SIMPLIFICATION OF THE GRID MODEL AND ITS IMPACT ON THE ANALYSIS OF ELECTRICAL POWER SYSTEMS

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ABSTRACT
This paper analyzes the impact of the use of Kron reduction on the state variables of a three-phase electrical system, even when it does not meet the necessary conditions for its application. Reduction is applied to a power line model to eliminate the equation corresponding to the neutral conductor of the line. The ATP program is used to model and simulate the behavior of an electrical system considering different degrees of disequilibrium as a reference for the comparison of results. The results show that under certain conditions of disequilibrium the Kron reduction can lead to significant errors in the state variables of the system.

Keywords: Electric power systems, Unbalance, Kron reduction.

INTRODUCTION

Electricity is an essential asset in modern life, for industry, commerce, services and the residential sector. Thus, it has to be offered with quality, which means continuous supply and appropriate characteristics for its consumption.

Ensuring such characteristics means continuously studying the Electric Power Systems (SEP), both in the projection and exploration stages, in order to predict their behavior. These studies relate a very high number of variables, due to the complexity and magnitude of the SEP.

Kirchhoff's and Ohm's laws, for example, allow the elaboration of systems of equations through which studies are conducted. It happens that systems can be relatively large and complex, involving significant time for their solution, even when computer programs are used. Because of this, various methods have been used to reduce the size of systems when they meet certain conditions. Some of the numerous applications of these

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methods are presented in [3-6, 12].

Power lines are one of the main components of SEP. When these systems are balanced, that is, symmetrical, transposed and load balanced, current does not circulate through the neutral conductor, so it can be disregarded in the analysis of the systems. Balanced systems then facilitate the use of simplified models for the power line.

However, if the systems are unbalanced, part of the current flows to the generators or transformers through the neutral, influencing the three phases, due to the effect of mutual inductances. Power distribution systems, even with balanced loads, show intrinsic imbalances due to the manifestation of different mutual inductances between phases. These inductances may not be very different, so that the error produced during the analysis by assuming that no current circulates through the neutral may be small. However, a network serving multiple consumers notices constant variations in phase load levels. In these conditions of imbalance, it is not correct to maintain the same model for the study of the system. This was the premise for the development of this work, which aimed to quantify the possible differences between both methodologies of analysis.

The ATP/EMTP electromagnetic phenomena analysis program [7] served as a tool for carrying out the simulations, and the proper and mutual inductances of the conductors of the power line were determined through Carson's equations [9]. The differences observed in the scenarios studied are associated with the value of the system imbalance.

REDUCTION IN SYSTEMS OF EQUATIONS

It is very common in engineering to formulate systems of equations to represent various phenomena that happen naturally, in industrial processes, among others.

A system of *n* independent equations, as shown in (1), where the number of unknowns is also *n*, can be represented in matrix form, such as (2):

$$\begin{aligned} &a_1x_1 + a_2x_2 + \ldots + a_nx_n = y_1 \\ &b_1x_1 + b_2x_2 + \ldots + b_nx_n = y_2 \\ &\ldots \\ &n_1x_1 + n_2x_2 + \ldots + n_nx_n = y_n \end{aligned} \tag{1}$$

$$[Y] = [C][X] \tag{2}$$

where *C* represents the matrix of the coefficients *a1*......*nn* of the independent variables *x*, and *Y* is the set of dependent variables. *C* has order *n* x *n* and vectors *X* and *Y* have order *n* x 1.

If at any time *m* dependent variables have a value of zero, the system can be reduced to *n-m* equations, due to the possibility of expressing *m* of the independent variables as a function of the other variables. For example, in a system of three equations, where *y2* = 0, we can express *x2* as a function of the variables *x1* and *x3*. That is:

$$x_2 = -\left(\frac{b_1}{b_2}x_1 + \frac{b_3}{b_2}x_3\right) \tag{3}$$

Substituting *x2* in the first and third equations of (1) we get:

$$\begin{aligned} \left(a_1 - \frac{a_2 b_1}{b_2}\right)x_1 + \left(a_3 - \frac{a_2 b_3}{b_2}\right)x_3 = y_1 \\ \left(c_1 - \frac{c_2 b_1}{b_2}\right)x_1 + \left(c_3 - \frac{c_2 b_3}{b_2}\right)x_3 = y_3 \end{aligned} \tag{4}$$

which is now a system of just two equations.

