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A SIMPLE MONTE CARLO SIMULATION OF RADIOACTIVE DECAY

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ABSTRACT
This work presents a simple Monte Carlo simulation of radioactive decay implemented in Fortran programming language. The simulation uses random number generation to model the probability of decay of unstable nuclei. Probabilities of 10%, 50%, and 90% were tested, and the results exhibit the expected exponential behavior, demonstrated by straight lines in semi-logarithmic plots. In addition to hypothetical cases, the decay of the radioactive isotope Carbon-14 was also modeled and analyzed, highlighting the method´s applicability to real-world scenarios. These findings demonstrate the effectiveness of the approach for examining statistical properties of decay processes and offer insight into the relationship between simulation parameters and physical observables.

Keywords: Nuclear Physics. Radioactive Decay. Monte Carlo Simulation.

UMA SIMULAÇÃO SIMPLES DE MONTE CARLO DE DECAIMENTO RADIOATIVO

RESUMO
Este trabalho apresenta uma simulação simples de Monte Carlo do decaimento radioativo implementada na linguagem de programação Fortran. A simulação utiliza geração de números aleatórios para modelar a probabilidade de decaimento de núcleos instáveis. Probabilidades de 10%, 50% e 90% foram testadas, e os resultados exibem o comportamento exponencial esperado, demonstrado por linhas retas em gráficos semilogarítmicos. Além de casos hipotéticos, o decaimento do isótopo radioativo Carbono-14 também foi modelado e analisado, destacando a aplicabilidade do método a cenários do mundo real. Esses resultados demonstram a eficácia da abordagem para examinar propriedades estatísticas de processos de decaimento e oferecem insights sobre a relação entre parâmetros de simulação e observáveis físicos.

Palavras-chave: Física Nuclear. Decaimento Radioativo. Simulação de Monte Carlo.

UNA SIMULACIÓN SIMPLE DE MONTE CARLO DE LA DESINTEGRACIÓN RADIACTIVA

RESUMEN
Este trabajo presenta una simulación Monte Carlo simple de la desintegración radiactiva, implementada en el lenguaje de programación Fortran. La simulación utiliza la generación

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de números aleatorios para modelar la probabilidad de desintegración de núcleos inestables. Se probaron probabilidades del 10%, 50% y 90%, y los resultados muestran el comportamiento exponencial esperado, representado por líneas rectas en gráficos semilogarítmicos. Además de los casos hipotéticos, también se modeló y analizó la desintegración del isótopo radiactivo Carbono-14, lo que destaca la aplicabilidad del método a escenarios reales. Estos hallazgos demuestran la eficacia del enfoque para examinar las propiedades estadísticas de los procesos de desintegración y ofrecen información sobre la relación entre los parámetros de simulación y las variables físicas observables.

Palabras clave: Física Nuclear. Desintegración Radiactiva. Simulación Monte Carlo.

1 INTRODUCTION

The discovery of radioactivity by Henri Becquerel in 1896 was a milestone in our understanding of atomic and nuclear phenomena (BECQUEREL, 1896). While investigating phosphorescent materials, Becquerel observed that uranium salts emitted rays capable of exposing photographic plates, even in the absence of sunlight. This unexpected result laid the foundation for the study of radioactive decay, a process by which unstable atomic nuclei spontaneously transform into more stable configurations, emitting energy in the form of particles or electromagnetic radiation.

Radioactive decay is inherently stochastic, each unstable nucleus has a certain likelihood of decaying over time, governed by its characteristic half-life (KRANE, 2014; CLOSE, 2015). This decay process plays a vital role in several scientific and technological fields. From powering nuclear reactors and treating cancer through radiotherapy, to dating ancient artifacts via radiocarbon dating, the applications of radioactive decay are both diverse and impactful (KNOLL, 2010; NICHOLS, 2012; KRANE, 2014).

Given the random nature of nuclear decay, simulations offer a powerful way to model and visualize these processes, especially in educational settings. One particularly effective and intuitive approach involves using dice or coins to represent the probabilistic behavior of atomic nuclei (KLEIN, 2010; BAKAÇ, 2011). In the first case, each die represents a nucleus, and rolling a specific number (such as a 3) corresponds to a decay event. Over successive rolls, the number of remaining *undecayed* dice can be tracked, mimicking the exponential decrease observed in real radioactive samples.

