Effects of a didactic sequence on the learning of functions in calculus mediated by the GeoGebra Platform and supported by the theory of objectification

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ABSTRACT

This work corresponds to an excerpt from the doctoral thesis, in which we address the use of technological artifacts developed in the GeoGebra Platform in the study of functions in higher education. Thus, in this article, we have as a general objective to analyze an activity carried out in a class of Differential and Integral Calculus, mediated by the GeoGebra Platform and supported for its elaboration in the Theory of Objectification. We intended that, based on the artifacts of this theory, this activity would enhance the study of the polynomial function. Students are indicated, by this theory, as the main actors encouraged by the teacher of the discipline, treating the content in a participatory and interactive way, aiming at both the updating and the materialization of knowledge, which Radford calls Joint Work. The methodological procedure we used in this research started from an educational action in which the students acted collectively, emphasizing interactive and collaborative actions. The tasks and activities gave prestige to problems in the real world and in the social environment of the students. As a result, we found that the students enhanced the study of functions from the interaction with the GeoGebra platform, validating the knowledge in a dynamic way, visualizing the changes in the values of the variables and perceiving the modifications of the dynamic constructions, which in the GeoGebra platform is called applications.

Keywords: Polynomial Function, GeoGebra Platform, Objectification Theory.

INTRODUCTION

Digital Theories of Information and Communication (DICT) are increasingly used in the educational context. As a didactic resource, they had a preponderantly notable importance in the recent pandemic period, which required remote teaching, a non-face-to-face modality, in which DICT is a fundamental element for its effectiveness. Of course, the importance remains, since remote teaching has brought some changes in the educational context, and is still used in some situations in teaching-learning, also contributing to the expansion of methodological possibilities for Education.

In general, there is a consensus among education professionals about the importance of didactic resources in general, and DICT in particular, in the teaching work and its potential to contribute to student learning. In this context, teaching platforms are inserted, such as GeoGebra, initially software, later evolving into a platform, in view of the potentialities.

Through textual searches in journals and websites of graduate programs of higher education institutions that research the relationship between content and technological resources and the GeoGebra platform, we found an increasing interest in the subject. In these discussions, we found that students have content difficulties in the study of functions, and especially when referring to visual and graphic representations of functions (BRAZ, 2020), which motivated this work.

In this sense, it is worth noting that both the updating and the materialization of the knowledge of the concepts of the functions, with regard to the Theory of Objectification, can be observed, at the moment of appropriation, by the student. It is highlighted, in fact, when we realize that the student has either updated himself at the time of contact with the artifacts, or even that he has assimilated, materializing knowledge, giving meaning and significance in the relationships with other concepts or with the real world.

From all these considerations, we carried out a research whose general objective was to analyze an activity carried out in a class of Differential and Integral Calculus, mediated by the GeoGebra Platform and supported for its elaboration in the Theory of Objectification.

THEORETICAL FOUNDATION

There are many researchers and scholars who are in favor of the use of didactic resources in the classroom (FREITAG, 2017), which includes digital communication technologies (MENEZES, BRAGA, SEIMETZ, 2019) and also playfulness (BRAGA et. al., 2019). Authors such as Libâneo (2017) and Menezes (2013) highlight the resources in their publications on Didactics, as precious elements for teachers in helping their daily work, including higher education.

In Mathematics Education, a specific context also concerning teaching, authors such as Rego and Rego (1999), Smole, Diniz and Milani (2006) and Menezes et al (2019) highlight the importance of didactic resources as aids in teaching by the teacher and facilitators of learning by the student. For the latter author, didactic resources, when well used, can act in a very positive and beneficial way in the teaching and learning process.

It is worth remembering that this requires prior organization, adequacy to the objectives, clarification of the contents, safety in handling, presentable conditions of use, among others (RÊGO and RÊGO, 1999). A didactic resource should serve as an aid to the teacher, and not an obstacle to classroom work (BRAGA et. al., 2019).

Discussions about the technologies used in the teaching and learning process, currently called digital information and communication technologies (DICT), have been growing within this context. With the dizzying technological advance, there is an increasing demand for the understanding and ways of using these elements more effectively for the educational process.

Skovsmose (2015, p. 14) describes that "Technology is not something additional that we can put aside, as if it were a piece, a hammer. We live in a technologically structured environment, a technonature." And, in turn, Mathematics is also part of this "technonature", as they were produced from various mathematical knowledge. Ponte (1995, p. 2) points out that the use of technologies in the teaching of Mathematics has brought several gains to the teaching and learning process, among them, "a growing interest in the realization of projects and activities of modeling, investigation and exploration by students, as a fundamental part of their mathematical experience".

