	The natural logarithm of total completed public investments in tourism-related projects (2000-2008).
Sant_Invest	The natural logarithm of total completed public investments in sanitary sewage-related projects (2000-2008).
Water_Invest	The natural logarithm of total completed public investments in supply water-related projects (2000-2008).
PubSecur_Invest	The natural logarithm of total completed public investments in public security-related projects (2000-2008).
FightHung_Invest	The natural logarithm of total completed public investments in fighting hunger-related projects (2000-2008).

Revista ARACÊ
ISSN: 2358-2472

Variable	Definition
Urban_Invest	The natural logarithm of total completed public investments in functional urban areas (2000-2008).
AgriDev_Invest	The natural logarithm of total completed public investments in agrarian development-related projects (2000-2008).
SVI_GR	The growth rate of Social Vulnerability Index (SVI) (2000-2010).
Gini_GR	The growth rate of Gini index (2000-2010).
IncomCapt_GR	The growth rate of the income per capita (2000-2010).
Popul_2010	The natural logarithm of populations of cities in 2010.

This table shows summary statistics of the main variables employed in this study. The table conveys the mean (Mean), standard deviation (S.D.), minimum (Min), Median, maximum (Max), and the number of observations (N). Table 1 presents information on variables.

Table 2

Summary statistics of variables

Variable	Mean	S.D.	Min	Median	Max	N
Public_Invest	0.4612891	0.4218684	0.0086743	0.3435897	3.917986	4627
Sport_Invest	0.0470449	0.0728166	0	0	0.9290753	4627
Tour_Invest	0.0330424	0.0985136	0	0	1.609438	4627
Sant_Invest	0.0025947	0.0314788	0	0	0.7617857	4627
Water_Invest	0.001238	0.0263723	0	0	1.5066	4627
PubSecur_Invest	0.0008951	0.0297456	0	0	1.225808	4627
FightHung_Invest	0.0003811	0.0149636	0	0	0.5877867	4627
Urban_Invest	0.1012864	0.1302225	0	0.0823797	1.662614	4627
AgriDev_Invest	0.0652533	0.0622873	0	0.0595417	0.749941	4627
SVI_GR	-0.2824601	0.0965401	-0.6276923	-0.2869023	0.258427	4627
Gini_GR	-0.0935633	0.112829	-0.5	-0.1	0.4634146	4627
IncomCapt_GR	0.5524892	0.3370027	-0.3223104	0.5129777	5.30329	4627
Popul_2010	9.417941	1.159534	6.703188	9.294498	16.23619	4627

Revista ARACÊ
ISSN: 2358-2472

This table presents the pairwise Pearson correlation coefficients between the main variables used in the empirical study. Table 1 explains the information of variables.

Table 3

Correlation of variables

Variables	Public_Invest	Sport_Invest	Tour_Invest	Sant_Invest	Water_Invest	PubSec_Invest	FightH_Invest	Urban_Invest
Sport_Invest	0.45							
	(0.00)							
Tour_Invest	0.42	0.14						
	(0.00)	(0.00)						
Sant_Invest	0.09	0.02	0.03					
	(0.00)	(0.19)	(0.05)					
Water_Invest	0.11	0.05	0.13	-0.00				
	(0.00)	(0.00)	(0.00)	(0.79)				
PubSecur_Invest	0.16	0.15	0.02	-0.00	-0.00			
	(0.00)	(0.00)	(0.13)	(0.87)	(0.92)			
FightHung_Invest	0.06	0.06	0.01	-0.00	-0.00	-0.00		
	(0.00)	(0.00)	(0.69)	(0.89)	(0.94)	(0.96)		
Urban_Invest	0.71	0.27	0.16	0.01	0.03	0.09	0.04	
	(0.00)	(0.00)	(0.00)	(0.56)	(0.08)	(0.00)	(0.02)	
AgriDev_Invest	0.31	0.07	0.03	0.03	0.01	0.04	0.01	0.07
	(0.00)	(0.00)	(0.02)	(0.04)	(0.71)	(0.02)	(0.61)	(0.00)

Information on the variables used in the empirical analysis are presented in Table 1 and related statistics are described in Table 2. Table 3 presents the pairwise Pearson correlation analysis of several public investment variables. Notably, we found that investments in sports facilities are often associated with broader urban development initiatives, with a strong positive correlation (0.71, $p < 0.001$). The results suggest that tourism-related projects frequently coincide with urban infrastructure improvements with a positive correlation (0.27, $p < 0.001$). Furthermore, water supply investment is positively correlated with both sports (0.11, $p < 0.001$) and tourism investment (0.05, $p < 0.001$), highlighting the interconnected nature of these investment areas. Investment in fighting hunger is positively and slightly correlated with urban infrastructure improvements (0.04, p <

Revista ARACÊ
ISSN: 2358-2472

0.002), implying that efforts to combat hunger are partially linked to urban development projects. These correlations underscore the integrated approach that is often required in public investment strategies to address multiple facets of social and economic development.

3 EMPIRICAL STUDY

This section analyzes the empirical results obtained from testing the noted hypotheses. The hypotheses can be empirically investigated using regression analysis, as follows:

$$SVI\_GR = \beta_0 + \beta_1 \text{Public\_Invest}_{2000-2008} + \text{Control variables} + \epsilon, \quad (9)$$

where the dependent variable SVI_GR is the growth rate of SVI from 2000 to 2010, and the main independent variable Public Invest is the natural logarithm of the concluded public investment from 2000 to 2008. Control variables, such as the growth rate of the Gini index (Gini_GR), growth rate of income per capita (IncomCapt_GR), Popul_2010, and the natural logarithm of the population of the city in 2010. To analyze the impact of public investment on the growth rate of SVI, we use two-year lagged information on public investment. Therefore, we used the concluded public investment from 2000 to 2008 to incorporate the influence of investment on the lives of individuals in the economy.

We estimated the relationship between the SVI growth rate and the public investment variable. To ensure accuracy, we considered other variables, such as the Gini index and income per capita growth rate. (L ewb e l , 2 0 1 2) method was used to address potential endogeneity issues. These issues arise from the interplay between the SVI and income variables, which mutually influence each other. Additionally, there is a concern of omitted variable bias, where unobserved factors can affect the dependent and independent variables.