Such didactic simulations provide students with a hands-on, visual representation of concepts that might otherwise seem abstract or counterintuitive. They not only illustrate the statistical nature of decay, but also foster a deeper understanding of half-life, randomness, and exponential decay curves.

An example that may be discussed in this context is the decay of carbon-14, an isotope used extensively in archaeological and geological dating. By simulating carbon-14 decay with dice, students can gain practical insights into how scientists determine the age of once-living materials, bridging theoretical knowledge with real-world applications.

Radioactive decay is a fundamental stochastic process in nuclear physics, characterized by the spontaneous transformation of unstable nuclei (KRANE, 2014; CLOSE, 2015). Computational modeling of this phenomenon is a powerful tool for both research and educational purposes. This work aims to simulate radioactive decay using an algorithm based on random sampling, implemented in the Fortran programming language.

2 METHODOLOGY

At the end of the 19th century, the experimental study of radioactivity showed that radioactive substances decay spontaneously and that this process follows an exponential law, i.e., the number of undecayed nuclei in a sample varies with time *t* as

$$N(t) = N_0 e^{-\lambda t}, \qquad (1)$$

Where:

N_0 is the initial quantity of nuclei, and λ is the decay constant. The λ quantity measures the probability per unit time for a nucleus to decay, and is a characteristic of each radionuclide. Defining the half-life $T_{1/2}$ of a radionuclide as the time to half of nuclei decays we obtain

$$T_{1/2} = \frac{\ln 2}{\lambda}. \qquad (2)$$

Table 1 lists the half-lives and decay constants for selected radionuclides, illustrating the wide range of decay timescales.

Table 1
Half-lives and decay constants of some radioactive nuclei (NNDC 2025).

Radionuclide	Half-life, $T_{1/2}$	Decay constant, λ [s^{-1}]
Protactinium-219	55 ns	1.26×10^{7}
Polonium-214	163.5 µs	4.24×10^{3}
Carbon-16	750 ms	9.24×10^{-1}
Oxygen-15	2.04 min	5.66×10^{-3}
Fluorine-18	1.83 h	1.05×10^{-4}
Iodine-131	8.02 d	1.00×10^{-6}
Cobalt-60	5.27 yr	4.17×10^{-9}
Carbon-14	5,686 yr	3.86×10^{-12}
Uranium-238	4.46×10^{9} yr	4.92×10^{-18}

Source: Author 2025

In this study, a computational model was developed to simulate the stochastic process of radioactive decay using the Monte Carlo method. The simulation was implemented in Fortran programming language, employing random number generation to replicate the probabilistic nature of nuclear decay events.

The fundamental assumption of the model is that each unstable nucleus has a constant probability of decaying within a discrete time step. We can obtain the relation between this probability, denoted by *p*, and the physical decay constant λ as follows.

After a time interval Δ*t* the number of undecayed nuclei is

$N(t + \Delta t) = N_0 e^{-\lambda(t+\Delta t)}$

or

$$N(t + \Delta t) = N_0 e^{-\lambda t} \times e^{-\lambda t} = N(t) e^{-\lambda.\Delta t}. \quad (3)$$

The fraction of nuclei that decays in the interval Δt is the probability p,

$$p = \frac{N(t) - N(t+\Delta t)}{N(t)} = 1 - e^{-\lambda.\Delta t}, \quad (4)$$

Where:

Δt represents the chosen time interval for the simulation. It should be noted that λ is the probability per unit time that a given nucleus will decay and does not depend on the time interval, whereas p is the probability that a nucleus actually decays within a finite time interval Δt, according to the expression above. In other words, λ represents the average probability per unit time that a single nucleus will decay and is independent of the choice of time interval - it is a fundamental physical property; p is the actual probability that a nucleus decays within a given finite time interval and depends on both λ and the length of the time interval Δt.

