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MALBA TAHAN: A LOOK AT DEFINITIONS IN MATHEMATICS

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ABSTRACT

This work emerges from observations related to the mathematical definitions contained in textbooks that, in general, are presented in a direct and technical way, without the concern of discussing other possibilities of presentation, such as those contained in the book The Problem of Definitions in Mathematics (1965), by Malba Tahan. Observing the possibility of presenting other ways of defining mathematical objects, the following question arises: What types of definitions are presented in the book The Problem of Definitions in Mathematics, by Malba Tahan and how do they appear in mathematics textbooks of the current high school? In order to obtain elements and arguments to support the answer to the question listed, it was established as a general objective to present definitions contained in high school mathematics textbooks that are in line with the definitions identified in the book The Problem of Definitions in Mathematics. The research can be characterized as exploratory, since it seeks to stratify different scenarios and perspectives, allowing familiarization and the development of diverse perceptions about the situation under study. Finally, as for the procedures, bibliographic, since it was based on a bibliographic review, both in physical and electronic media. From the analysis of textbooks, a set of definitions contained in high school textbooks was listed, according to the classification established by Malba Tahan. Therefore, the research evidenced the predominance of mathematical definitions presented in a technical and direct way in high school textbooks. The classification proposed by Malba Tahan in The Problem of Definitions in Mathematics proved to be a relevant tool to broaden the understanding and reflection on how definitions can be better worked. The study highlights the need to consider approaches that favor the contextualization and construction of meaning by students, contributing to more enriching pedagogical practices in the

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teaching of Mathematics. Finally, we recommend the full reading of Malba Tahan's book, which includes other topics, including the study of point, line, angles, polygonal lines.

Keywords: Mathematics, Mathematics Teaching, Malba Tahan, Definitions in Mathematics.

INTRODUCTION

One of the great challenges in teaching mathematics over time has been how to present concepts in a way that is understandable and accessible to students. In this context, Malba Tahan, pseudonym of the Brazilian educator Júlio César de Mello e Souza, stands out as one of the most innovative and creative disseminators of mathematics. In her work *The Problem of Definitions in Mathematics*, Malba Tahan takes a nuanced approach, using engaging narratives and examples to make mathematical definitions more accessible and interesting for students.

On the other hand, current textbooks mostly follow a more formal and structured approach, reflecting contemporary trends in mathematics teaching. These definitions are often presented in a straightforward and technical way, with a focus on conceptual precision and academic rigor. The central question she addressed was: *What types of definitions are presented in the book The Problem of Definitions in Mathematics, by Malba Tahan and how do they appear in mathematics textbooks of current high school*?

In order to obtain elements and arguments that answer the questioning listed, the objective was to *present definitions contained in high school mathematics textbooks that are in line with the definitions identified in the book The Problem of Definitions in Mathematics*. To contemplate the general objective and to mark the paths of the research, the following specific objectives were established: i) To read the book *The Problem of Definitions in Mathematics*, by Malba Tahan, with a view to identifying the types of definitions; ii) To identify definitions in high school mathematics textbooks, in order to compare them with the definitions contained in the book taken as a basis (iii) To present approximations and distances between the definitions identified in current textbooks and those identified in Malba Tahan's book.

Research was defined as applied in terms of its nature, as it seeks to generate knowledge with practical application, involving solving specific problems and directed to specific interests. As for the approach, it is a qualitative research, with the aim of developing a deeper understanding of issues or problems that cannot be quantified only through numbers and data obtained through questionnaires.

Still on the research with regard to the objectives, it is exploratory, since it seeks to stratify different scenarios and perspectives, allowing familiarization and the development of diverse perceptions about the situation under study. Finally, as for the procedures, bibliographic, since it was based on a bibliographic review, both in physical and electronic media.

The choice to compare the mathematical definitions presented by Malba Tahan with the definitions present in current textbooks is justified by the need to reflect on pedagogical approaches in mathematics teaching over time. On the other hand, Malba Tahan is widely recognized for his innovative didactic style, using narratives and creative examples to make mathematics more accessible and interesting. In contrast, contemporary textbooks follow a more formal and technical line, which reflects changes in educational guidelines and curriculum requirements.

In addition, the research offered the opportunity to rescue and value the work of Malba Tahan, whose contribution to mathematics education is often underestimated in the current context. By bringing out their ideas and comparing them with contemporary pedagogical practices, they emerge as a complementary form of teaching, with enriching potential associated with teaching materials.

In short, the justification lies in the importance of analyzing the pedagogical approaches in the teaching of mathematics and in the search for alternatives that can make the teaching process feasible and that make learning more effective and meaningful. Malba Tahan's work offers a unique and creative perspective that, when compared to current methods, can contribute to the improvement of mathematics teaching.