In the event that *y1* and *y3* are known, the system solution will be given by:

$$\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_1 - \frac{a_2 b_1}{b_2} & a_3 - \frac{a_2 b_3}{b_2} \\ c_1 - \frac{c_2 b_1}{b_2} & c_3 - \frac{c_2 b_3}{b_2} \end{bmatrix}^{-} \begin{bmatrix} y_1 \\ y_3 \end{bmatrix} \tag{5}$$

It is proven that *y2* does not explicitly participate in the solution of the system of equations, however, the value of *x2* will be known, through (3), once x1 and *x3 are determined*.

In the example, the second equation of (1) was not necessary for the solution of the system of equations, since the system was reduced, the matrix was reduced, which is always important for reducing the complexity of the calculations and reducing the time to obtain the solution. Currently, the use of computer programs has greatly reduced this time, however, whenever possible it is convenient to reduce the order of the matrix, as such time is inversely proportional to its order [8].

With *y2* = 0 we obtained (3), where *x2* was placed as a function of the other variables. However, *x1* or *x3* could have been chosen and, in this case, substituted in the original system of equations to obtain (4). In any case, as in the example, it is usual to choose the variable whose position in the equations coincides with the number of the equation that will be eliminated. This process of eliminating equations is known as *Kron reduction* in electrical engineering [10]. With such a reduction, the elements of the new *matrix C* are determined as:

$$c_{jk} = c_{jk} - \frac{c_{jp} c_{pk}}{c_{pp}} \tag{6}$$

where *p* is the number of the equation being eliminated from the original system. For example, for *p* = 2 we get the following matrix:

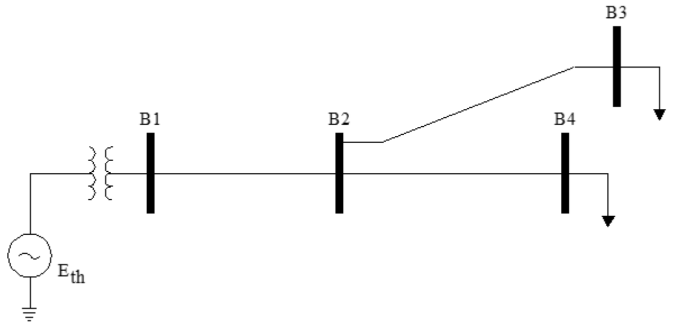
$$C = \begin{bmatrix} a_1 - \frac{a_2 b_1}{b_2} & 0 & a_3 - \frac{a_2 b_3}{b_2} \\ 0 & 0 & 0 \\ c_1 - \frac{c_2 b_1}{b_2} & 0 & c_3 - \frac{c_2 b_3}{b_2} \end{bmatrix} \tag{7}$$

With (7) and (5) it is observed that the element *pp* acted as a "pivot", eliminating the row and the column that intersected in it.

THE KRON REDUCTION IN ELECTRICAL POWER SYSTEMS

To exemplify the application of Kron reduction, the four-bar system in Fig.1 is considered, which is part of an electrical network.

Fig. 1. Single-line diagram of electrical power system.



The application of Kirchhoff's law to the meshes of the network results in a system of equations, summarized in matrix as:

$$[\dot{V}] = [\bar{Z}_{bus}][\dot{I}] \tag{8}$$

where *Ve I* represent the vectors of the voltages and currents of the bars, respectively, and *Zbus* represents the impedance matrix of the same. For convenience, in the system of equations, the numbering of the equations corresponds to the number of the bars in Fig.1.

With the information of the impedances of the lines, the Zbus matrix is determined, according to the procedure shown in [11], resulting in:

$$[\bar{Z}_{bus}] = \begin{bmatrix} 0 - 16{,}75i & 0 + 11{,}75i & 0 + 2{,}50i & 0 + 2{,}50i \\ 0 + 11{,}75i & 0 - 19{,}25i & 0 + 2{,}50i & 0 + 5{,}00i \\ 0 + 2{,}50i & 0 + 2{,}50i & 0 - 5{,}80i & 0 + 0{,}00i \\ 0 + 2{,}50i & 0 + 5{,}00i & 0 + 0{,}00i & 0 - 8{,}30i \end{bmatrix} \text{pu}$$