The algorithm presents the following steps:

i. Initialization. A population of *N* identical radioactive nuclei is defined. The decay probability p per time step is calculated based on the desired decay constant or half-life. Parameter p is an input.

ii. Random number generation. For each nucleus at each time step, a pseudo-random number r uniformly distributed in the interval [0,1) is generated using Fortran's built-in random number function. The quality of this pseudo-random number generator is crucial for the accuracy of Monte Carlo simulations. A good pseudo-random number generator ensures that the generated numbers are truly uniformly distributed and do not exhibit repeating patterns or biases over the simulation's duration. This fidelity to true randomness is essential for accurately representing the stochastic nature of radioactive decay and for obtaining reliable statistical outcomes.

iii. Decay decision. A nucleus is considered to have decayed during the time step if the condition $r < p$ is satisfied. Otherwise, the nucleus remains undecayed for that interval.

iv. Time evolution. The simulation iteratively updates the number of undecayed nuclei over successive time steps. At each step, the number of undecayed nuclei is recorded, allowing for the construction of a decay curve.

In Figure 1 is depicted the flowchart of the simulation. The process begins with initialization (defining the population *N* and decay probability *p*), followed by iterative time steps where each nucleus undergoes a decay decision based on a random number *r*. If $r < p$, the nucleus decays; otherwise, it remains unchanged. The simulation continues until all nuclei decay, recording the undecayed count at each step to generate the decay curve *N(t)*. To analyze the decay behavior, the results were plotted as the number of undecayed nuclei *N* versus time using a semi-logarithmic scale, with the logarithm of *N* on the vertical axis and linear time on the horizontal axis. Given that the theoretical decay law follows an exponential form as shown in (1), the data are expected to form a straight line when plotted in semi-logarithmic axes. A linear regression was performed on the logarithmic data to extract the slope, which corresponds to -1 times the decay constant λ. From the obtained λ, the half-life $T_{1/2}$ was calculated using the relation given in (2). This procedure was repeated for different values of the probability *p*, allowing the examination of how the decay constant and half-life correlate with the chosen probability per time step, further validating the consistency between the stochastic model and the analytical decay law.

This approach provides a numerical visualization of the exponential radioactive decay behavior and enables verification of the statistical properties inherent to the decay process. The Monte Carlo technique captures the inherent randomness of individual decay events while reproducing the deterministic exponential trend when considering large populations.

The Fortran program simulates the radioactive decay of a set of nuclei. Simulations were performed for 1 million hypothetical nuclei and for $p = 0.1$, 0.5, and 0.9. The real radioactive Carbon-14 case was also studied.

