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# THE MAGIC OF NUMBERS IN THE SIMPLICITY OF JOHN NAPIER'S LOGARITHMS

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**ABSTRACT**
John Napier, Baron Merchiston (1550–1617), was a Scottish mathematician and theologian whose contributions revolutionized science and mathematics. According to Fernandez, Tomas, and Tamaro (2004), Napier combined his Protestant faith with a critical approach to the Catholic Church, expressed in his theological work A Plaine Discovery of the Whole Revelation of Saint John. Although this work was notable in its time, its legacy endures mainly for the development of logarithms, a fundamental tool in the simplification of complex mathematical calculations.

**Keywords:** Logarithms. Mathematics. Astronomy. Simplification. Reform. Protestant.

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## INTRODUCTION

This article presents the history, life and contributions of John Napier (1550-1617) was an influential Scottish mathematician, physicist and astronomer known for his notable contributions to the field of mathematics, in particular for his invention of logarithms and his influence on the development of mathematics in his time.

John Napier

Napier was born in Merchiston, Scotland, to a noble family. Throughout his life, he became interested in a wide range of disciplines, including theology, astronomy, and mathematics. However, he is best known for his work in mathematics, which culminated in the invention of logarithms in the first half of the 17th century.

The main idea of this document is to show the important contributions of this Scottish mathematician among his fundamental contributions are Napier's logarithms, published in his work "Mirifici Logarithmorum Canonis Description" in 1614, they allowed complex calculations, such as multiplication and division, to be simplified, transforming them into simpler operations of addition and subtraction. This invention revolutionized mathematics and navigation, as it facilitated the calculations necessary for astronomy and cartography, among other disciplines.

Napier's logarithms also paved the way for the further development of calculus and trigonometry, influencing mathematicians and scientists of future generations. His legacy lives on in mathematics and science in general, and his contribution to the simplification of complex calculations is a fundamental part of the history of mathematics.

## CONCEPTUAL THEORY

After more than twenty years of research, Napier published in 1614 his work Mirifici Logarithmorum Canonis Descriptio, introducing logarithms. This innovation transformed multiplication and division operations into addition and subtraction, facilitating calculations in fields such as astronomy and navigation. According to the Dictionary of National Biography (2013), Henry Briggs, an English mathematician, collaborated with Napier to develop base-10 logarithms, which became popular throughout Europe.

John Napier also excelled in trigonometry and astronomy, creating trigonometric logarithms that simplified calculations in these areas. In addition, he contributed to the design of mathematical tools, such as logarithmic tables and Napier's rules, expounded in his work Rabdologiae seu Numerationis per Virgulas Libri Duo (1617). Napier Born in a time marked by religious tensions, Napier was a fervent Protestant. According to Joseph Frederick Scott (2009), he dedicated a large part of his life to defending his beliefs, expressed in his work Plaine Discovery. This attitude reflected the religious transformations in Scotland during the Protestant Reformation.

Although his initial recognition was limited compared to contemporaries such as Galileo Galilei, Napier's impact grew over time. His work was instrumental in the transition from medieval to modern mathematics. Napier's contributions to logarithms and the simplification of mathematical calculations are still valued today, cementing his place in the history of science. According to Bradley, Micheael (2006). "Without a doubt, his greatest contribution in the field of mathematics was the concept of logarithm. Napier studied about them between 1590 and 1617. The first work he published in this sense was Mirifici Logarithmorum Canonis Descriptio (Description of an Admirable Table of Logarithms) in 1614. There he describes how to use logarithms to solve problems with triangles and gives a table of logarithms. In 1619 his son^ Robert posthumously published Mirifici logarithmorum canonis constructio (Construction of an Admirable Table of Logarithms), where he explains how the construction of the
table of logarithms".

Although in the beginning he called logarithms artificial numbers, he himself would later create the name by which they are known today, by combining the Greek words "logos" (proportion) and "arithmos" (number).

Napier's discovery was an immediate success, both in mathematics and astronomy. Some of the pioneers to follow his work were Henry Briggs and John Speidell. Johannes Kepler dedicated a 1620 publication to Napier, stating that logarithms were the central idea for discovering the third law of the motion of the planets.

A quote from Pierre-Simon Laplace mentions and honors Napier's discovery and application of logarithms.

According to Pierre-Simon Laplace (2013) "With the reduction of the work from several months of calculation to a few days, the invention of logarithms seems to have doubled the lifespan of astronomers".

According to Susana B. Impellizere de Córdoba. (2005). "In the sixteenth and early seventeenth centuries, as mathematics developed, enormous difficulties of a practical computational nature were experienced. These difficulties were concentrated around the problems of making tables of trigonometric functions, the determination of the value of, the search for simple and reliable algorithms for determining the roots of equations with given numerical coefficients, among others, the calculations were carried out only by hand".