Utilizing heteroskedasticity in the data, we apply (L ewb e l , 2 0 1 2) approach to gener- ate internal instruments. One way to start is to verify the existence of heteroscedasticity using tests such as the Breusch-Pagan test (Ha l u n g a e t a l . , 2 0 1 7) . When heteroskedasticity is detected, instruments can be generated by multiplying the endogenous regressors (Gini index and income per capita growth) by their respective heteroskedasticity. We used these instruments in a two-stage least squares (2SLS) regression. During the initial phase, the regression of endogenous variables on instruments occurs, whereas exogenous variables are used to obtain predicted values. During the second

Revista ARACÊ
ISSN: 2358-2472

stage, the predicted values are used in the regression analysis to estimate their impact on the growth rate of the SVI.

By employing this approach, we can mitigate the inherent bias and inconsistency resulting from endogeneity, thereby yielding a more accurate assessment of the impact of income inequality and economic growth on social vulnerability. The approach of using internal instruments derived from (L ewb e l , 2 0 1 2) provides a practical and reliable solution to address this endogenous estimation challenge.

This table presents the estimates. The regression model is $SVI\_GR = \beta_0 + \beta_1 \text{Public\_Invest}_{2000-2008} + \text{Control variables} + \epsilon$, where the dependent variable SVI_GR is the growth rate of SVI from 2000 to 2010, the main independent variable Public Invest is the natural logarithm of concluded public investment from 2000 until 2008 and control variables such as the growth rate of Gini index Gini_GR, the growth rate of income per capita IncomCapt_GR, Popul 2010, the natural logarithm of Population of the city in 2010. We estimate an instrumental variables regression model based on (Lewbel, 2012) method. All models include constant, region-fixed effects. Standard errors are in parentheses. * indicates significance ($p < 0.05$). Table 4 presents information on variables.

Table 4

Determinants of impacts of public investment on svi

	Pool	**Pool Control**	**Fixed**	**Fixed Control**	**IV Fixed Control**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
Public Invest	-0.0073	-0.0172*	-0.0136*	-0.0146*	-0.0132*
	(0.0028)	(0.0032)	(0.0034)	(0.0038)	(0.0032)
Gini GR		0.2515*		0.2235*	0.1291*
		(0.0156)		(0.0134)	(0.0312)
IncomCapt GR		-0.0351*		-0.0662*	-0.0561*
		(0.0053)		(0.0048)	(0.0102)
Popul 2010		0.0040		-0.0032*	-0.0023
		(0.0014)		(0.0015)	(0.0016)
Constant	-0.2790*	-0.2690*	-0.3188*	-0.2316*	-0.2559*
	(0.0021)	(0.0146)	(0.0050)	(0.0145)	(0.0192)
FE: Regions	No	No	Yes	Yes	Yes
Observations	4629	4629	4629	4629	4629
Adjusted R^2	0.001	0.077	0.018	0.096	0.215

Revista ARACÊ
ISSN: 2358-2472

We test first hypothesis, which states that public investment benefits the economy by lowering its socioeconomic vulnerability. The results, shown in Table 4, demonstrate the significant impact of public investments on the SVI in Brazil, focusing on various model specifications. Across all models, Public Invest exhibited a negative and statistically significant coefficient, indicating that increased public investment is associated with reduced social vulnerability. Specifically, the coefficient ranges from -0.0073 ($p < 0.01$) in column (1), the pool regression model without control variables, to -0.0172 ($p < 0.001$) in column (2), the pool regression with the control variables, suggesting that controlling for additional variables strengthens the observed relationship. Including regional fixed effects in columns(3) to (5) further confirms this negative relationship, with the coefficients remaining significant. Moreover, we control for income inequality, per capita income, and city population size. The Gini GR variable, representing income inequality, consistently shows a positive and significant relationship with the SVI across all relevant models, highlighting that higher income inequality correlates with increased social vulnerability. The instrumental variable (IV) regression model in column (5) uses the Lewbel (2012) method. We will continue to use this model in the remainder of this paper.

Table 5

Regional analysis of impacts of public investment on svi

	Center_West	**North**	**North_East**	**South**	**South_East**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
Public Invest	-0.0033	-0.0524*	-0.0173	-0.0044	0.0048
	(0.0100)	(0.0125)	(0.0053)	(0.0076)	(0.0051)
Controls	Yes	Yes	Yes	Yes	Yes
Observations	409	303	1427	1107	1383
Adjusted R^2	0.055	0.185	0.077	0.101	0.058

This table presents the estimates. The regression model is $SVI\_GR = \beta_0 + \beta_1 \text{Public\_Invest}_{2000-2008} + \text{Control variables} + \epsilon$, where the dependent variable SVI_GR is the growth rate of SVI from 2000 to 2010, the main independent variable Public Invest is the natural logarithm of concluded public investment from 2000 until 2008 and control variables such as the growth rate of Gini index Gini_GR, the growth rate of income per capita IncomCapt_GR, Popul 2010, the natural logarithm of Population of the city in 2010. We estimate an instrumental variables regression model based on (Lewbel, 2012) method. All

Revista ARACÊ
ISSN: 2358-2472

models include constant. Standard errors are in parentheses. * indicates $p < 0.05$. Table 5 presents information on variables

The first hypothesis was tested for each region in Brazil. The results in Table 5 highlight the regional variations in the impact of public investments on the SVI across different regions. The coefficient of public investment is negative and statistically significant in the North (-0.0524, p < 0.001) and North-East (-0.0173, p < 0.01), indicating that increased public investment is particularly effective in reducing social vulnerability in these areas. This suggests that these regions benefit more from public investments, owing to their higher initial levels of social vulnerability and greater need for infrastructure and social service improvements. In contrast, the coefficients for public investment in the Central- West, South, and South-East regions were not statistically significant, suggesting that public investment in these regions does not significantly reduce social vulnerability. This could be due to a variety of factors, including lower levels of social vulnerability, better existing infrastructure, or differences in the efficiency of investment implementation. These findings are consistent with Brazil's well-documented social and economic dis- parities. According to the IBGE, the North and North-East regions have the country's highest levels of poverty and social inequality levels. For instance, the North-East has a poverty rate of 43.5 compared to the national average of 25.4 is also the highest in the North-East and North regions at 0.545 and 0.539, respectively, compared with a national average of 0.515. These variations highlight the differing effectiveness and impact of public investment in reducing social vulnerability across Brazil and emphasize the need for region-specific strategies and tailored approaches to public spending to maximize social benefits.