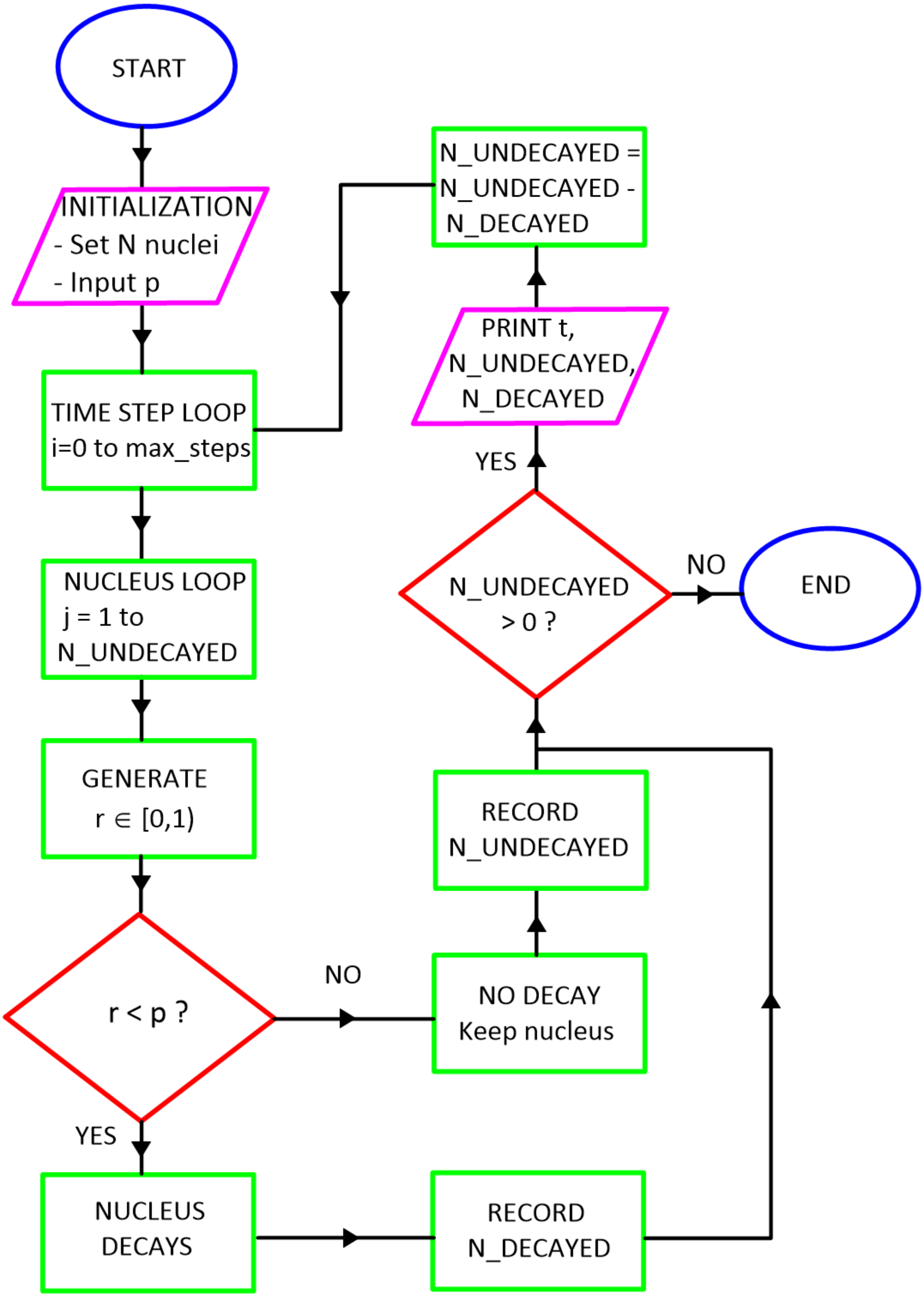
3 RESULTS AND DISCUSSION

3.1 A HYPOTHETICAL NUCLEUS

Simulations were run for one million hypothetical nuclei with $p = 0.1$, 0.5, and 0.9. Figure 2 shows the decay curves, highlighting the exponential trend. The number of undecayed

