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HISTORICAL EXCERPTS ON THE CONSTITUTION OF THE CONCEPT OF INTEGRAL

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ABSTRACT
Mathematics is still seen as abstract, and difficult and presents challenges in its teaching. Our main point is to present a pedagogical approach to the use of history in teaching, specifically in teaching the concept of integral. The aim is to explore the use of the History of Mathematics as a pedagogical resource, highlighting its relevance in teaching based on Chaquiam (2023). Thus, the objective is to present excerpts from the history of mathematics related to the concept of integral, based on the diagram by Chaquiam (2023), associated with classroom activities. The methodology involves a literature review and analysis of pedagogical practices, including those based on the historical use of mathematics in the teaching of Mathematics. Results indicate that the History of Mathematics can enrich the teaching process, making it more interactive and contextualized. In short, the importance of considering the History of Mathematics as a pedagogical resource and its contributions to the initial and continuing education of Mathematics teachers is reaffirmed.

Keywords: History of Mathematics. History in the Teaching of Mathematics. Integral Calculus.

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INTRODUCTION

Mathematics teaching in Brazil faces profound challenges, as it is one of the subjects that presents the most difficulties for students, many of whom consider it abstract and disconnected from their daily experiences. This distance can create a significant barrier to effective learning since the lack of connection between the content and the student's reality can reduce their interest and motivation. An innovative and promising approach to overcome this obstacle is the integration of the History of Mathematics as a pedagogical resource. By including History in teaching, students are allowed to see Mathematics not only as a set of abstract formulas and concepts but as a human construction, with a rich history of discoveries, errors, and advances over the centuries, involving a series of thinkers and cultures. This perspective allows for a more meaningful approach, making the subject more accessible and interesting.

Among the current trends in Mathematics Education, some approaches stand out as essential for the transformation of teaching, such as Critical Education. This perspective proposes a reformulation of the teaching-learning process, seeking to contextualize the content and involve students more actively, leading them to reflect critically on the knowledge they are acquiring and its applications in the real world. In addition, Interdisciplinarity has consolidated itself as an effective strategy, as it integrates Mathematics with other areas of knowledge, promoting a more holistic view of the content and allowing students to perceive the multiple connections between the different disciplines. The History of Mathematics, when used in this context, not only provides a new way of presenting the content but also stimulates students' curiosity, providing extra motivation by showing how Mathematics was developed over time and how it directly impacts our daily lives.

The general objective is to present excerpts from the History of Mathematics related to the constitution of the concept of Integral, linked by Chaquiam's methodological diagram (2023), associated with the activities that will be applied in the classroom. The research will be organized into different sections, which will address the importance of integrating the History of Mathematics into teaching, its effects on the learning process, and how it can be implemented practically in classrooms, to promote a deeper and more engaging understanding of the subject. The expectation is that, by exploring this path, we can offer new tools to educators and contribute to the formation of a new generation of students who are more engaged with Mathematics.

The teaching of Mathematics in Brazil faces challenges that make it difficult for students to fully understand it. The subject, often approached as abstract and difficult, often follows a path far removed from students' daily experiences. However, this view can result in difficulties in understanding learning and resistance on the part of students, which creates barriers in the educational process. In the context addressed, the History of Mathematics emerges as a very effective and concrete strategy for transforming the approach to teaching. According to Mello and Fonseca (2018), the History of Mathematics not only adds richness to the content, but also humanizes the subject, allowing students to see mathematicians as real people, with flaws and successes, and not as distant and unattainable figures. They state that "the History of Mathematics has great pedagogical potential, as it allows students to perceive Mathematics as a human construction, the result of a trajectory of advances, failures and, above all, creativity" (Mello & Fonseca, 2018, p. 123).

The National Curricular Guidelines for the Training of Basic Education Teachers at the Higher Education Level (Opinion CNE/CP/09/2001) establish fundamental principles that guide the training of future teachers. Among the aspects highlighted is the need for a direct connection between theoretical training and professional practice, to transform the traditional teaching model. Thus, the aim is to integrate specific and pedagogical knowledge, which is fundamental for teaching.