For Menezes et al (2019), the teaching of mathematics through DICT provides the teacher with an additional tool to work in the classroom in different contexts and they have favored significant advances, both in terms of the understanding of mathematical concepts and content and in the improvement of the teacher's teaching practice.

As teachers working in licentiate, bachelor's, mathematics, professional master's and specialization courses in mathematics, in the face-to-face and distance learning modalities, we share the concerns regarding the insertion of DICT in the teaching and learning process of this discipline. From this perspective, since the conception of the program, the computer has been a valuable didactic resource for more meaningful educational practices aligned with the demands of society that increasingly uses technological devices (BRAGA et al, 2019).

Official documents aimed at teaching, including the National Curriculum Parameters and the Law of Guidelines and Bases for National Education (LDBEN), also encourage the use of computers and other virtual technologies and platforms. Currently, the National Common Curricular Base (BRASIL, 2018) reaffirms these guidelines.

The Theory of Objectification makes a great contribution to the collaborative work among students, mediated by the teacher's guidance in group tasks, applicable in the teaching of mathematics. We chose to work with this theory and its artifices, with the purpose of presenting a scenario that allows us to develop educational actions in the classroom with a collaborative, participatory and interactive nature, with the purpose of materializing or updating knowledge.

In this perspective, we created with the use of the artifacts of the GeoGebra platform an educational action in which students and teachers work for the materialization of knowledge in a joint, collaborative, participatory and interactive action, validating the Joint Work.

Menezes, Nascimento and Magalhães. (2001) state that:

> Teachers and students need to be more attuned to a common theme, which is the training of higher education professionals, including mathematics teachers in general, and the deepening of mathematical thinking, including the teaching-learning of Calculus I, in particular. (2001, p.12)

According to the authors in the citation, they deal with the relationship between teachers and students with the objective of studying a content, treating knowledge by highlighting mathematical thinking for the study of concepts of the discipline of Differential and Integral Calculus. In this sense, we highlight what Radford (2010; 2014), in his works on the Theory of Objectification, treats as Joint Work, a collaborative action in tune with the development of the knowledge to be studied.

For this researcher, the fundamental idea is that study can be associated with knowledge, the object, and with the subject, with the subjectivity of being.

In view of this concept, the development of the theory of objectification both affects the subject's relationship with knowledge, materializing in knowledge, and de-characterizes the subject with new conceptions.

Thus, we understand that in this theory, learning is not limited to knowing, but also with the new posture of the subject, a new being, who thinks and criticizes when necessary. This theory fundamentally supported this work, as it signals a teaching practice connected with students and that they must be attuned to the same level of the teaching and learning process. This attitude enables the development of a skill, thinking mathematically, in a reflective and critical way.

Through readings in books, articles, dissertations and theses, we found that scholars such as Radford (2010; 2014) affirm about the process of interaction of the subject in a historical and cultural context that mental models, means of communication and actions with signs and artifacts result in knowledge, as a form of representation of knowledge by students.

Starting from the difficulties of the student to think mathematically, because they have not supposedly been introduced to the aforementioned learning model since basic education and to act from this perspective, we think of a change in teaching practice in which the knowledge on stage is also the responsibility of the students, in a collaborative, participatory and interactive way.

The content of Geometry, having less prominence in basic education to the detriment of Algebra, can lead the student to not be motivated to think about the contents of mathematics in any dimension, by the teaching practice adopted in the classroom.

Thus, in order to establish collaborative, participatory and interactive work in the development of content in the classroom, we treated the contents together with a change in teaching practice, which was also informed to the students. We are moving in this direction.

METHODOLOGICAL PROCEDURES

Our research was qualitative and exploratory. The sample consisted of twelve students from the mathematics course at a public university, who volunteered, these students were at the time of the research taking the discipline of Differential and Integral Calculus I. The textbook of the discipline and who guided the activities was Stewart (2016).

Thus, we adopted, in our empirical research, with the ideas in force in the Theory chosen to guide the work, which are: teaching practice, where we think and elaborate the problems that allow the discussion among the students with the objective of reaching the answer and exchanging experiences with the teacher to together objectify the knowledge by updating in knowledge, and according to which the teacher must make it clear to the students that in order for the objectification process to occur they they are at the same level in the teaching and learning process.