Figure 1

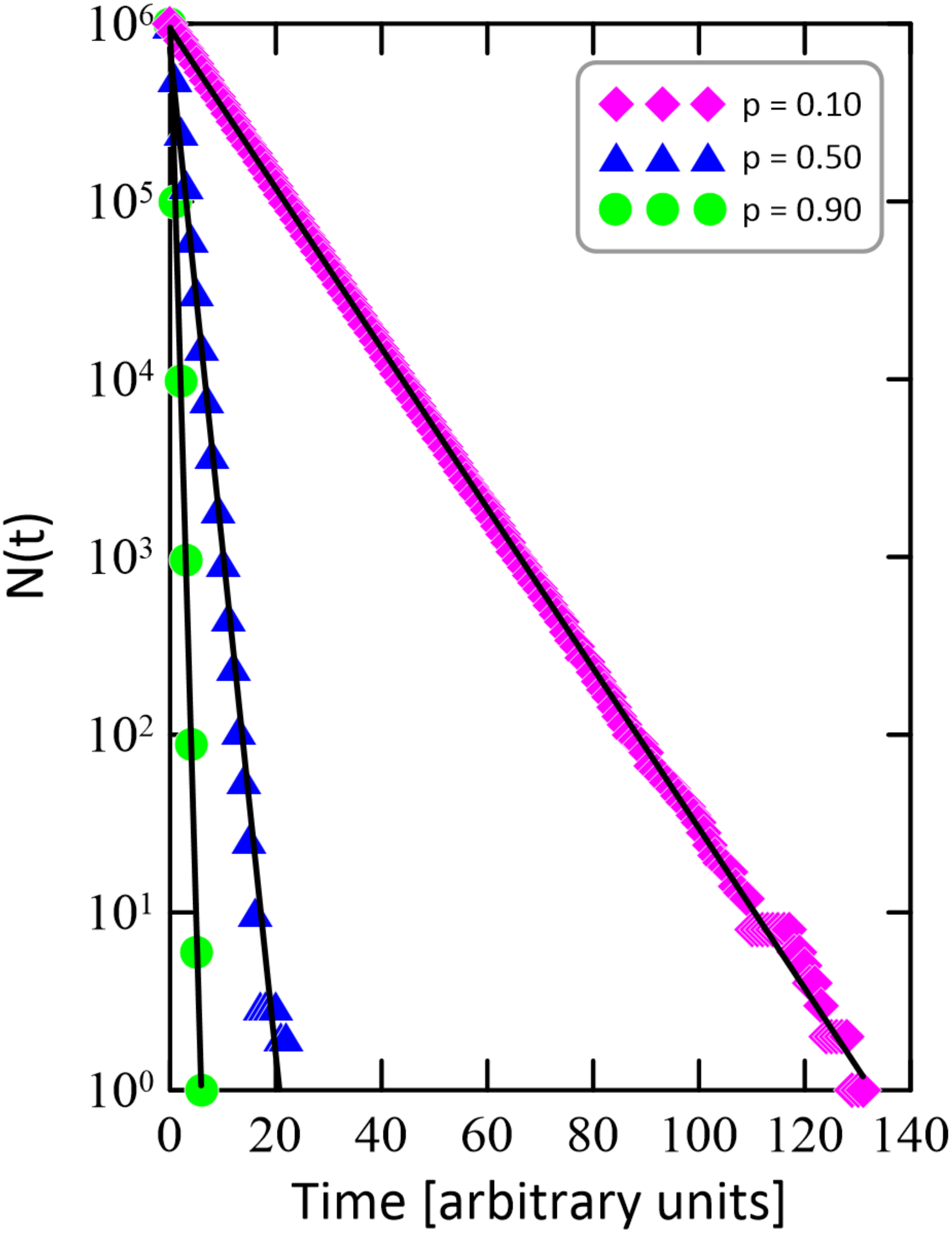
Flowchart of the Monte Carlo simulation for radioactive decay.



Source: Author 2025

Figure 2

Radioactive decay is simulated with different probabilities for 10^6 initial nuclei. Solid lines are fitted curves to the data.



Source: Author 2025

nuclei *N(t)* was recorded as a function of simulated time. The y-axis is shown on a logarithmic scale, highlighting the exponential nature of the decay. It is clearly seen the effect of the values of decay probabilities on the behavior of curves. Decay is extremely fast for p = 0.9 and is slow for p = 0.1.

Decay constant for each probability was obtained by fitting the data set to the equation

$$\ln N = a + bt, \quad (5)$$

Where:

$b = -\lambda$.

Inserting the λ values in (2) we obtain the half-lives 6.688, 1.063, and 0.296, respectively for p = 0.1, 0.5, and 0.9. The linearity observed in the semi-logarithmic plot is consistent with the theoretical model of exponential decay.

Considering small fluctuations in the simulated data are expected due to the random nature of the process. The approach used proves effective in illustrating the average behavior of the system.

3.2 APPLICATION FOR A REAL CASE: CARBON-14 DECAY

The radioactive isotope Carbon-14 (C-14) has a half-live of 5,686 years (NNDC 2025). From (2) we obtain