According to the document, teacher training courses should promote a solid base of didactic pedagogical knowledge, especially with regard to mathematics. These courses should also encourage the connection between academic training and classroom practice, in addition to investigative practices that combine theory and practice. The National Curricular Guidelines for Bachelor's and Teaching Degree courses in Mathematics (CNE/CES Opinion 1,302/2001) also define the curricular organization and establish the desired profile of graduates, highlighting the skills and abilities to be developed throughout the course.

The Curricular Guidelines The National Curricular Guidelines for the Mathematics course in higher education are important for academic organization, with emphasis on the disciplines of differential and integral calculus. These guidelines aim to achieve essential content for the development of specific skills and abilities, especially in the case of a Bachelor's Degree in Mathematics.

In the context of Differential and Integral Calculus, the National Curricular Guidelines highlight the importance of promoting mathematical reasoning and the ability to solve complex problems. The approach involves both the manipulation technique, and calculations, such as derivatives, integrals, limits, and others, as well as conceptual understanding, with applications in different areas. The contents of the (CDI) are structured to develop logical-mathematical thinking, associated with modeling, and connecting theory and practice. In addition, the National Curricular Guidelines encourage the use of technology and graphic representations that are fundamental to understanding and applying concepts in modern contexts.

By immersing themselves in the historical study of Mathematics, students can understand that the concepts they use today did not emerge in isolation, but rather as a result of centuries of experimentation, discoveries, and improvements. Furthermore, History helps to put into context concepts that often seem disconnected from students' reality. Lima (2020) highlights that "introducing the History of Mathematics in classes is a powerful way to contextualize mathematical knowledge, making it more accessible and relevant to students' lives" (p. 75). The inclusion of this historical approach can help students understand mathematics as a constantly evolving tool that has been used over the centuries to solve practical problems, impacting a wide range of areas of human knowledge.

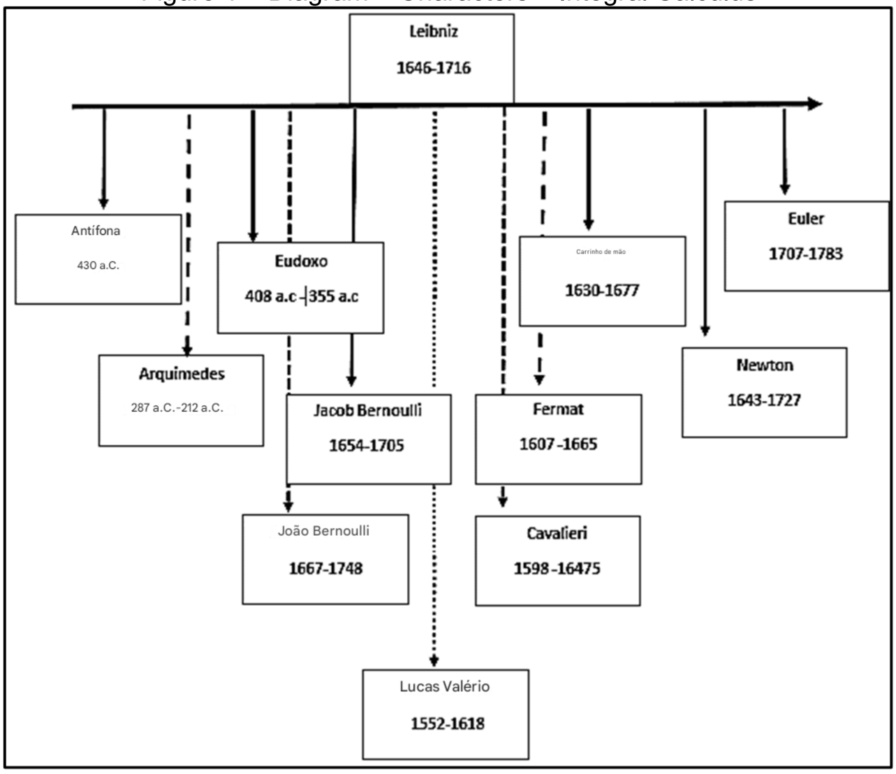
Studying the origin and development of mathematical concepts does not only mean learning about mathematicians but also understanding the fundamental role of mathematics in today's world. The historical methodology brings more meaningful learning by putting concepts into perspective, showing their relevance and application in different historical contexts.