This table presents the estimates. The regression model is $SVI\_GR = \beta_0 + \beta_1 \text{Public\_Invest}_{2000-2008} + \text{Control variables} + \epsilon$, where the dependent variable SVI_GR is the growth rate of SVI from 2000 to 2010, the main independent variable Public Invest is the natural logarithm of concluded public investment from 2000 until 2008 and control variables such as the growth rate of Gini index Gini_GR, the growth rate of income per capita IncomCapt_GR, Popul 2010, the natural logarithm of Population of the city in 2010. We estimate an instrumental variables regression model based on (Lewbel, 2012) method. All

Revista ARACÊ
ISSN: 2358-2472

models include constant. Standard errors are in parentheses. * indicates $p < 0.05$. Table 6 presents information on variables.

Table 6

Regional analysis of impacts of public investment on svi

	Center_West	North	North_East	South	South_East
	(1)	**(2)**	**(3)**	**(4)**	**(5)**
Public Invest	-0.0033	-0.0524*	-0.0173	-0.0044	0.0048
	(0.0100)	(0.0125)	(0.0053)	(0.0076)	(0.0051)
Controls	Yes	Yes	Yes	Yes	Yes
Observations	409	303	1427	1107	1383
Adjusted R^2	0.055	0.185	0.077	0.101	0.058

To better understand the regional variations in the results presented in Table 6, we determined the economic situation of each region. We consider economic factors such as the human development index (HDI), life expectancy, and income per capita to construct binary variables indicating whether a factor is above the median. Areas with economic factors above the median are considered high-income areas; otherwise, they are considered low-income areas. Table 6 provides a detailed breakdown of the distribution of the HDI, life expectancy, and income per capita across different regions in Brazil. The data reveal significant regional disparities in these vital socioeconomic indicators.

Regarding HDI, the North-East region exhibited the highest percentage of low HDI (95.77 followed by the North (88.6 Conversely, the South and South-East regions showed much lower percentages of low HDI (15.25 Central-West region also showed a moderately low HDI (41.73 further highlights these regional differences. The North-East (97.32 have the highest percentages of low life expectancy, emphasizing the need for targeted health interventions, while the South and South-East have significantly lower percentages of low life expectancy (13.79 healthcare services. The per-capita income distribution also follows a similar pattern. The North-East (96.19 capita, indicating significant economic disparity, whereas the South and South-East have significantly lower percentages of low income per capita (19.15 reflecting more vital economic conditions. The Central-West region shows a moderate rate of low-income per capita at 28.4.

This table presents the estimates The regression model is $\text{SVI\_GR} = \beta_0 + \beta_1 \text{Public\_Invest}_{2000-2008} + \text{Control variables} + \epsilon$, where the dependent variable SVI_GR is the

Revista ARACÊ
ISSN: 2358-2472

growth rate of SVI from 2000 to 2010, the main independent variables are the logarithm of concluded sport, tourism, urban and agrarian public investments from 2000 until 2008 and control variables such as the growth rate of Gini_index Gini_GR, the growth rate of income per capita IncomCapt_GR, Popul 2010, the natural logarithm of Population of the city in 2010. We estimate an instrumental variables regression model based on (Lewbel, 2012) method. All models include constant. Standard errors are in parentheses * $p < 0.05$. Table 7 presents information on variables

Table 7

Determinants of regional impacts of public investment on svi in low- and high-income areas

	Human Development Index		**Life Expectation**		**Income per capita**	
2-3 (lr)4-5 (lr)6-7	**Low**	**High**	**Low**	**High**	**Low**	**High**
	(1)	**(2)**	**(3)**	**(4)**	**(5)**	**(6)**
Public Invest	-0.0221*	0.0045	-0.0207*	0.0002	-0.0234*	0.0026
	(0.0051)	(0.0038)	(0.0044)	(0.0044)	(0.0052)	(0.0038)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
FE: Regions	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2326	2303	2320	2309	2315	2314
Adjusted R^2	0.204	0.127	0.216	0.102	0.225	0.107

Next, we tested the second hypothesis, which states that public investment benefits the economy by lowering the socioeconomic vulnerability of the low-income economy rather than that of the high-income economy. The results in Table 7 reveal significant variations in the impact of public investments on the SVI based on different socioeconomic indicators such as the HDI, life expectancy, and income per capita. Public investment has a statistically significant negative effect on the SVI in regions with low HDI (-0.0221, $p < 0.001$), low life expectancy (-0.0207, $p < 0.001$), and low income per capita (- 0.0234, $p < 0.001$). These findings indicate that public investment is particularly effective at reducing social vulnerability in areas with lower levels of human development, life expectancy, and income. In contrast, the coefficients for public investment are not statistically significant in regions with high HDI, life expectancy, and income per capita. This suggests that the impact of public investment on reducing social vulnerability is less pronounced in more developed areas with better socioeconomic conditions. This figure shows the distribution of the growth rate of the SVI (SVI GR) (Top) and public investments (Public Invest) (Down) for high and low life expectancy areas. Table 8 presents information on the included variables.

Revista ARACÊ
ISSN: 2358-2472

This table presents the estimates. The regression model is $SVI\_GR = \beta_0 + \beta_1 \text{Public\_Invest}_{2000-2008} + \text{Control variables} + \epsilon$, where the dependent variable SVI_GR is the growth rate of SVI from 2000 to 2010. We estimate an instrumental variables regression model based on (Lewbel, 2012) method. All models include constant. Standard errors are in parentheses. * $p < 0.05$. Table 8 presents information on variables.