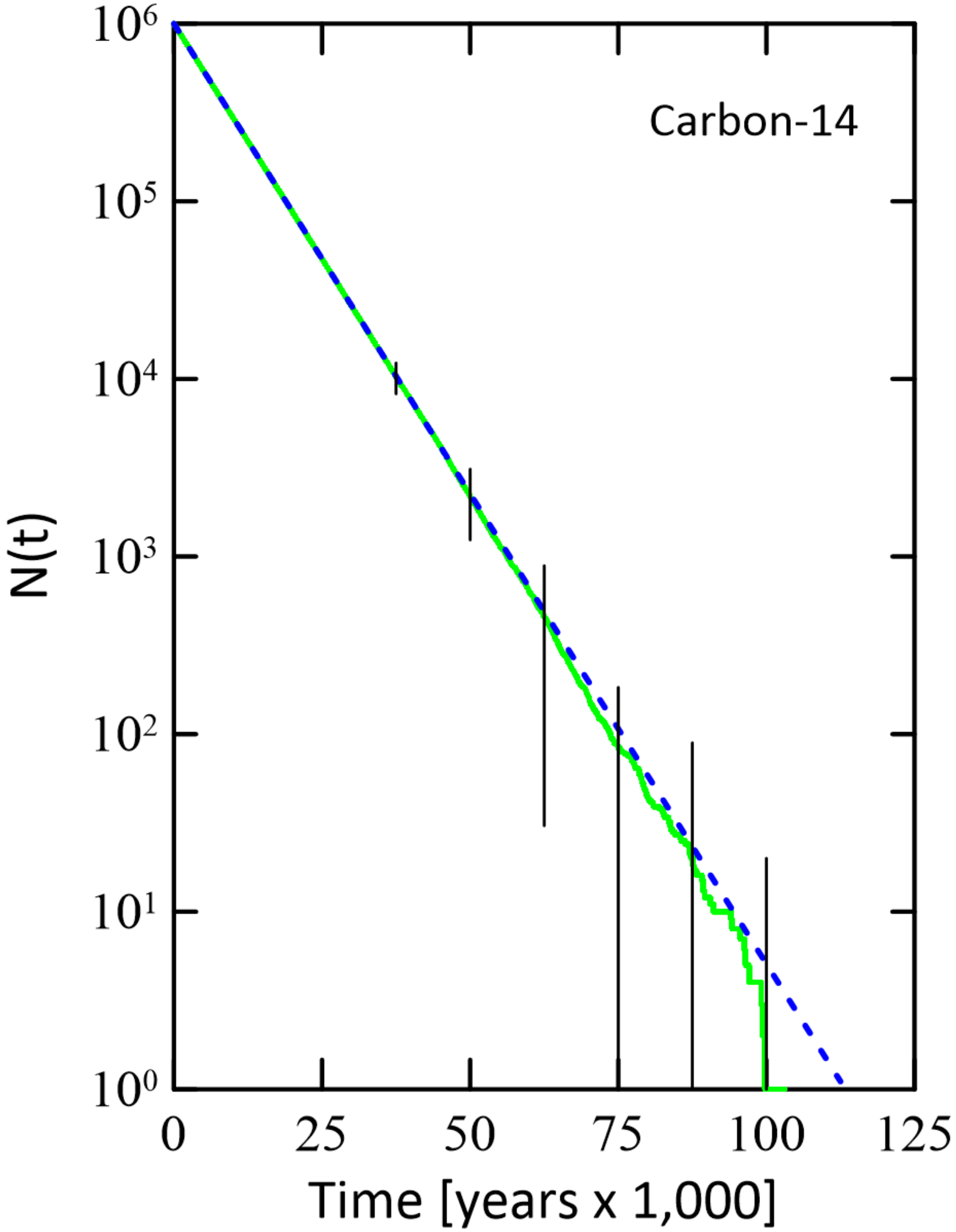
$$\lambda = \frac{\ln 2}{T_{1/2}} = 1.22 \times 10^{-4}\ years^{-1},$$

and from (4), the annual (Δt = 1) probability of decay is $p = 1.22 \times 10^{-4}$. It should be noted that for small Δt ($\lambda\Delta t \ll 1$), the probability p approximates the fraction of nuclei decaying in Δt, $p \approx \lambda\Delta t$ (obtained from the Taylor expansion of (4)).

In Figure 3 the results of simulation for C-14 are shown. The slope of the simulated curve gives $\lambda = 1.275 \times 10^{-4}$ year^{-1}, which corresponds to a half-life $T_{1/2}$ = 5,437 years. It represents a difference of 4.4% when compared to the reported value (NNDC 2025), which can be ascribed to the fluctuations in results due to the low quantity of remaining nuclei at the end of simulated curve. The difference from reported value diminishes when we consider the simulation only up to 10, 100, 1,000 and 10,000 remaining nuclei, as summarized in Table 2.

Figure 3

Carbon-14 radioactive decay simulation. Solid line represents simulated results and dashed line represents results obtained directly from Equation (1). Some error bars are shown and to make the growth of relative uncertainty visible, they were multiplied by a factor of 10 without changing the central value of the data point.