However, contrary to what is currently taught in calculus courses, the concept of integration was developed before differentiation. The integral arose from problems related to lengths, areas, and volumes, while differentiation was created to solve questions about the tangents of curves. It is likely that, due to the practical needs of the time, such as determining areas of irregular terrain, integration was prioritized. Only years later, after the emergence of both concepts, integration, and differentiation were understood as inverse operations of each other.

Unlike the historical context, where the integral was developed to solve practical problems such as calculating areas of volume, today it is explored in broader applications, including probability, physics, and engineering. This change allows us to understand the geometric meaning of the integral and its practical usefulness in modeling real phenomena.

The methodology proposed by Chaquiam (2022), known as a methodological diagram, seeks a structured approach to integrate the History of Mathematics into teaching. It proposes that Mathematics be taught sequentially and chronologically so that students can follow the development of mathematical knowledge over time. Chaquiam (2022) argues that "organizing mathematics as a field of historical knowledge allows students to view its discoveries continuously, appreciating the complexity of its origins and the impact of ideas over the centuries" (p. 58). The goal of this methodology is to transform the way mathematics is taught, offering students a broader and more contextualized understanding of the subject, and connecting historical findings to current mathematical concepts. This approach suggests that the history of mathematics should not be just supplementary content, but rather a fundamental resource in the teaching process. By adopting a historical timeline, teachers can highlight the main figures and discoveries that have marked mathematics over the centuries, such as Pythagoras, Archimedes, Newton, and Leibniz, among others. Almeida (2019) notes that "by applying Chaquiam's methodological diagram, mathematics teaching becomes more dynamic, connecting students to the living history of the subject and allowing for more interactive and meaningful learning" (p. 42).

Figure 1 – Diagram – Characters – Integral Calculus



Source: Adapted from Chaquiam (2023)

The use of the timeline proposed by Chaquiam provides a clear view of how mathematical ideas interconnect and evolve throughout history. It allows students to perceive the interactions between mathematical discoveries and historical contexts, helping to connect mathematical concepts with the social, cultural, and technological realities of each historical period.

According to Souza and Costa (2021), "the use of a historical timeline allows students to see Mathematics as a collective product of humanity, developed over many centuries by different cultures and thinkers" (p. 90). This timeline serves not only as an organization of the content but as a way to show how Mathematics continually evolves and intertwines with the advances of society. The analysis of important historical milestones, such as the emergence of natural numbers or the development of calculus, allows students to perceive Mathematics as a living and constantly changing discipline.

To illustrate the evolution of Mathematics, a historical timeline is suggested, covering everything from ancient civilizations to the most recent mathematical developments. The aim is to highlight the discoveries and key figures that drove the progress of Mathematics. Throughout the History of Mathematics, several figures have contributed to the development of mathematics, especially in the construction of the foundations of integral and differential calculus. Antiphon and Eudoxus, in Ancient Greece, explored the method of exhaustion, which consisted of approximating areas and volumes through geometric figures, anticipating the concept of limit, essential for calculus. Antiphon of Athens (430 BC) arrived at the following method in the area of calculating a circle: by successively doubling the number of sides of a regular polygon, inscribed in a circle, the difference between the areas of the circle and the polygons would eventually be exhausted. Eudoxus (370 BC), seems to have been the first to completely solve the problem of incommensurable quantities, thus arriving at the theory of proportions, which applied to both commensurable and incommensurable quantities. This theory developed by Eudoxus is found in Euclid's book V. Archimedes, perfecting this technique, managed to calculate areas and volumes of solids such as spheres and cylinders, which made him stand out as one of the pioneers of integral calculus (Boyer, 1996, p. 83). He highlights that "it was a great invention, standing out in mathematics, physics, astronomy, and engineering, but he was always more interested in the product of his thoughts than in his mechanical devices". In his studies to calculated the area and volume of irregular regions. The idea is based on.

> (...) cut the corresponding region into a very large number of thin strips or parallel slices and (mentally) hang these pieces from one end of a lever, in such a way as to establish equilibrium with a figure of known area or volume and centroid. (Eves, 2004, p. 422).

Using this method, he arrived at results that led to definite integrals, present in the early texts on calculus. Later, in the Renaissance, mathematicians such as Luca Valério and Bonaventura Cavalieri expanded these ideas by studying volumes and areas through infinite sums, forming a more consolidated basis for modern calculus. Pierre de Fermat, in turn, developed methods for finding maxima and minima, a fundamental contribution to differential calculus, paving the way for the calculation of derivatives.