Table 8

Regional analysis of impacts of different public investment on SVI

	ALL	Center West	North	North East	South	South East
Panel I: Sport investment						
Sport Invest	-0.0441*	-0.0233	-0.1276*	-0.0840	0.0326	-0.0042
	(0.0178)	(0.0560)	(0.0513)	(0.0294)	(0.0379)	(0.0317)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4629	409	303	1427	1107	1383
Adjusted R^2	0.214	0.049	0.116	0.076	0.100	0.053
Panel II: Tourism investment						
Tour Invest	-0.0495*	-0.1149	-0.1451	-0.0624	0.0055	0.0175
	(0.0124)	(0.0361)	(0.0445)	(0.0200)	(0.0261)	(0.0280)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4629	409	303	1427	1107	1383
Adjusted R^2	0.215	0.070	0.111	0.078	0.101	0.057
Panel III: Urban investment						
Urban Invest	-0.0196*	-0.0170	-0.1041	-0.0459*	-0.0205	0.0314*
	(0.0099)	(0.0312)	(0.0335)	(0.0183)	(0.0268)	(0.0127)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4629	409	303	1427	1107	1383
Adjusted R^2	0.213	0.045	0.139	0.076	0.102	0.060
Panel IV: Agrarian development investment						
AgriDev Invest	0.0249	-0.0652	0.0638	0.0838*	-0.1292*	0.0248
	(0.0219)	(0.0670)	(0.0883)	(0.0353)	(0.0617)	(0.0345)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	4629	409	303	1427	1107	1383
Adjusted R^2	0.213	0.042	0.088	0.077	0.103	0.052

This table presents the estimates. The regression model is $SVI\_GR = \beta_0 + \beta_1 \text{Public\_Invest}_{2000-2008} + \text{Control variables} + \epsilon$, where the dependent variable SVI_GR is the growth rate of SVI from 2000 to 2010. We estimate an instrumental variables regression

Revista ARACÊ
ISSN: 2358-2472

model based on (Lewbel, 2012) method. All models include constant, region-fixed effects. Standard errors are in parentheses. * $p < 0.05$. Table 9 presents information on variables.

Table 9

Determinants of impacts of different public investments on SVI in low-income and high-income areas

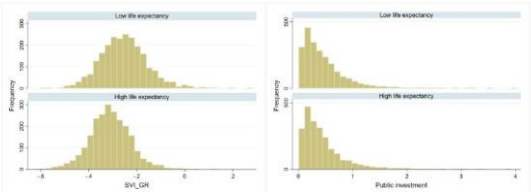
Panel I: Sport investment

	Human Dev. Index		**Life Expectation**		**Income per capita**	
2-3 (lr)4-5 (lr)6-7	**Low**	**High**	**Low**	**High**	**Low**	**High**
Sport Invest	-0.0972*	0.0250	-0.0774*	0.0194	-0.0791	-0.0078
	(0.0265)	(0.0209)	(0.0231)	(0.0233)	(0.0278)	(0.0222)
Controls&FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2326	2303	2320	2309	2315	2314
Adjusted R^2	0.204	0.127	0.214	0.101	0.222	0.108

	Human Dev. Index		**Life Expectation**		**Income per capita**	
2-3 (lr)4-5 (lr)6-7	**Low**	**High**	**Low**	**High**	**Low**	**High**
Tour Invest	-0.0824*	-0.0099	-0.0726*	-0.0138	-0.0896*	-0.0114
	(0.0209)	(0.0149)	(0.0194)	(0.0149)	(0.0230)	(0.0133)
Controls&FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2326	2303	2320	2309	2315	2314
Adjusted R^2	0.203	0.127	0.214	0.102	0.224	0.109

Figure 1

This figure shows the distribution of the growth rate of SVI (SVI GR)(Top) and public investments (Public Invest) (Down) for high and low life expectancy areas. Table 1 presents information on variable



Revista ARACÊ
ISSN: 2358-2472

4 DISTRIBUTIONS OF THE GROWTH RATE OF SVI AND PUBLIC INVESTMENTS FOR HIGH AND LOW LIFE EXPECTANCY AREAS

As mentioned earlier, the main findings reveal that public investment positively impacts the economy by reducing the socioeconomic vulnerability of low-income areas more than that of high-income areas. These results raise another question: whether the primary drivers of these diverse effects are related to diverse levels of public investment in low- and high-income areas? Figure 1 presents a comparative analysis of the SVI growth rate and public investment, grouped by low and high life expectancy. The top graph illustrates the distribution of the SVI GR, while the bottom graph displays the distribution of public investments. The SVI GR distribution for cities with low life expectancy showed a roughly normal distribution centered around -0.3, indicating a general decrease in social vulnerability. The peak frequency was slightly skewed towards higher negative values, suggesting that a considerable number of cities experienced a significant reduction in social vulnerability. Conversely, cities with high life expectancy exhibited a normal distribution, but centered closer to -0.4, suggesting that these cities also saw reductions in social vulnerability. However, the peak frequency was more pronounced, indicating a higher concentration of cities with moderate decreases in vulnerability.

The distributions of public investment in cities with low and high life expectancies reveal highly skewed distributions, with most cities receiving smaller investments. No significant differences were observed between the two distributions. We also applied a series of statistical tests and did not find a substantial discrepancy between the two distributions. The comparative analysis of SVI GR and public investment distributions highlights several key insights. Areas with low and high life expectancy generally experience reductions in social vulnerability of varying magnitudes; however, this diverse effect is not due to inequalities in public funding.

We examined the first hypothesis to understand the regional benefits of different public investments in the economy by reducing socioeconomic vulnerability. The results from Table 8, Panel I illustrate the differential impact of sports investments on SVI across various regions in Brazil, and the remaining results are presented in Internet Appendix. The anal- ysis reveals that sports investments have a statistically significant negative impact on the SVI at the national level (ALL), indicating that these investments effectively reduce social vulnerability across the country. Specifically, the coefficient of sports investments in the national model is negative and significant, highlighting its potential to improve social

Revista ARACÊ
ISSN: 2358-2472

outcomes. However, when disaggregated by region, the effectiveness of sports investment varies. In the North, the coefficient of sports investments is negative and statistically significant, suggesting that these investments are particularly beneficial in reducing social vulnerability in this region. This is consistent with the higher levels of social vulnerability and need for recreational infrastructure in the North.