According to Susana B. Impellizere de Córdoba (2005). "The decimal and natural logarithms that are currently used do not use the same base as Napier's logarithms, although in their honor natural logarithms are called Neperian. It took Napier twenty years of work to reason about the properties and existence of logarithms. He must have reflected on the successions of powers of a given number, which had been published in Stifel's Arithmetica completa fifty years earlier and in the works of Archimedes. (The notation we use today for powers was only introduced by Descartes after Napier's death.) Napier observed that the products or quotients of the powers correspond respectively to the sums or differences of the indices or exponents of the powers themselves. That's where the idea of substituting each multiplication with a sum came from."

The problem I found was that, if I used a succession of integer powers of an integer base, for example, two, it was not useful for calculation because the large gaps between successive terms make the interpolation too imprecise.

The knowledge, through John Craig, of the use of the method of 15 prostapheresis used by astronomers in Denmark, encouraged him to redouble his efforts and finally published in 1614 his work Mirifici logarithmontm canonis descriptio (description of the wonderful rule of logarithms).

The central idea of Napier's work was the following: to make the terms of a geometric progression formed by the integer powers of a given number very close to each other, it is necessary to take that number very close to one.

Napier decided to take 1 − 10−7 = 0.99999999 as the given number; then the terms of the (decreasing) progression of increasing whole powers are very close to each other. He multiplied all the powers by 107, to avoid the use of decimals, so if

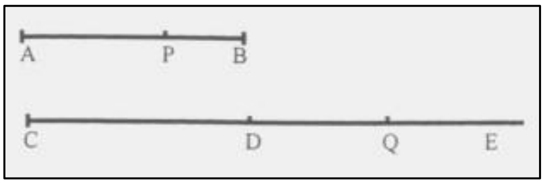
$N = 10^7 (1 - \frac{1}{10^7})^L$, [illegible]

será 0, el logaritmo de $10^7 (1 - \frac{1}{10^7}) = 9999999$ será 1.

Al dividir los números y los logaritmos por $10^7$, obtendríamos prácticamente un sistema de logaritmos de base $\frac{1}{e}$, ya que $(1 - \frac{1}{10^7})^{10^7}$ no diferencia demasiado del $\lim_{n\to\infty} (1 - \frac{1}{n})^n = \frac{1}{e}$.

According to Susana B. (2005) "Napier did not use the basic idea of a logarithm system, since its definition is different from ours. Napier explained in geometric form the correspondence between two successions of numbers, one in arithmetic progression and the other in geometric progression, using the concept of two points moving along different straight lines, one with uniform velocity, and the other with accelerated velocity.

Suppose a given segment $\overline{AB}$ and a semiline $\overline{CDE}$, a point P starting from A and moving along $\overline{AB}$ with variable velocity decreasing in proportion to its distance from B; and that the point Q starts at the same time from C and moves along the halfline $\overline{CDE}$ with uniform velocity equal to the initial velocity of the point P; then Napier calls the variable distance $\overline{CQ}$ the logarithm of the distance $\overline{PB}$.




This way of defining logarithms is in accordance with the previous definition. For the proof, let $\overline{PB}$ = x and $\overline{CQ}$ = y . If we take $\overline{AB}$ = 107 initial velocity of P,  then, in today's language it is equivalent to the equation $^{dx}$ = −x and $^{dy}$ = 107, com

$x_0 = 10^7$

Entonces $\frac{dy}{dt} = -\frac{10^7}{x}$ o bien $y = -10^7 lncx$ donde la constante c se determina a partir de las condiciones iniciales y resulta $c = 10^{-7}$, asi pues, $y = -10^{-7} ln \frac{x}{10^7}$ o bien $\frac{y}{10^7} = log_{\frac{1}{e}} \frac{x}{10^7}$.

Es decir, que si las distancias $\overline{PB}$ y $\overline{CQ}$ estuvieran divididas por $10^7$ entonces la definición de Napier nos conduciría precisamente a un sistema de logaritmos de $\frac{1}{e}$, tal como decíamos anteriormente.

According to Dolciani, Berman, Wooton (2004). "At first Napier called his power indices or exponents "artificial numbers" or "relation numbers" (from the union of the Greek words: λ o and θ – relation αρtφμοε – number). He chose this name to emphasize that logarithms are auxiliary numbers that measure relationships between the corresponding numbers."

Despite the general idea of the continuous numerical scale, Napier's logarithms were still tables comparing the values of two progressions: arithmetic and geometric.