The advancement of integral and differential calculus continued with decisive contributions from Isaac Newton and Gottfried Wilhelm Leibniz in the 17th century. Isaac Newton formulated the Fundamental Theorem of Calculus, applying it to the laws of physics to describe phenomena such as gravity. Although Newton was not the first to use differentiation and integration, his legacy in calculus was to have consolidated a general algorithm applicable to all functions, whether algebraic or transcendental (BOYER, 1996). Leibniz, in turn, created the mathematical notation used to this day, making calculus accessible and practical. However, the creation of calculus generated an immense controversy between Newton and Leibniz. Newton had studied before Leibniz, who was the first to publish his results, so it is believed that both developed their research independently. The brothers Johann and Jacob Bernoulli deepened the study of calculus, with Johann introducing the term "integral" and Jacob applying it to probabilities and infinite series. Furthermore, Leonhard Euler consolidated these significant contributions by developing mathematical analysis and applying calculus to a wide range of problems in physics and number theory, establishing mathematics as an important tool for science. In calculus, the Beta and Gamma functions of advanced calculus are assigned to it, the differential equation of E uler, the idea is to solve linear differential equations, with constant coefficients, distinguishing them between homogeneous and non-homogeneous.

SHORTCUTS FROM THE HISTORY OF INTEGRAL CALCULUS

Below I present biographical traits of characters who contributed to the constitution of the concept of integral until its formalization, highlighting the contribution of each one, in the sequence presented in the diagram. We begin with Antiphon (Around 430 BC), a Greek philosopher who supposedly lived in the fourth century before Christ, was a sophist and master of rhetoric and persuasion, founder of psychoanalysis. Considered one of the first to work with the idea of exhaustion, which anticipates calculus. Antiphon considered the following method for calculating the area of a circle: using successive doublings of the number of sides of a regular polygon, inscribed in a circle, the difference between the areas of the circle and the polygons would eventually be exhausted. Archimedes of Syracuse (287 BC - 212 BC), a mathematician and great black inventor, was born in the Greek colony of Syracuse, in Sicily, Italy. Archimedes died in the same city where he was born, on the day that Syracuse was taken by Rome. He developed methods for calculating areas and volumes using exhaustion. According to Archimedes, Eudoxus provided the lemma that today bears Archimedes' name, sometimes called Archimedes' axiom, and which served as the basis for the method of exhaustion, the Greek equivalent of integral calculus. (Boyer, 1996, p.62)

Eudoxus of Cnidus (408 BC - 355 BC) was a mathematician from the 4th century BC who participated in the Platonic Academy of Athens and became one of the most important mathematicians of the Hellenic period. His works, however, were lost, and knowledge of his work is studied today through the records of other mathematicians who succeeded him – especially in Euclid's Elements (BOYER 1968, p. 126-132). Among his main contributions to the field of Mathematics are the theory of proportions and the Eudoxus-Archimedes Principle, applied in the Method of Exhaustion. Eudoxus of Cnidus emphasized purely geometric methods for calculating areas and volumes, and his system can be considered a precursor of integral calculus.

Luca Valerio (1552 - 1618), was born in Naples but raised in Greece. His studies were deepened in Italy, and he became a member of the Academia dei Lincei, of which Galileo was also a member, known for his work on areas and volumes, applying similar methods to calculus. Also born in Italy, in the city of Milan, was Bonaventura Francesco Cavalieri (1598 - 1647). Among his works, the treatise Geometria Indivisibilibus, published in 1635, stands out. In this work, he presents the method of indivisibles, which is similar to the ideas developed by Archimedes for calculating areas and volumes, contributing to the advancement of integration (Eves, 2004). Cavalieri discovered that if two plane figures can be compressed between parallel straight lines in such a way that they have identical vertical sections in each segment, then the figures have the same area.