In contrast, the coefficients for sports investment in the Central-West, North-East, South, and South-East regions are not statistically significant, indicating that sports investments do not have a measurable impact on reducing social vulnerability in these regions. This could be due to several factors, such as better existing sports infrastructure, different regional priorities, or varying levels of investment effectiveness. Regional variations in the impact of sports investment underscore the importance of considering local contexts when designing and implementing public investment strategies. While sports investments can reduce social vulnerability at the national level and in specific regions, such as the north, other regions may require different types of investments or additional measures to achieve similar social outcomes.

We continue to examine the second hypothesis to understand the benefits of different public investments in the economy comparing low-and high-income areas. The results in Table 9 provide a comprehensive analysis of the determinants of the impact of public investments on the SVI across different socioeconomic dimensions, including HDI, life expectancy, and income per capita; the rest of the results are presented in Internet Appendix. Panel I shows that sports investment significantly reduces social vulnerability in regions with low HDI, life expectancy, and income per capita. These findings indicate that sports investments are particularly effective in underdeveloped areas with significant needs. By contrast, regions with high HDI, life expectancy, and income per capita do not show substantial impact from sports investment, suggesting that these regions may already have adequate sports infrastructure or other pressing priorities. Panel II shows that tourism investment has a similar pattern, with such investments significantly and negatively impacting the SVI in regions with low HDI, life expectancy, and income per capita.

However, in regions with higher HDI, life expectancy, and income per capita, the impacts were not statistically significant, indicating fewer relative benefits from tourism investment in these more developed areas. Panel III shows that urban investment effectively reduces social vulnerability in regions with low HDI, life expectancy, and income per capita. Interestingly, urban investments also have a significant positive impact in areas with high HDI

Revista ARACÊ
ISSN: 2358-2472

(0.0276, p < 0.01) and income per capita (0.0262, p < 0.01), suggesting that even in more developed regions, urban infrastructure improvements can yield substantial social benefits. Panel IV shows that agricultural investments do not have statistically significant impacts on SVI across the various socioeconomic dimensions analyzed, except for a small positive effect in regions with low HDI and life expectancy. This finding indicates that, while agrarian development is essential, its direct impact on reducing social vulnerability may be less pronounced or may require complementary investments to be more effective.

This table presents the estimates. The regression model is $SVI\_GR = \beta_0 + \beta_1 \text{Public\_Invest}_{2000-2008} + \text{Control variables} + \epsilon$, where the dependent variable SVI_GR is the growth rate of SVI from 2000 to 2010. We estimate an instrumental variables regression model based on (Lewbel, 2012) method. All models include constant. Standard errors are in parentheses. * indicates $p < 0.05$. Table 10 presents information on variables.

Table 10

Determinants of impacts of public investment on SVI of cities with low life expectancy

	ALL	Center West	North	North East	South	South East
Panel I: Sport investment						
Sport Invest	-0.0774*	-0.1477	-0.1446*	-0.0629*	0.0020	-0.0593
	(0.0231)	(0.0823)	(0.0568)	(0.0307)	(0.1089)	(0.0573)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2320	148	278	1388	153	353
Adjusted R^2	0.214	0.100	0.139	0.069	0.025	0.007
Panel II: Tourism investment						
Tour Invest	-0.0726*	-0.0742	-0.1741*	-0.0608	0.0864	0.0828
	(0.0194)	(0.1276)	(0.0466)	(0.0223)	(0.1448)	(0.0711)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2320	148	278	1388	153	353
Adjusted R^2	0.214	0.118	0.119	0.077	-0.435	0.002
Panel III: Urban investment						
Urban Invest	-0.0557*	-0.0224	-0.1208	-0.0435*	-0.0292	0.0542
	(0.0138)	(0.0829)	(0.0394)	(0.0181)	(0.1729)	(0.0364)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2320	148	278	1388	153	353
Adjusted R^2	0.215	0.081	0.158	0.074	-0.373	-0.097
Panel IV: Agrarian development investment						
AgriDev Invest	0.0510	-0.0834	0.0663	0.0710*	-0.5159	-0.0371

Revista ARACÊ
ISSN: 2358-2472

	ALL	Center West	North	North East	South	South East
	(0.0313)	(0.1307)	(0.0945)	(0.0351)	(0.1830)	(0.0955)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Observations	2320	148	278	1388	153	353
Adjusted R^2	0.212	0.073	0.072	0.078	0.081	0.019

Finally, we restrict our research to understanding the impact of various types of public investments on SVI in cities with low life expectancy across different regions. The results are presented in Tables 10 and in Internet Appendix. In this regard, Panel I shows that sports investment has a significant negative impact on the SVI at the national level (ALL), indicating that such investments are effective in reducing social vulnerability in cities with low life expectancy. The North region also showed a significant negative impact (-0.1446, $p < 0.05$), similar to the North-East (-0.0629, $p < 0.05$). However, the effect was insignificant in the Central-West, South, and South-East regions, suggesting that the effectiveness of sports investments may vary based on regional context and existing infrastructure. Panel II reveals that tourism investments significantly reduces SVI at the national level (ALL).

The North-East region shows a powerful and significant negative impacts, highlighting the importance of tourism development in these areas. Conversely, the impact is insignificant in the Central-West, South, and South-East regions, and the adjusted R-squared values are lower, indicating less explanatory power for the model in these regions.

Summary and discussion

This study investigated the distributive impact of public investments on the SVI across Brazil's heterogeneous regional economies and socioeconomic circumstances in Brazil. The results demonstrate notable asymmetric policy returns in the efficacy of public interventions, underscoring the necessity of focused approaches to mitigate social vulnerability. Our analysis indicates that targeted public interventions can effectively decrease structural vulnerability, especially in less-developed areas with greater baseline levels of necessity. Investments in sports infrastructure, tourism, and urban development have decreased the SVI, particularly in the North and North-East regions, where socioeconomic conditions are most challenging. These regions receive more significant advantages from public investment because they initially have higher levels of socioeconomic vulnerability and are in greater need of infrastructure and social service improvements. These geographically stratified outcomes reflect Brazil's core-periphery dynamics and align with the well-documented social and economic inequalities among different regions in Brazil, as emphasized by the IBGE.