The students' performance took place in groups, aiming to socialize their conceptions and experiences with their colleagues so that together and with the support of the teacher they present their difficulties to objectify knowledge; the artifacts, in our case, built on the GeoGebra Platform, free of charge, chosen for its quality brought to the visualization of the constructions elaborated therein. The possibility of changing the values of the variables is a great differential in the teaching and learning process for both teachers and students. This Platform allows the student to build models and analyze their constructions, visualizing changes instantly. The students were given the activities and tasks, described in the appendix of this article.

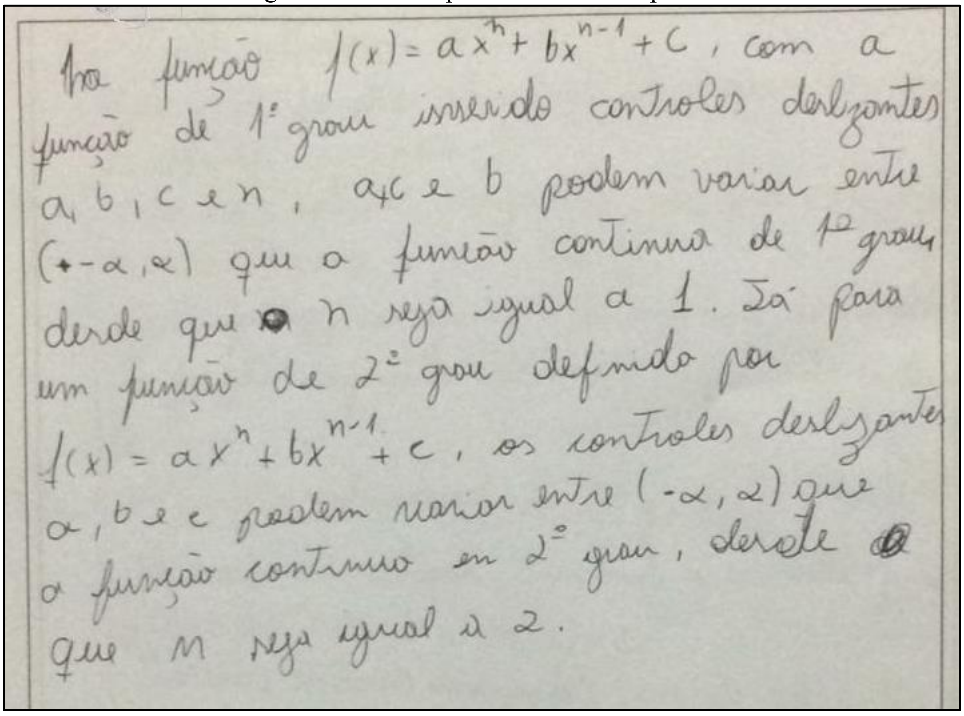
Both the following activities and tasks refer to what we will address in the educational actions with the students. They were elaborated with the intention of dealing with the ability of the formation of mathematical algebraic thinking in the teaching and learning process of the polynomial function. The activity model can be found in appendix A. The data constituted by the records of the students' speeches were categorized and then analyzed according to Bardin's (2016) content analysis

ANALYSIS OF AND DISCUSSION OF DATA AND RESULTS

First of all, we consider it worth noting that the excerpts presented by the students in the resolution of the activities were monitored by means of instruments developed with the purpose of building our data for analysis and subsequent presentation of the data. Thus, in the first productions, we see the emphasis of Algebra, leaving Geometry in the background in the approach to resolutions.

Moreover, we notice the update favored with the artifacts in the GeoGebra Platform, and expressed in the representations of the polynomial function, as highlighted below:

Figure 01. Transcript of a student's response.



Na função $f(x) = ax^n + bx^{n-1} + c$, com a
função de 1º grau inserido controles deslizantes
a, b, c e n, a, c e b podem variar entre
$(+-\alpha, \alpha)$ que a função continua de 1º grau,
desde que n seja igual a 1. Já para
um função de 2º grau definido por
$f(x) = ax^n + bx^{n-1} + c$, os controles deslizantes
a, b e c podem variar entre $(-\alpha, \alpha)$ que
a função continua em 2º grau, desde
que n seja igual a 2.

Source: Researchers' protocols

In the answer presented by this student, we found that he both actualizes and materializes the proposed knowledge. In the case of the update, it was verified by the relationship with the possibility of building other families of functions using the same method, associated with the use of the slider, a tool of the GeoGebra platform. With regard to the materialization of the knowledge of functions, we can see by the awakening from the possibility of studying the behavior of the coefficients "b" and "c" of polynomial functions, for example. A fact that rarely happens in the study of function families in both elementary and high school. Usually emphasis is placed on the variation of the values for the coefficient "a", angular in the polynomial function of the first degree and the behavior of the parabola in the polynomial function of the second degree, for example.