Pierre de Fermat (1607 - 1665) was born on August 17, 1601, in Beaumont-de Lomages, France, and died on January 12, 1665, in Castres, also in France. He carried out important research and discoveries, such as analytical geometry, number theory, probability theory, calculus, optics, and especially the famous “Fermat's Last Theorem”. Contributions to mathematics are declared in the themes: Analytical Geometry, Number Theory, Probability Theory. He developed methods for calculating maxima and minima, which were fundamental to differential calculus. Isaac Barrow (1630 - 1677) was born in London in 1630 and died in 1677 after contracting a malignant fever. He was famous at school for being a terrible student. He was Newton's teacher and a pioneer in the fundamental theorem of calculus. Barrow's most important mathematical work is Lectione Geometrica, published in 1670, which deals with tangents. This work was later revised by Newton and contributed to the development of differential calculus. (Eves, 2004). A student of Barrow, Isaac Newton (1643 - 1727) is widely recognized as one of the most influential scientists of all time. Isaac Newton was born in 1643 in the United Kingdom. Famous for his “Law of Universal

Gravitation”, he described differential and integral calculus, formulating the fundamental theorem of calculus. Although Newton was not the first to use differentiation and integration, his legacy in calculus was to have consolidated a general algorithm applicable to all functions, whether algebraic or transcendental (BOYER, 1996). Newton died in 1727, in the city of London, in the United Kingdom.

The flux method published in 1736 considered that a curve was generated from the movement of a point. Starting from this assumption, the coordinates of this point become variables. Thus, Newton called it a flux, and its rate of variation is called a flux. By relating the fluxes to their fluxes, we arrive at what we today call the differential action. (EVES, 2004)

At the same time, Gottfried Wilhelm Leibniz (1646 - 1716) emerged. He was born in Germany and raised by his mother, who passed on rigorous knowledge. He entered the Nicolau school when he was only seven years old and acquired knowledge in a self-taught manner. He studied integral calculus and binary calculus, which would later be important for the establishment of computer programs. He used the symbol ∫ for integral for the first time in 1675. This symbol was derived from the first letter of the word sum. His first article on integrals is dated 1684 (Eves, 2004). Independently of Newton, he developed differential and integral calculus with the notations we still use today. He died alone, a victim of a gout attack, far from the spotlight of the aristocracy that accompanied him for part of his life. Jacob Bernoulli (Jacques Bernoulli; 1654 - 1705), was born in 1655. Like his brother Johann Bernoulli, a mathematician, they were early scholars of calculus and carried out experiments on vast problems. They contributed to calculus and probability theory. The word integral was first used in 1690, in a resolution of the problem, published in Acta eruditorum, on the isochronous or tautochronous curve (a curve in which the time taken by an object to slide without friction in uniform gravity to its minimum point is independent of its starting point). In 1696 Jacob Bernoulli solved the equation, which today bears his name. Another member of this family is Johann Bernoulli (1667 - 1748), a Swiss mathematician born in Basel, a Swiss university center. He even defended a doctoral thesis (1690) on effervescence and fermentation, as his father wanted him to be a doctor. However, the following year he discovered that he was so interested in calculus that he wrote two books that were only published later. He is considered one of the important founders of calculus. He popularized calculus and introduced the term “integral”. Johann became a professor at the University of Groningen in 1697 and, after his brother died in 1705, he succeeded him at the University of Basel, where he remained for the rest of his days (EVES, 2004).

Johann Bernoulli played a fundamental role in the history of calculus by spreading the potential of this new mathematical tool throughout Europe. It was through materials provided by Johann to L'Hospital (1661-1704) that he published the first text on calculus. In addition to his involvement with calculus, Johann contributed enormously to other fields of mathematics. (RIBEIRO, 2015).

Finally, Leonhard Euler (1707 - 1783), born in 1707 in Basel, Switzerland, Euler died in 1783 in Russia, having had his first concepts of mathematics taught to him by his father. At the age of 16, he obtained his Master of Arts degree, with a philosophy dissertation that he bought the natural philosophy system of Isaac Newton and René Descartes.

He devoted himself to mathematics in general, acknowledging numerous contributions to modern mathematics, in addition to his contributions to calculus: the analogy between infinitesimal calculus and the calculus of finite differences, when he discussed in detail all the formal aspects of Differential and Integral Calculus of the time. Euler was considered the master of mathematics of the 18th century. There is no branch of mathematics in which his name does not appear (Eves, 2004).