Revista ARACÊ
ISSN: 2358-2472

The North-East and northern areas, characterized by elevated poverty rates and pronounced social disparities, exhibit the most substantial declines in social vulnerability resulting from public interventions.

More precisely, sports investments have a notable effect on decreasing SVI at the national level and in the North and North-East regions. However, these investments insignificantly influence the Central-West, South, and South-East regions. This implies that areas with greater social vulnerability and less advanced infrastructure experience more significant advantages from these investments. Similarly, tourist investments have a notable adverse effect on the SVI at the national level, particularly in the North-East region, emphasizing the potential of tourist development in enhancing social outcomes in economically depressed areas. Investments in urban areas significantly reduced socioeconomic vulnerability in the North and North-East regions. This highlights the need to enhance the urban infrastructure to address these vulnerabilities.

However, the effect of investments in agricultural growth differed across regions. Although agricultural investments had a positive and substantial impact in the North-East, they had a negative and significant effect in the South. This indicates that regional agricultural practices and economic conditions influence the success of agrarian investments. These findings are aligned with the current body of research on the impact of public spending on fostering social and economic progress.

Further research should examine the complex relationship between government socioeconomic goals, the distribution of public resources, and factors that influence this process. This study emphasizes the importance of examining the relationships between political incentives and institutional capacity, which ultimately affects the achievement of socioeconomic goals through the distribution of public investment. By thoroughly examining these interactions, further analysis can uncover a more detailed understanding of efficient strategies for enhancing the allocation of resources and promoting fair and sustainable socioeconomic growth. This could illuminate pathways for institutional reforms to enhance the equity- socioeconomic efficiency tradeoff in public investment.

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Revista ARACÊ
ISSN: 2358-2472

APPENDIX

Proof of Proposition 1

Proof of Proposition 1. We define the objective function of the central planner as maximization of the social welfare function W:

$$\max_{I_H, I_L}\left[U(AK^{\alpha}L^{\beta}I_H^{\gamma}I_L^{\delta}) - (\phi(I_H) + \psi(I_L))\right]$$

We set up the Lagrangian function to solve the above Optimization Problem:

$$\mathcal{L} = U(AK^{\alpha}L^{\beta}I_H^{\gamma}I_L^{\delta}) - (\phi(I_H) + \psi(I_L)) + \lambda(I - I_H - I_L)$$

First-order Conditions of the Lagrangian function:

$$\frac{\partial \mathcal{L}}{\partial I_H} = U_Y \cdot \gamma AK^{\alpha}L^{\beta}I_H^{\gamma-1}I_L^{\delta} - \phi'(I_H) - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial I_L} = U_Y \cdot \delta AK^{\alpha}L^{\beta}I_H^{\gamma}I_L^{\delta-1} - \psi'(I_L) - \lambda = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - I_H - I_L = 0$$

We solve the first-order conditions simultaneously to find the optimal I_H and I_L. We equate the first and second marginal conditions for λ and we can get:

$$U_Y \cdot \gamma AK^{\alpha}L^{\beta}I_H^{\gamma-1}I_L^{\delta} - \phi'(I_H) = U_Y \cdot \delta AK^{\alpha}L^{\beta}I_H^{\gamma}I_L^{\delta-1} - \psi'(I_L)$$

We simplify and rearrange the above expression to find the ratio $\frac{I_H}{I_L}$:

$$\gamma I_H^{\gamma-1}I_L^{\delta} - \frac{\phi'(I_H)}{AK^{\alpha}L^{\beta}} = \delta I_H^{\gamma}I_L^{\delta-1} - \frac{\psi'(I_L)}{AK^{\alpha}L^{\beta}}$$
$$\frac{I_H}{I_L} = \frac{\gamma}{\delta} \cdot \frac{\phi'(I_H)}{\psi'(I_L)}$$

We use the budget constraint $I_H + I_L = I$, and we substitute:

$$I_H = \left(\frac{\gamma}{\delta} \cdot \frac{\phi'(I_H)}{\psi'(I_L)}\right) I_L.$$

We solve for I_H and I_L as follows:

$$I_L = \frac{I}{1 + \frac{\gamma}{\delta} \cdot \frac{\phi'(I_H)}{\psi'(I_L)}}, \quad I_H = I - \frac{I}{1 + \frac{\gamma}{\delta} \cdot \frac{\phi'(I_H)}{\psi'(I_L)}}.$$

We obtain optimal allocation of public investments I_H for high-income sections and I_L for low-income sections with ensuring the maximization of social welfare by balancing economic output and reducing social vulnerability. The optimal allocation of public investments I_H and I_L depend on the marginal utilities and marginal impacts of public investment on social vulnerability.

Revista ARACÊ
ISSN: 2358-2472

If γ (the output elasticity of investment in high-income section) is high, a larger fraction of the public investment may be allocated to high-income sections, provided the marginal utility of output exceeds the marginal increase in social vulnerability. However, if δ (the output elasticity of investment in low-income section) is high and $\psi'(I_L)$ (the marginal reduction in vulnerability for low-income investment) is significant, more public investment may be allocated to low-income sections.

Proof of Proposition [prop_2]. We continue with proof of Proposition [prop_1] and analyze the marginal reduction in social Vulnerability. We recall social vulnerability V, inspired by (Castel, 1995):

$$V = \phi(I_H) + \psi(I_L),$$

where $\phi(I_H)$ and $\psi(I_L)$ are functions where investments reduce vulnerability with substituting I_H and I_L obtained from Proposition [prop_1].