In view of the answer, the student presents his conceptions about a representation of the polynomial functions of the first and second degrees. To develop the suggested knowledge, the student opted for the construction through the GeoGebra platform of artifacts that are the sliders. These artifacts allow students to vary the values of the variables involved correlated with the sliders in the visualization of the construction. A fact that favors the understanding of knowledge through the visualization of the construction advancing in the appropriation of knowledge.

With this procedure built through the artifacts developed in the GeoGebra platform, it allowed the student an immediate visualization of the graphic variations represented in the viewing window in a dynamic way. A fact that would not be perceived in the classroom, on the blackboard and pencil as well as in the students' notebooks and pencils. The dynamic mode of visualization of the construction is one of the potentialities of the GeoGebra platform.

Thus, we articulate sociocultural activities for the teaching of the polynomial function, through visual representation, and a proposal for collaborative work among students directing the objective to the appropriation of knowledge.

Based on this scenario, we started to deal with the students, participants of the research, this new posture with the content studied, functions. Thus, the students began to reflect and among themselves to dialogue about the variations that are presented in each family of functions, for example, in the polynomial function of the first degree we have both the variations that can occur for the values of the variables "x" and "y" and for the variations of the coefficients, in the case of the algebraic representations of the functions. Thus, considering this analysis, there is an advance in the study of this content, as well as the relationship with others that present algebraic representations for the representation of experimental data that need to be modeled by means of formulas.

Ponte and Serrazina (1998) point to the relationship of technologies as a means of exchanging information in various situations, and treat it as a tool for collaborative work.

In fact, these technologies (i) constitute a privileged means of access to information, (ii) are a fundamental instrument for thinking, creating, communicating and intervening in numerous situations, (iii) they constitute a very useful tool for collaborative work and (iv) represent a support for human development in the personal, social, cultural, recreational, civic and professional dimensions. (p.02).

In our view, one of the aspects of social insertion that can favor the paradigm shift and the relationship of problems associated with the use of technologies in Higher Education, justifies an advance in teaching practice.

It is worth noting that the dialogue between students provided by this situation, object of the research, enhanced the debate on the study of mathematical models for experimental data, treatment and solutions to real-world problems.

Considering the relationship between technologies and mathematical contents, Ubiratan D'Ambrósio (1996) made the following statement:

Throughout the evolution of humanity, Mathematics and technology have developed in close association, in a relationship that we could say symbiotic. Technology understood as the convergence of knowledge (science) and doing (technique), and mathematics are intrinsic to the solidary search for survival and transcendence. The generation of mathematical knowledge cannot, therefore, be dissociated from the available technology. (D'AMBRÓSIO, 1996).

The relationship between natural sciences and mathematics and technologies favors teaching and learning, as long as they are through educational actions planned and articulated to be worked on in the classroom environment. This may be a relevant differential in changing teaching practice. In view of this panorama, we justify the use of the GeoGebra platform, appropriating its commands and potentialities, to develop our educational actions with students in the classroom environment, favoring the approximation with knowledge.

In view of this, we found that one of the objectives of our research was contemplated, with regard to the materialization and updating of knowledge with the articulation that was presented in the excerpts of the students, participants in this research.

For the development of the activities, we proposed to the students the use of the tools of the GeoGebra platform as well as its applications. In this way, it allowed the student to understand the objective of the questions and to discuss the forms of resolution with their colleagues. The constructions in the application also allowed us to advance towards the objectification of knowledge. Therefore, elements of the theory used were highlighted in the development and resolution of the activities. The gestures, the signals, and the insights and feedbacks perceived in these stages lead us to understand which elements of the theory were achieved, justifying the study.

In this sense, we start from the principle that learning can be enhanced with rigor through artifacts developed by technological tools as in the case of the GeoGebra platform. This platform allows us to rigorously work on learning in an interactive, collaborative and participatory way, providing the appropriation of knowledge, under a proposal of dynamic representation of content. The fundamental elements of the content are then constructed and reconstructed, setting knowledge in motion, materializing it.

Considering a methodological proposal, we can cite researchers such as Dantas (2016), who points to one of the most important contributions of the GeoGebra platform, the visualization of constructions and mathematical models and mathematical objects that enhance mathematical thinking. With the support of this important pedagogical tool, students can visualize the construction of the Polynomial Function in different situations and better understand the mathematical representation of the concepts involved.