CLASSROOM ACTIVITIES

The activities presented below were developed using the excerpt presented. In this sense, read the text before solving. The use of the timeline proposed by Chaquiam (2023) provides a clear view of how mathematical ideas interconnect and evolve throughout history. It allows students to perceive the interactions between mathematical discoveries and historical contexts, helping to connect mathematical concepts with the social, cultural, and technological realities of each historical period.

Based on the use of the History of Mathematics as a pedagogical resource, several activities can be proposed to stimulate active and reflective learning among students. These activities, aimed at higher education, aim to develop a critical reflection on the development of Mathematics and its practical applications.

These activities aim to engage students in the process of constructing mathematical knowledge, allowing them to understand not only the concepts but also the contexts and reasons why these concepts were developed. By incorporating the History of Mathematics into teaching, students can perceive the subject in a more interesting and relevant way, establishing a connection with the contemporary world and their own experiences. As Barbosa (2020, p. 102) points out, "the proposed activities should transcend the simple memorization of formulas, providing a deeper and more contextualized understanding of mathematical concepts".

However, the activities enable the student to understand and plan the interpretations of the concepts that are presented in the historical context, developing rational and analytical thinking about the evolution of mathematical demonstrations.

Using Leibniz's notation, solve the following proposal:

• Suppose we want to calculate the area under the curve of the function $f(x) = x^2$ between $x = 0$ and $x = 3$. According to Leibniz's idea, what is the area under the curve?

• The famous project developed by Leibniz is the project of the sum of an infinite series for the approximation of π. Leibniz discovered that the following infinite series converges to the value of π\4. Using this idea of Leibniz, what is the sum of the first 4 terms?

FINAL CONSIDERATIONS

Incorporating the History of Mathematics into teaching represents an innovative and effective strategy to face the challenges that traditionally hinder the understanding of this subject. By inserting Mathematics within a historical context, it becomes possible to humanize it, bringing it closer to the students' everyday reality. In this way, History not only makes Mathematics more accessible and understandable but also broadens students' view of this area of knowledge, revealing its evolution over time and highlighting the importance of the contributions of different mathematicians and their contributions to the development and constitution of the concepts we use.

For example, Archimedes' ideas on exhaustion inspired later methods, such as Cavalieri's principle, and these, in turn, influenced the work of Leibniz and Newton in integral calculus. Furthermore, they contribute to the identification of specific results. A diagram allows us to isolate the contribution of each mathematician in the development of the integral, such as the use of infinite series by Fermat, the formalization of the integral by Newton and Leibniz, and the refinement made by Euler and Bernoulli. This helps to understand how the methods used have been added together over the centuries.

This work showed that the adoption of methodologies such as Chaquiam's (2022) methodological diagram and the use of a historical timeline offers students a deeper and more holistic understanding of Mathematics. These methods allow students to view Mathematics not as an isolated discipline, but as a collective construction that has developed over centuries, shaped by different thinkers and concepts. By adopting this historical approach, students can perceive Mathematics in a way that is more connected to their daily lives, in addition to realizing how this science applies to concrete problems and how its knowledge has evolved according to the needs and discoveries of each era.

By involving students in practical activities, such as the analysis of mathematical concepts throughout history in activities that seek to relate them to historical figures and their contributions to Mathematics, inserted within a historical context of the collaborative timeline, they are challenged to make connections between the concepts learned and the historical, social and cultural contexts in which they were developed. These activities not only deepen the understanding of mathematical concepts but also make learning more relevant, dynamic, and lasting, promoting active learning.

The introduction of the History of Mathematics in teaching has also favored the development of several skills in students, such as research capacity, critical reflection, and collaborative work. By engaging students in hands-on activities, such as analyzing mathematical concepts throughout history or constructing a collaborative timeline, they are challenged to make connections between the mathematical concepts they have learned and the historical, social, and cultural contexts in which they were developed. Mathematics, viewed through its historical evolution, ceases to be a set of abstract formulas and becomes understood as a dynamic and relevant field that directly impacts society and the modern world.

Ultimately, this study highlights the importance of transforming the teaching of Mathematics into a more interactive and contextualized practice that goes beyond the traditional focus of memorizing formulas and calculations. The History of Mathematics, therefore, is not just a complement to the content, but a way that allows students to perceive the beauty and applicability of this science.

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