Source: Author 2025

Table 2

Differences in half-lives of C-14 when different quantities of final N undecayed nuclei are considered in simulation.

N	Simulated $T_{1/2}$ [years]	Difference [%]
1	5,437	4.4
10	5,553	2.3
100	5,584	1.8
1,000	5,655	0.5
10,000	5,680	0.1

Source: Author 2025

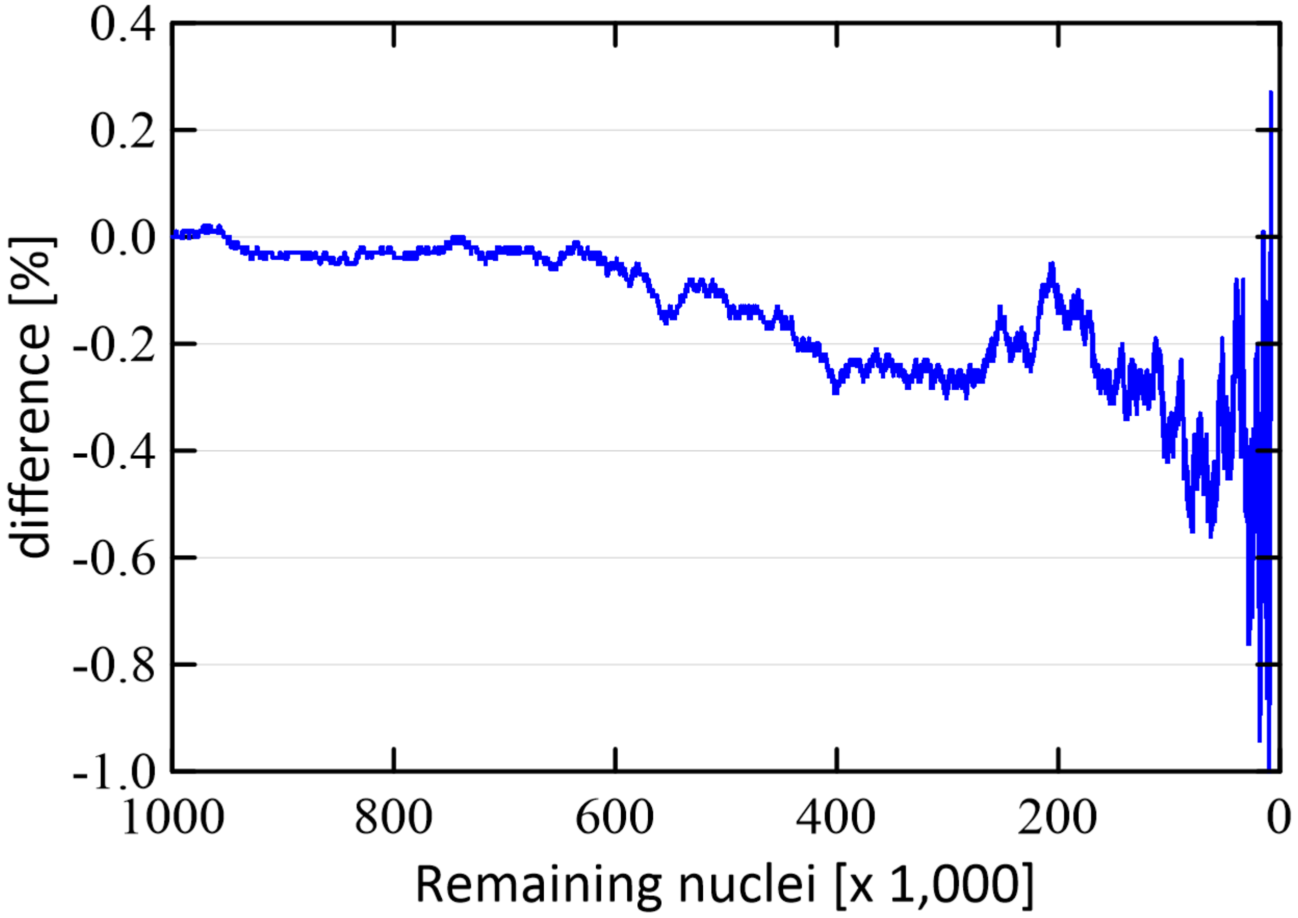
3.2.1 Probability over specific time periods

- Young samples. At 5,000 years a significant number of C-14 nuclei remains (~ 54% left).
- Older samples. At 60,000 years the number of C-14 nuclei is nearly undetectable (< 0.067% of original amount remains). In radiocarbon dating, the fraction of remaining C-14 is used to infer the age of organic materials. The simulation confirms that even after 50,000 years, only a negligible fraction of C-14 remains, limiting the effective dating range to about 10 or 11 half-lives (~ 60,000 years).