According to Fig.1, in which only bars 3 and 4 have charge, the following current vector is considered:

$$[\dot{I}] = \begin{bmatrix} 0 \\ 0 \\ 1{,}00\angle -90^\circ \\ 0{,}68\angle -135^\circ \end{bmatrix} \text{pu}$$

with which we obtain the following bar voltages:

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_3 \\ \dot{V}_4 \end{bmatrix} = \begin{bmatrix} 0{,}9691\angle -162^\circ \\ 0{,}9673\angle -161^\circ \\ 0{,}9996\angle -165^\circ \\ 0{,}9487\angle -159^\circ \end{bmatrix} \text{pu}$$

For certain studies of the electrical system, bars 1 and 2 are of lesser importance, since no sensitive load is placed on them, as well as no generator. According to Kron reduction, the first and second equations can be eliminated from (8).

After the elimination and determination of the new *Zbus matrix*, the stresses result in:

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_3 \\ \dot{V}_4 \end{bmatrix} = \begin{bmatrix} 0,9691\angle -162° \\ 0,9996\angle -165° \\ 0,9487\angle -159° \end{bmatrix} \text{pu}$$

It is observed that the stresses of bars 1, 3 and 4 obtained after reduction are equal to those obtained in the original system, which confirms the validity and usefulness of the method.

For comparison purposes, we eliminate the third equation of (8), obtaining the following results, which deviate significantly from the original values:

$$\begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \dot{V}_4 \end{bmatrix} = \begin{bmatrix} 0,4330\angle -135° \\ 0,4364\angle -135° \\ 0,4752\angle -135° \end{bmatrix} \text{pu} \qquad (9)$$

THE KRON REDUCTION IN THE MODEL OF POWER LINES IN THREE-PHASE SYSTEMS

The power line is one of the main components of electrical power systems. Depending on its length, long, medium or short, it has matching models. In the case of distribution systems where the distances between the substation and the loads are short, the line model can be considered as being only that of an impedance [1].

For three-phase systems with neutral, the line model, per unit length, is given by:

$$\left[\overline{Z}_l\right] = \begin{bmatrix} \overline{Z}_{aa} & \overline{Z}_{ab} & \overline{Z}_{ac} & \overline{Z}_{an} \\ \overline{Z}_{ba} & \overline{Z}_{bb} & \overline{Z}_{bc} & \overline{Z}_{cn} \\ \overline{Z}_{ca} & \overline{Z}_{cb} & \overline{Z}_{cc} & \overline{Z}_{cn} \\ \overline{Z}_{na} & \overline{Z}_{nb} & \overline{Z}_{nc} & \overline{Z}_{nn} \end{bmatrix} \Omega/\text{u}$$

where the *element Zii* corresponds to the proper impedance of phase *i* and *Zij* to the mutual impedance between phases *i* and *j* (*Zij* = *Zji*).

In a line section, the voltage drop in the conductor corresponding to phase *a*, for example, is given by:

$$\overline{V}_{aa'} = \overline{Z}_{aa}\overline{I}_a + \overline{Z}_{ab}\overline{I}_b + \overline{Z}_{ac}\overline{I}_c - \overline{Z}_{an}\overline{I}_n \qquad (10)$$

And in general:

$$\begin{bmatrix} \bar{V}_{aa'} \\ \bar{V}_{bb'} \\ \bar{V}_{cc'} \\ \bar{V}_{nn'} \end{bmatrix} = \begin{bmatrix} \bar{Z}_{aa} & \bar{Z}_{ab} & \bar{Z}_{ac} & \bar{Z}_{an} \\ \bar{Z}_{ba} & \bar{Z}_{bb} & \bar{Z}_{bc} & \bar{Z}_{cn} \\ \bar{Z}_{ca} & \bar{Z}_{cb} & \bar{Z}_{cc} & \bar{Z}_{cn} \\ \bar{Z}_{na} & \bar{Z}_{nb} & \bar{Z}_{nc} & \bar{Z}_{nn} \end{bmatrix} \begin{bmatrix} \bar{I}_{a} \\ \bar{I}_{b} \\ \bar{I}_{c} \\ -\bar{I}_{n} \end{bmatrix} \tag{11}$$