To demonstrate that the total vulnerability function V decreases with respect to I_H and I_L, we need to take the partial derivatives of V with respect to I_H and I_L and show that these derivatives are negative. Given the total vulnerability function:

$$V = \phi\left(\frac{\gamma I}{\delta(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)})}\right) + \psi\left(\frac{I}{1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}}\right).$$

Let's denote the terms inside the functions for simplicity:

$$A = \frac{\gamma I}{\delta(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)})}, \quad B = \frac{I}{1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}}.$$

In the first step, we estimate partial derivative with respect to I_H. First, we need to express $\frac{\partial A}{\partial I_H}$ and $\frac{\partial B}{\partial I_H}$.

$$\frac{\partial A}{\partial I_H} = \frac{\partial}{\partial I_H}\left(\frac{\gamma I}{\delta(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)})}\right) = \frac{\gamma I}{\delta}\cdot\left(\frac{-\frac{\gamma}{\delta}\cdot\frac{\partial}{\partial I_H}\left(\frac{\phi'(I_H)}{\psi'(I_L)}\right)}{\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}\right).$$

Since $\frac{\partial}{\partial I_H}\left(\frac{\phi'(I_H)}{\psi'(I_L)}\right) = \frac{\phi''(I_H)\cdot\psi'(I_L)}{\psi'(I_L)^2} = \frac{\phi''(I_H)}{\psi'(I_L)}$:

$$\frac{\partial A}{\partial I_H} = -\frac{\gamma^2 I\cdot\phi''(I_H)}{\delta^2\psi'(I_L)\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}.$$

Taking the derivative of B with respect to I_H:

Revista ARACÊ
ISSN: 2358-2472

$$\frac{\partial B}{\partial I_H} = \frac{\partial}{\partial I_H}\left(\frac{I}{1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}}\right) = -\frac{\gamma I\cdot\phi''(I_H)}{\delta\psi'(I_L)\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}.$$

Now, we use these results to find the partial derivatives of V with respect to I_H:

$$\frac{\partial V}{\partial I_H} = \phi'(A)\cdot\frac{\partial A}{\partial I_H} + \psi'(B)\cdot\frac{\partial B}{\partial I_H}.$$

Substitute the derivatives:

$$\frac{\partial V}{\partial I_H} = \phi'(A)\cdot\left(-\frac{\gamma^2 I\cdot\phi''(I_H)}{\delta^2\psi'(I_L)\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}\right) + \psi'(B)\cdot\left(-\frac{\gamma I\cdot\phi''(I_H)}{\delta\psi'(I_L)\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}\right)$$

Factor out the common term:

$$\frac{\partial V}{\partial I_H} = -\frac{\gamma I\cdot\phi''(I_H)}{\psi'(I_L)\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}\cdot\left(\frac{\gamma}{\delta}\cdot\phi'(A)+\psi'(B)\right).$$

Since $\phi''(I_H) < 0$, $-\phi''(I_H) > 0$. Therefore: $\frac{\partial V}{\partial I_H} < 0$.

Next, we estimate partial derivative with respect to I_L, therefore, the derivative of A with respect to I_L:

$$\frac{\partial A}{\partial I_L} = \frac{\partial}{\partial I_L}\left(\frac{\gamma I}{\delta\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)}\right) = \frac{\gamma I}{\delta}\cdot\left(\frac{-\frac{\gamma}{\delta}\cdot\frac{\partial}{\partial I_L}\left(\frac{\phi'(I_H)}{\psi'(I_L)}\right)}{\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}\right) = \frac{\gamma^2 I\cdot\phi'(I_H)\cdot\psi''(I_L)}{\delta^2\psi'(I_L)^2\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}.$$

Taking the derivative of B with respect to I_L:

$$\frac{\partial B}{\partial I_L} = \frac{\partial}{\partial I_L}\left(\frac{I}{1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}}\right) = \frac{\gamma I\cdot\phi'(I_H)\cdot\psi''(I_L)}{\delta\psi'(I_L)^2\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}.$$

Now, we use these results to find the partial derivatives of V with respect to I_L:

$$\frac{\partial V}{\partial I_L} = \phi'(A)\cdot\frac{\partial A}{\partial I_L} + \psi'(B)\cdot\frac{\partial B}{\partial I_L}.$$

Substitute the derivatives:

$$\frac{\partial V}{\partial I_L} = \phi'(A)\cdot\left(\frac{\gamma^2 I\cdot\phi'(I_H)\cdot\psi''(I_L)}{\delta^2\psi'(I_L)^2\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}\right) + \psi'(B)\cdot\left(\frac{\gamma I\cdot\phi'(I_H)\cdot\psi''(I_L)}{\delta\psi'(I_L)^2\left(1+\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)^2}\right)$$

Factor out the common term:

Revista ARACÊ
ISSN: 2358-2472

$$\frac{\partial V}{\partial I_L} = \frac{\gamma I \cdot \phi'(I_H) \cdot \psi''(I_L)}{\psi'(I_L)^2 \left(1 + \frac{\gamma}{\delta} \cdot \frac{\phi'(I_H)}{\psi'(I_L)}\right)^2} \cdot \left(\frac{\gamma}{\delta} \cdot \phi'(A) + \psi'(B)\right).$$

Since $\psi''(I_L) < 0$, $\psi''(I_L) < 0$. Therefore: $\frac{\partial V}{\partial I_L} < 0$. The partial derivatives $\frac{\partial V}{\partial I_H}$ and $\frac{\partial V}{\partial I_L}$ are both negative, showing that the total vulnerability function V decreases with respect to I_H and I_L. This shows that the optimal allocation of public investments minimizes total socioeconomic vulnerability. ▫

Proof of Proposition [prop_3]. First, we continue with proof of Proposition [prop_1] and analyze the marginal reduction in social Vulnerability. We derive the total reduction in social vulnerability for an incremental increase in public investment.