Figure 4 indicates that the differences between simulated and calculated results from Equation (1) remain less than 1% until the number of undecayed nuclei is about 3,500. Figure 4 clearly shows statistical fluctuations when the number of remaining nuclei is low. The difference depicted in this figure is primarily a manifestation of statistical noise inherent in any stochastic process when observed with limited sample sizes. As the number of undecayed nuclei N decreases, the relative magnitude of these random fluctuations increases. This phenomenon is a direct consequence of the statistical nature of radioactive decay; while the decay constant λ governs the average decay rate, individual decay events are inherently probabilistic. When N is large, these individual random events average out, leading to a smooth, predictable exponential curve. However, as N becomes small, the effect of each individual decay event, or lack thereof within a given time step, becomes more pronounced

Figure 4

Percentage difference between theoretical results of the number of C-14 remaining nuclei obtained directly from Equation (1) and simulation.



Source: Author 2025

relative to the total number of remaining nuclei. This leads to larger relative deviations from the theoretical exponential curve. In terms of statistical theory, for a binomial process like radioactive decay, the standard deviation is proportional to $\sqrt{N}$. Therefore, the relative uncertainty (standard deviation divided by *N*) is proportional to $1/\sqrt{N}$, which clearly shows why the relative error increases as *N* decreases.

Each unstable nucleus has a constant, independent probability of decaying in a given time interval. Thus, each nucleus can be treated as a Bernoulli trial with two possible outcomes, decay (success) or does not decay (failure). For a population of *N(t)* undecayed nuclei at time *t*, the number of decays in one time step follows a binomial distribution:

$$P(k) = \binom{N(t)}{k} p^k (1-p)^{N(t)-k}, \quad (6)$$

Where:

p is the probability of success in a single trial, *k* is the number of successes, and $\binom{N(t)}{k}$ is the number of ways to choose *k* successes from *N* trials.

The standard deviation for the number of decays is given by

$$\sigma_k = \sqrt{N(t)p(1-p)}. \quad (7)$$

When *p* is small (as is typical in radioactive decay per small time interval), this simplifies to

$$\sigma_k \approx \sqrt{N(t)p}. \quad (8)$$

As the sample decays, *N(t)* decreases, which reduces the absolute standard deviation but increases the relative uncertainty:

$$\frac{\sigma_k}{N(t)} \approx = \frac{1}{\sqrt{N(t)}}. \quad (9)$$

In real-world radiometric measurements, the region of low remaining nuclei is precisely where counting statistics become the dominant source of uncertainty, necessitating longer measurement times or more sensitive detectors to achieve acceptable precision in age determination. These increased fluctuations at very low numbers of undecayed nuclei have significant implications for radiocarbon dating, particularly for very old samples. As the amount of remaining Carbon-14 becomes extremely small, the inherent random nature of individual decay events leads to a greater relative uncertainty in the measured quantity of undecayed nuclei. This increased statistical noise translates directly into a larger uncertainty in the calculated age of the sample, effectively limiting the reliable dating range of radiocarbon methods (around 60,000 years). Beyond this range, the statistical fluctuations become so prominent that they can render precise age determination highly challenging or unreliable.

4 CONCLUSION

The computational simulation clearly demonstrates the exponential nature of radioactive decay. The methodology based on random numbers is easy to implement and yields results consistent with theory. This work highlights the potential of stochastic simulation applied to physics and also demonstrates that even simple Monte Carlo models can accurately replicate complex natural processes, offering valuable insights for students and researchers alike. The methodology is adaptable to more complex decay schemes, potentially supporting advanced nuclear physics researches or safety assessments.

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