SYMMETRICAL, TRANSPOSED AND BALANCED SYSTEMS

When the system is symmetrical and transposed, that is, with its three phases of equal impedance, and when the effect of their mutual impedances has been canceled during the construction of the line, the voltage at the terminal of the loads is symmetrical, i.e.:

$$\begin{bmatrix} \dot{V}_a \\ \dot{V}_b \\ \dot{V}_c \end{bmatrix} = \begin{bmatrix} V\angle\theta \\ V\angle\theta - 120° \\ V\angle\theta + 120° \end{bmatrix} \text{V}$$

where *V* and θ symbolize the relationship of the amplitude and angle of the voltage at the substation member with the corresponding quantities of the impedance from the line to the load bar.

Being symmetrical and transposed, the line model is simplified to:

$$\left[\bar{Z}_l\right] = \begin{bmatrix} \bar{Z}_p & 0 & 0 & 0 \\ 0 & \bar{Z}_p & 0 & 0 \\ 0 & 0 & \bar{Z}_p & 0 \\ 0 & 0 & 0 & \bar{Z}_p \end{bmatrix} \Omega/\text{u}$$

where *Zp* identifies the proper impedance of the phase and neutral conductors. If the Y-load is also balanced and grounded, there will be no voltage drop in the neutral, because no current will circulate through it. In this case (11) it is simplified to:

$$\begin{bmatrix} \dot{V}_{aa'} \\ \dot{V}_{bb'} \\ \dot{V}_{cc'} \end{bmatrix} = \begin{bmatrix} \bar{Z}_p & 0 & 0 \\ 0 & \bar{Z}_p & 0 \\ 0 & 0 & \bar{Z}_p \end{bmatrix} \begin{bmatrix} \dot{I}_a \\ \dot{I}_b \\ \dot{I}_c \end{bmatrix} \tag{12}$$

which in practice is equivalent to having made a Kron reduction in the corresponding system of equations.

UNBALANCED SYSTEMS

Under the above conditions, the three-phase system can be "reduced" to a single-phase system. This has been practiced since the beginning of the analysis of three-phase electrical power systems, also aiming to simplify calculations and speed up the obtaining of results.

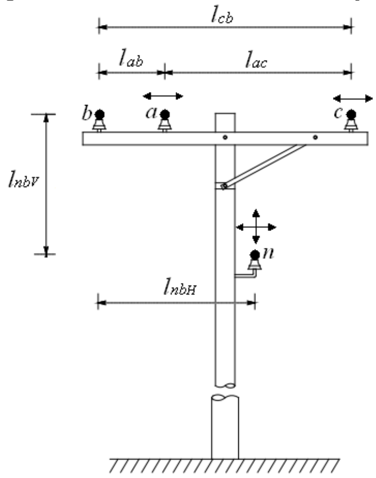
However, the distribution systems have shown that they are "naturally" unbalanced, because it is practically impossible for the loads to be equal in all three phases. They are also asymmetrical, because in addition to the possibility of having impedances of different phases, the most common thing is that the line is not crossed, which does not eliminate the effect of mutual impedances between conductors.

Under these conditions the current circulating through the neutral is not zero, nor is the voltage drop in it zero, and therefore the fourth equation of (11) should not be eliminated through a Kron reduction process, because the results would not be reliable, just as they were not in (9).

ATP PROGRAM. CASE STUDIES

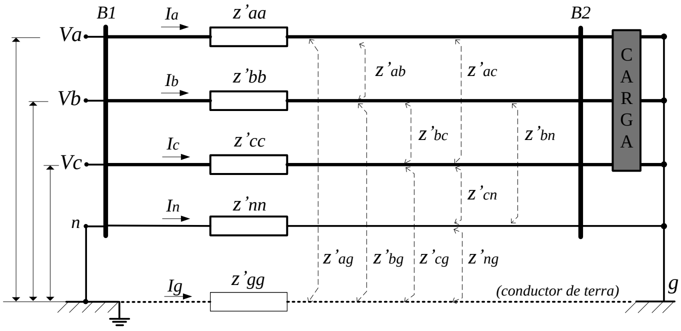
Fig. 2 shows the arrangement of the four conductors in the feeder used in this study. The conductors are of type 336,400 26/7 ACSR for the phases and 4/0 6/1 ACSR for the neutral, whose characteristics are specified in [1].