LIFE AND WORK OF MALBA TAHAN

Júlio César de Mello e Souza, better known by his pseudonym Malba Tahan, was born in Rio de Janeiro, on May 6, 1895. With his family, he moved to Queluz, São Paulo, where he spent his childhood until he was 10 years old, when he returned to Rio de Janeiro and, in 1906, he entered the Military College, but left his career after three years, however, he got a scholarship at Colégio Pedro II (Oliveira, 2018). With a strong vocation for teaching, he graduated as a primary school teacher at the Normal School of the Federal District and, in 1913, graduated in Civil Engineering from the Polytechnic School. He began his career as a teacher at the Externato do Colégio Pedro II, where he had studied (Oliveira, 2018).

Júlio César de Mello e Souza created the pseudonym Malba Tahan as an autonomous character, attributing to him a fictional biography. According to this narrative, Malba Tahan, whose name means "miller of the oasis", would be an Arab writer born on May 6, 1885 in the village of Muzalit, near Mecca. Júlio César presented himself as the translator of the works of this fictitious Arab author, feeding the idea that everything he wrote was the result of oriental culture (Oliveira, 2018).

In 1925, he married Nair Marques da Costa, with whom he had three children: Rubens Sérgio, a Navy officer; Maria Sônia, painter; and Ivan Gil, architect. Malba Tahan died on June 18, 1974, at the age of 79, victim of acute pulmonary edema and coronary thrombosis, while staying with his wife at the Hotel Boa Viagem, in Recife. His body was transferred to Rio de Janeiro, where he was buried. (Oliveira, 2018)

According to Salles (2015), from a young age, Júlio César was fascinated by literature and the impact of stories on the human spirit. At the age of 24, working as an office-boy and translator of war correspondence at the newspaper O Imparcial, in Rio de Janeiro, he decided to submit a short story of his authorship for publication. However, the editor left the text untouched on his desk.

Frustrated, Júlio took the story back, changed his signature from J.C. Mello e Souza to the fictional R.V. Slady, and presented it again as being by an unknown American writer. The next day, to his surprise, the short story *The Story of the Eight Loaves* was published prominently, in two columns and with a special frame (Salles, 2015)

In view of this, Júlio reflected: "When it is J. C. Mello e Souza, lead on top! When is R.V. Slady, featured on the front page...!?". It was this episode that inspired him to create the pseudonym Malba Tahan, an identity that would mark his career and win over the Brazilian public. The story was reported by him in testimonies to the Museum of Image and Sound of Rio de Janeiro (1973) and shared with friends and admirers throughout his life (Salles, 2015, p. 3).

According to Oliveira (2001), Malba Tahan has published more than 120 books, covering various topics such as oriental tales, recreational mathematics, mathematics didactics and other genres. Among its editorial initiatives, the magazine Damião stood out, dedicated to the social readjustment of leprosy patients in Brazil, widely distributed to professionals and institutions in Portugal and Brazil for a decade.

In addition, Oliveira (2001) points out that for five years, he edited the magazine Al-Kwarizmi, focused on mathematical recreations. He was a frequent contributor to magazines and newspapers of large circulation, including O Imparcial, O Jornal, O Diário da Noite, O Cruzeiro, O Correio da Manhã, Folha de São Paulo, Diário de Notícias and Jornal do Brasil.

Some of his works have been translated and published abroad, with a family representative in the United States in charge of copyright. In Brazil, Sônia Pereira's son-in-law manages contacts with publishers for the republication of her books and organizes issues related to copyright among the descendants.

Júlio Cesar died on 06/18/1974, when he was in Recife at the invitation of the Department of Education and Culture, teaching the courses for teachers "The art of reading and telling stories" and "Games and recreations in the teaching of mathematics", being buried in Rio de Janeiro, in the São Francisco Xavier Cemetery, Caju neighborhood.

Malba Tahan wrote a wide variety of works, many of which have become classics, especially in Brazil. His books address mathematics, oriental tales, didactics and mathematical recreations, some of which are presented below.

OTHER WORKS BY MALBA TAHAN

Below, we list Malba Tahan's best-known works, followed by mathematical and recreational works, plus oriental tales and philosophies and, finally, other contributions.

Quadro 1: Other works by Malba Tahan

THE BEST KNOWN	
The Man Who Calculated	His most famous work combines adventures with mathematical problems solved by the protagonist Beremiz Samir, showing the beauty and logic of mathematics.
A Thousand Endless Stories	A collection of short stories that explore ethical, cultural and philosophical values, set in the Arab universe.
Legends of Heaven and Earth	A collection of fantastic narratives that mix history and fiction with life lessons.
Maktub – Stories That Teach	A series of inspiring stories, focusing on wisdom and morals.
MATHEMATICS AND RECREATION	
Fun and Curious Math	A book that makes mathematical concepts more accessible and interesting through stories, riddles and curiosities.
Didactics of Mathematics	A manual dedicated to teachers, with guidance on how to teach mathematics creatively and effectively.
Math for Fun	It addresses mathematical questions and problems in a playful way.
Mathematical Riddles and Riddles	It presents mathematical challenges in the form of games and problems.
Fun Geometry	Explore geometry through engaging narratives.
ORIENTAL TALES AND PHILOSOPHY	
The Shadow of the Rainbow	Tales that mix fantasy and philosophical reflections.
The Wonders of the Arabic Tale	Inspired by *the Arabian Nights*, with stories full of mystery and wisdom
OTHER CONTRIBUTIONS	
The Heavens of Allah	A collection of stories that highlight the values of Arab culture.
Nasrudin Stories	Comedic and reflective accounts of the iconic character Nasrudin.
Salim the Magician	A story that mixes magic, adventure and mathematics.