In the theory of objectification, in order for objectification to occur in fact, knowledge must be objectified in the form of knowledge. The materialization or updating occurs at the moment when the student realizes that the potentiality provided in the artifacts suggested by GeoGebra enables the advancement of knowledge. In theory, the student creates his strategies, together with his classmates and the teacher. It enabled collective, participatory and interactive discussion, valuing everyone's work and socializing the responsibilities in the teaching and learning process.

In this way, we identify that at the time of solving the activities, we have the elements, joint work, the presence of gestures and signs, the debates, the explanations, all with the objective of achieving knowledge. Thus, we understand that by adopting the theory of objectification as a foundation for our activities, it enabled the advancement of students in the materialization of knowledge. Therefore, the collaborative work, where the student expressed his ideas, proved to contribute to a better understanding of the subject, and graphic interpretation of the situation, and the use of the artifacts built by GeoGebra in the tasks acted, in fact, as an aid to its understanding.

FINAL CONSIDERATIONS

We highlight here some relevant aspects regarding the relationship between teaching and learning, related to teaching practice, should also be taken into account, not only the knowledge should be on the scene but also the teacher's posture to develop the content in a dynamic and participatory way with the students. We observed, in this research, that the students showed that they want to learn and seek information as long as it is in a motivating, dynamic and updated way with the social and cultural environment, because knowledge can be developed in this perspective.

Based on this premise, we found the motivation of the students to participate and make decisions in the study of the proposed content, because they feel like a character in the scenario that was proposed to them in the activity. From the moment they realize that the activity is part of their world, and that the experimental data must be collected and treated by them, the activity takes on another dimension. At this moment, they are not "created" or "ready-made" data, but in fact of the reality of each one, as it deals with specific issues and the reality of each one. An example to be considered deals with the activity of the cistern. In this activity, the student must treat the data of their reality, they can and even should interact with colleagues to understand and analyze the data, but the answers are specific and individual. The fact is that each one has a specific consumption of water for personal hygiene, drinking and household cleaning. Thus, the answers will not be the same, the consumption or even the waste of water will be different for each of the participants and also for society in general.

We understand that it is important that teachers can approach knowledge with the support, participation, collaboration and interaction of students, causing a materialization and/or updating of knowledge in a dynamic way, as seen in this research, how an educational action can be developed in a virtual teaching environment with the use of artifacts to update knowledge.

Thus, the use of the GeoGebra platform enhances the development of activity when it requires experimental data to be treated from the analysis of its variations. This, among other actions, can be developed through dynamic constructions elaborated with the tools of the GeoGebra platform, enhancing the study and providing discussions with the results presented.

Therefore, we must develop other actions and we hope to contribute to teachers understanding the way in which the Theory of Objectification places knowledge to be treated in the classroom, in the idea of giving an objective to knowledge through an update resulting in knowledge.

And yet, the use of the GeoGebra platform to treat experimental data from real-world problems enhances discussion, debate, interaction, collaboration, paradigm shift, participation and, finally, the visual representation of experimental data in a dynamic way.

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APPENDIX A: APPLIED ACTIVITY WITH STUDENTS

Activity: - According to the website of the Ministry of Social Development – MDS, "the cistern for human consumption is designed to meet basic needs (drinking, cooking and personal hygiene) of a family of up to five people for eight months, the normal period of drought in the semi-arid region". It is a social technology – knowledge developed and shared in the community itself – simple and low-cost, which captures rainwater. It is a masonry reservoir that stores rainwater captured by a system of gutters interconnected to it, installed on the roof.

The Cisterns Program also supports the construction of social technologies for access to water to expand the conditions for farming families to produce food for self-consumption and also for the sale of surpluses at local fairs or in institutional purchasing programs, such as the Food Acquisition Program (PAA) and the National School Feeding Program (PNAE)." (MDS, 2018)[1]

In view of this:

What should be the best ratio between the volume and the radius of the base of this tank for a capacity of 250 liters, and 800 liters?

What should be the best ratio between the volume and the height of the cistern for a family that needs 50,000 liters to sustain itself for eight months?

Establish an algebraic representation from the proposed relationship between volume and radius?

Establish an algebraic representation from the proposed relationship between volume and pitch?

Write down your understanding of the generalization of the polynomial function proposed in the activity.

[1] Accessed the website on 08-13-2018. http://mds.gov.br/area-de-imprensa/noticias/2017/agosto/programa-cisternas-e-uma-das-tres-melhores-politicas-publicas-do-mundo