Fig. 2. Feeder structure for analysis.



To simplify the analysis, the two-bar system in Fig. 3 is considered, whose line is 1 km long.

Fig. 3. Electrical network for analysis.



The proper and mutual impedances of the conductors represented in Fig. 2 and Fig. 3 are determined using Carson's equations and result in being:

$$\left[\bar{Z}_l\right] = \begin{bmatrix} 0{,}249+0{,}878i & 0{,}059+0{,}529i & 0{,}059+0{,}451i & 0{,}059+0{,}446i \\ 0{,}059+0{,}529i & 0{,}249+0{,}878i & 0{,}059+0{,}485i & 0{,}059+0{,}459i \\ 0{,}059+0{,}451i & 0{,}059+0{,}485i & 0{,}249+0{,}878i & 0{,}059+0{,}457i \\ 0{,}059+0{,}446i & 0{,}059+0{,}459i & 0{,}059+0{,}457i & 0{,}427+0{,}961i \end{bmatrix} \quad (13)$$

Because the distances between the conductors in the feeder are not the same, the mutual impedances between the phases, and between these and the neutral are not equal. In this way, the system will experience a certain degree of imbalance, which reduces the possibility of the neutral current being zero.

Although the simplicity of the circuit would allow an analytical study, it was preferred to use the ATP program to obtain the results, considering that this circuit is part of other more complex circuits that have been studied with the help of this program.

The grid generator is three-phase and symmetrical with a line voltage of 13.8 kV and 60 Hz. The transformer consists of three single-phase transformers, with a reactance of j0.475 Ω. For simplicity, grounding inductances are not considered in the generator or in the load.

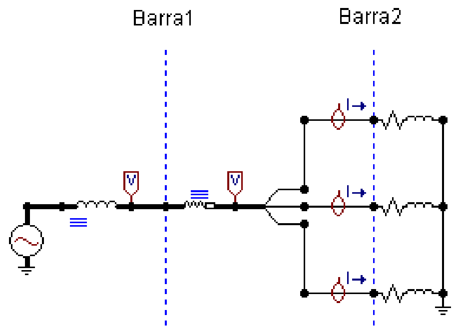
Table I shows the characteristics of the loads used for the analysis. Their values were purposely selected to reveal the differences that were being sought.

TABLE I - Characteristics of the system loads

Phase	Apparent Power (kVA)		
	Scenario 1	Scenario 2	Scenario 3
the	3582+j2314	149.5 + j46.0	141.5 + j15.7
b	3634 + J2348	3466+j2239	5718 + J3812
c	3615+J2335	391.1 + J120.9	147.2 + J16.4
Note:	0.01% imbalance	1.00% imbalance	1.85% imbalance

Fig. 4 shows the electrical circuit implemented in the ATPdraw program. For the studies, the electrical network received as parameters the data of (13) or those resulting from the application of the Kron reduction by eliminating the equation corresponding to the neutral (line 4 of *Zl*). These configurations are called "4 wires" and "3 wires", respectively.

Fig. 4. Electrical circuit implemented in the ATPdraw program.



Tables II to IV show the results of the bar voltages and currents in the scenarios studied for the real system (4-wire circuit) and after the Kron reduction (3-wire circuit).

TABLE II - Phase voltages and currents rms for scenario 1

Phase	4 wires			3 wires		
	Vb1 (V)	Vb2 (V)	I (A)	Vb1 (V)	Vb2 (V)	I (A)
a	7816	7575	562,98	7816	7575	563,09
b	7814	7627	567,29	7814	7629	567,07
c	7814	7611	565,40	7814	7608	565,53

B1: Barra 1; B2: Barra 2

TABLE III - Phase voltages and currents rms for scenario 2

Phase	4 wires			3 wires		
	Vb1 (V)	Vb2 (V)	I (A)	Vb1 (V)	Vb2 (V)	I (A)
a	7965	7941	19,70	7965	7942	19,46
b	7819	7606	542,47	7815	7476	555,71
c	7960	7958	51,44	7960	8086	50,82

B1: Barra 1; B2: Barra 2

TABLE IV - Phase voltages and currents rms for scenario 3

Phase	4 wires			3 wires		
	Vb1 (V)	Vb2 (V)	I (A)	Vb1 (V)	Vb2 (V)	I (A)
a	7967	7936	17,94	7967	7940	17,54
b	7692	7317	939,22	7681	7074	980,98
c	7966	7987	18,54	7966	8219	18,15

B1: Barra 1; B2: Barra 2

The system imbalance was determined according to NEMA, MG1-1978 [2]:

$$\delta = \frac{\left| \max\left(V_{ab} - \overline{V}, V_{bc} - \overline{V}, V_{ac} - \overline{V}\right) \right|}{\left| \overline{V} \right|} \tag{13}$$

where $\overline{V}$ is the average value of the line voltages.