Let ΔI_H and ΔI_L be a small increase in public investment in high-income section and in low-income section, respectively. We apply Taylor series around I_H and I_L for expanding the vulnerability functions for high-income and low-income sections as follows:

$$\phi(I_H + \Delta I_H) \approx \phi(I_H) + \phi'(I_H)\Delta I_H + \frac{1}{2}\phi''(I_H)(\Delta I_H)^2,$$
$$\psi(I_L + \Delta I_L) \approx \psi(I_L) + \psi'(I_L)\Delta I_L + \frac{1}{2}\psi''(I_L)(\Delta I_L)^2.$$

Let's assume that $\Delta I_H = \Delta I_L = \Delta I$. Then, the vulnerability reduction becomes:

$$\Delta V_H \approx \phi'(I_H)\Delta I + \frac{1}{2}\phi''(I_H)(\Delta I)^2,$$
$$\Delta V_L \approx \psi'(I_L)\Delta I + \frac{1}{2}\psi''(I_L)(\Delta I)^2.$$

Next, we derive two expressions: $\psi'(I_L)$ and $\phi'(I_H)$. From the ratio:

$$\frac{I_H}{I_L} = \frac{\gamma}{\delta} \cdot \frac{\phi'(I_H)}{\psi'(I_L)}.$$

The solutions for $\psi'(I_L)$ and $\phi'(I_H)$ as follows:

$$\psi'(I_L) = \frac{\gamma}{\delta} \cdot \frac{I_L}{I_H} \cdot \phi'(I_H), \quad \phi'(I_H) = \frac{\delta}{\gamma} \cdot \frac{I_H}{I_L} \cdot \psi'(I_L).$$

In next step, we calculate $\psi''(I_L)$ and $\phi''(I_H)$. We have the ratio of public investments for high-income and low-income sections as $\frac{I_H}{I_L} = \frac{\gamma}{\delta} \cdot \frac{\phi'(I_H)}{\psi'(I_L)}$, we continue by differentiating the given equation with respect to I_H and I_L:

(a) Let's differentiate this equation respect to I_H. From left side:

$$\frac{\partial}{\partial I_H}\left(\frac{I_H}{I_L}\right) = \frac{1}{I_L}.$$

From right side: with using the quotient rule:

Revista ARACÊ
ISSN: 2358-2472

$$\frac{\partial}{\partial I_H}\left(\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)=\frac{\gamma}{\delta}\cdot\frac{\psi'(I_L)\phi''(I_H)}{[\psi'(I_L)]^2}.$$

Next, we combine both parts:

$$\frac{1}{I_L}=\frac{\gamma}{\delta}\cdot\frac{\psi'(I_L)\phi''(I_H)}{[\psi'(I_L)]^2}.$$

With solving for $\phi''(I_H)$, we obtain:

$$\phi''(I_H)=\frac{\delta}{\gamma}\cdot\frac{\psi'(I_L)}{I_L}.$$

(b) Let's differentiate with respect to I_L: From left side:

$$\frac{\partial}{\partial I_L}\left(\frac{I_H}{I_L}\right)=-\frac{I_H}{I_L^2}.$$

Right side: using the quotient rule:

$$\frac{\partial}{\partial I_L}\left(\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)}{\psi'(I_L)}\right)=-\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)\psi''(I_L)}{[\psi'(I_L)]^2}.$$

So:

$$-\frac{I_H}{I_L^2}=-\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)\psi''(I_L)}{[\psi'(I_L)]^2}.$$

Solving for $\psi''(I_L)$:

$$\psi''(I_L)=\frac{\delta}{\gamma}\cdot\frac{I_H}{I_L^2}\cdot\frac{[\psi'(I_L)]^2}{\phi'(I_H)}.$$

Finally, to compare ΔV_L and ΔV_H and show that ΔV_L is less than ΔV_H, we'll substitute the derived expressions for $\psi'(I_L)$, $\phi'(I_H)$, $\psi''(I_L)$, and $\phi''(I_H)$ into the Taylor expansions for changes in social vulnerability.

$$\Delta V_L=\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)\cdot I_L}{I_H}\cdot\Delta I+\frac{1}{2}\left(\frac{\delta}{\gamma}\cdot\frac{I_H}{I_L^2}\cdot\frac{[\psi'(I_L)]^2}{\phi'(I_H)}\right)(\Delta I)^2,$$
$$\Delta V_H=\frac{\delta}{\gamma}\cdot\frac{I_H}{I_L}\cdot\psi'(I_L)\cdot\Delta I+\frac{1}{2}\left(\frac{\delta}{\gamma}\cdot\frac{\psi'(I_L)}{I_L}\right)(\Delta I)^2.$$

We simplify ΔV_L with substituting $\psi'(I_L)$ into ΔV_L, similarly, substitute $\phi'(I_H)$ into ΔV_H.

$$\Delta V_L=\left(\frac{\gamma}{\delta}\cdot\frac{\phi'(I_H)\cdot I_L}{I_H}\right)\Delta I+\frac{1}{2}\left(\frac{\gamma\cdot\phi'(I_H)}{\delta\cdot I_H}\right)(\Delta I)^2,$$
$$\Delta V_H=\phi'(I_H)\Delta I+\frac{1}{2}\left(\frac{\phi'(I_H)}{I_L}\right)(\Delta I)^2.$$

To compare ΔV_L and ΔV_H in their entirety and show that increasing public investment in low-income sections reduces social vulnerability more significantly than increasing

Revista ARACÊ
ISSN: 2358-2472

investment in high-income sections, we need to compare the first and second-order terms of ΔV_L and ΔV_H as follows:

(a) First-order terms (ΔI): Since $\frac{\gamma}{\delta} \leq 1$, and $I_H \geq I_L$, $\psi'(I_L)$ can potentially be smaller than $\phi'(I_H)$. However, given that $\psi'(I_L)$ scales with I_L, it may actually be larger for sufficiently small I_L. Therefore, we infer the first-order term for ΔV_L may be relatively large due to the multiplication with I_L.

(b) Second-order terms $(\Delta I)^2$: The term $\frac{\gamma}{\delta} \cdot \frac{1}{I_H}$ in ΔV_L might be smaller than $\frac{1}{I_L}$ in ΔV_H, since $I_H \geq I_L$. Therefore, we infer the second-order term for ΔV_L is smaller due to the division by I_H (potentially larger than I_L).

Overall, even though $\psi'(I_L)$ is scaled down by $\frac{\gamma}{\delta}$, the effect on reducing social vulnerability in low-income sections could still be significant due to the larger impact of the first-order term and the smaller scaling effect of the second-order term. Thus, we can conclude that increasing public investment in low-income sections ΔV_L can reduce social vulnerability more significantly than in high-income sections ΔV_H with given economic conditions.