The progression that Neper studied were arithmetic progressions that began with 0 and geometric progressions that began with 1

| Progresión Aritmética | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Progresión Geométrica | 1 | 10 | 100 | 1 000 | 10 000 | 100 000 | 1 000 000 |

Let's look at the following progressions: we have the arithmetic progression (0,1,2,3,4,5 ... ) and the geometric progression (1, 10,100, 1000 ... ).

### FIRST RULE

If we add two terms of the arithmetic progression, I get the same term of the same progression, which is equivalent to multiplying the corresponding terms of the geometric progression, for example:

$$2 + 4 = 6$$
$$100 \cdot 1{,}000 = 1{,}000{,}000$$

Adding 2 + 4 = 6, which is equivalent to multiplying their corresponding terms in the geometric progression, that is, if multiplying the corresponding to 2, which would be 100, multiplied by the corresponding to 10,000, we obtain 1,000,000, which would be the corresponding to 6 in the geometric progression.

### SECOND RULE

If I subtract two terms from the arithmetic progression, I get another term from it, which corresponds to the quotient of the two terms of the corresponding geometric progression, for example:

$$6 - 4 = 2$$
$$\frac{1\,000\,000}{10\,000} = 100$$

If 6 is subtracted from 4, 2 is obtained, that is, it would be equivalent for the geometric progression to divide 1,000,000 with 10,000 and thus obtain the number 100, which would correspond to 2 in geometric progression.

## RULE NUMBER 3

If we take a term of the arithmetical progression as multiplying and any number as a multiplier, we obtain a product such that its corresponding term of the geometric one is the result of taking as a basis the result raised to the equal power of the multiplier.

I mean:

$$2 \cdot 3 = 6$$

$$100^3 = 1\,000\,000$$

If we have 2, which is the term of the arithmetic progression, and we multiply it by 3, the result is 6, which would be the same as raising the 100 to that same 3, and it will result in the corresponding 6 in the geometric one, that is, 1,000,000.

## RULE NUMBER 4

If one term of the arithmetic progression is divided by any quantity, one obtains as a quotient another of the same progression corresponding to one of the geometric progression, which is the root of the quantity corresponding to the one taken as a dividend in the arithmetic progression.

I mean:

$$\frac{6}{2} = 3$$

$$\sqrt{1\,000\,000} = 1\,000$$

If we take the 6 of the arithmetic progression and divide it by 2, we get the 3 of that same progression, which would be equivalent to taking the square root of 1,000,000 from the corresponding 3 in the geometric one, therefore we get 1,000.

| Progresión Aritmética | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Progresión Geométrica | 1 | 10 | 100 | 1 000 | 10 000 | 100 000 | 1 000 000 |

Neper called logarithms the terms of a geometric progression that begin with one, the corresponding terms of the arithmetic progression that begin with 0, that is, if the logarithm of 1 is equal to 0, the logarithm of 10 is equal to 10, so on, the logarithm of 10,000 is equal to 4, the logarithm of 1,000,000 is equal to 6.

Logarithms, from the word composed of two Greek words logos (or reason) and arithmos (or number). In view of the demands of the astronomy of the time, as we have already said, Napier's table was formed by the logarithms of trigonometric functions. First of all, a separate column was formed by the logarithms of the sines of the angles of the first quadrant, chosen with intervals of 1. They also gave the values of the logarithms of the cosines (as sines of the complementary angles). In a special column, under the heading of "difference" were placed the differences of the logarithms of the sines of the complementary angles, that is, the logarithms of the tangents. Napier knew that the logarithms of inverse trigonometric functions were obtained simply by a change of sign.

## FINAL THOUGHTS

John Napier was an important figure in the history of mathematics and science, whose contributions transcended his time. His development of logarithms, presented in his work Mirifici Logarithmorum Canonis Descriptio (1614), marked a before and after in the simplification of complex calculations, transforming multiplication and division into addition and subtraction. This innovation greatly facilitated calculations in fields such as astronomy and navigation, and was further consolidated with the collaboration of Henry Briggs, who helped develop base-10 logarithms.

In addition to his work in mathematics, Napier excelled in trigonometry and astronomy, and his contribution to the design of mathematical tools such as logarithmic tables and Napier's rules was of great importance. His critical approach to the Catholic Church and his Protestant fervor also influenced his life and work, reflecting the religious tensions of his day.

Despite not receiving immediate recognition from other contemporaries such as Galileo Galilei, Napier's impact grew over time, and his work was essential to the passage from medieval to modern mathematics. Today, Napier's contributions remain pivotal in science and mathematics, cementing his legacy in history.

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