When the system is balanced, or when the unbalance is negligible, as in scenario 1, there are virtually no differences between voltages or phase currents, regardless of the type of configuration employed. This is what Table I shows, where the voltages and currents in bar 2 (where the loads are connected) are practically the same for 4 or 3 wires. Under these conditions the neutral current is zero or very low, and the elimination of the corresponding conductor, to apply the Kron reduction, has no significant effects on the system.

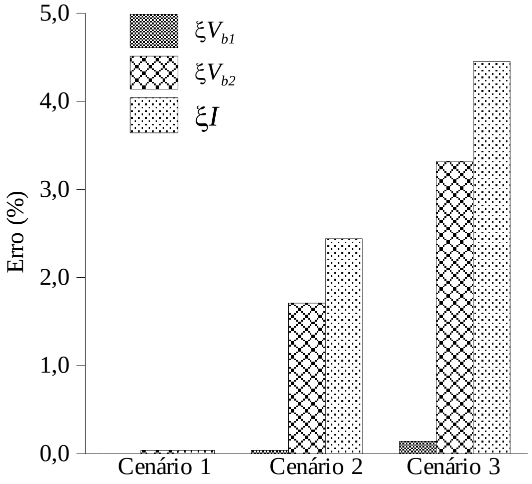
However, when Kron reduction is performed in systems with a high degree of unbalance, as in scenarios 2 and 3, the voltages and currents in the load bar do not reflect the actual values of the system (which would correspond to the 4-wire configuration). In the two scenarios shown, the values are higher and/or lower than the real ones, with greater differences in the responsible phase of the imbalance.

Table V shows the relative error between the 4-wire and 3-wire configurations for the phase voltages and currents. Fig. 5 shows the maximum values.

TABLE V - Relative percentage errors between 4 and 3 wire configurations

Phase	Scenario 1			Scenario 2			Scenario 3		
	ζVb1	*ζVb2*	*gI*	*ζVb1*	*ζVb2*	*gI*	*ζVb1*	*ζVb2*	*gI*
a	0,00	0,00	0,02	0,00	0,01	1,21	0,00	0,04	2,22
b	0,00	0,03	0,04	0,04	1,71	2,44	0,14	3,32	4,45
c	0,00	0,04	0,02	0,01	1,60	1,19	0,00	2,90	2,09

Fig. 5. Maximum relative errors in phase voltages and currents.



CONCLUSIONS

Knowledge of load voltages is important in reliability and power quality studies, among others. Likewise, the knowledge of the currents allows the dimensioning of protection systems.

The Kron reduction applied to power lines contributes to an easier and faster analysis of the systems and has no significant effect on their quantities, when the system is balanced, or when the imbalance is negligible.

This work analyzed a system with an almost balanced three-phase load and it was proven that in this condition the differences are minimal. However, when the load became almost single-phase, significantly unbalancing the system, the results of applying the Kron reduction deviated from the real values, the greater the imbalance of the system.

Since the analyzed system may be only a section of a larger system, it is understood that in this more complex system, the "error propagation", due to Kron reduction, can report gross results for the state variables of the system.

For the conditions analyzed in this work, the maximum relative error in the phase current proved to be approximately 2.5 times the value of the system unbalance, and 1.8 times the phase voltage at the load.

In view of the effect of the line's characteristics on the error, its longitude was doubled, resulting in a maximum relative error of 8.2%, i.e., approximately 4.4 times the value of the system's imbalance.

The levels of imbalance analyzed in this work are acceptable and likely to manifest themselves in any system. However, higher levels are also possible, so that the *unbalance-line* combination can report significant errors when performing Kron reduction to simplify their analysis.

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