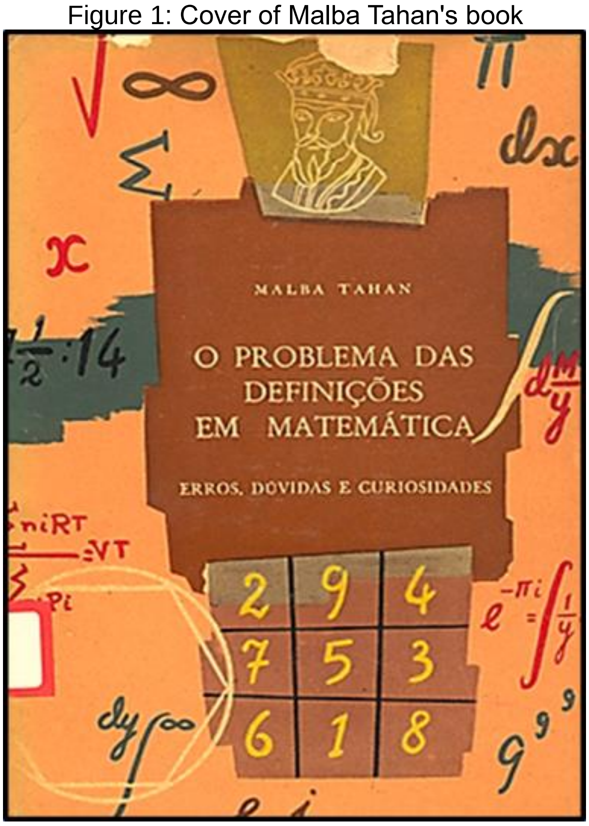
Source: Prepared by the authors, 2024.

Malba Tahan also left a legacy through magazines and newspaper articles, in addition to her contributions to the popularization of mathematics in an engaging and accessible way.

THE BOOK "THE PROBLEM OF DEFINITIONS IN MATHEMATICS"

Malba Tahan, in the book The Problem of Definitions in Mathematics, addresses in a reflective and didactic way the importance of definitions in the teaching and understanding of mathematics. The author, known for his creative and accessible style, explores the difficulties students face when dealing with abstract concepts and how precision in definitions can facilitate learning. He highlights the role of definitions as a basis for logical reasoning and the construction of mathematical knowledge, offering practical examples and suggestions for clearer and more effective teaching. The book is a valuable contribution to educators and students, promoting a critical view of the way mathematical concepts are presented and understood. It is a reading that combines the simplicity of Malba Tahan's narrative style with the depth of a pedagogical and epistemological analysis.

Figure 1: Cover of Malba Tahan's book



Source: Authors' collection (2024)

THE DEFINITIONS OF THE BOOK

We will present, below, some of the definitions contained in the book *The Problem of Definitions in Mathematics*.

Nominal or Explicit Definition

The nominal definition is formed according to the Aristotelian rule. And to clarify the reader about the concept of nominal definition, we will cite two more examples, one collected in Algebra and the other in Geometry. Here is the first example:

1) Two equations are said to be equivalent when every root of one is also of the other.

In this definition we observe:

a. Defined concept: Equivalent equations.
b. Close genus: equations.
c. Specific difference: every root of one is also a root of another.

There would be greater precision if at the end of the definition the expression "and vice versa" were added.

Now a second example taken from Geometry:

2) A geometric place is the set of points that enjoy the same property.

In this definition, adopted by good teachers, which is found in our excellent textbooks, we highlight:

a. Defined concept: Geometric place.
b. Next gender: Set of points.
c. Specific difference: The points that enjoy the same property.

Thus, as we have shown, the two definitions formulated according to the Aristotelian rule.

Descriptive Definition

The descriptive definition consists in the presentation of the being we wish to define, so that we can, from the data of the definition, have a precise idea of its form and attributes. And of frequent employment in Geometry. Let's take an example from the book by the late professor Tales Mello Carvalho (1916-1961):

> A prism is a polyhedron that has two equal and parallel faces (bases) and the others are parallelograms in a number equal to the number of sides of the equal polygons (Carvalho, 1950, p. 420).

What did the mathematician do to define the prism? He described this polyhedron; indicated the main elements contained in it. He therefore presented a definition that we call descriptive.

Definition by Concept Extension

In certain cases, by extension of an already known concept, we can obtain the definition of a new concept. Thus, we define the power m of a number (where m is integer and positive) and, by simple extension of concept, we can arrive at the definition of power one, zero power, negative power, etc.

Let us present, as an example, the case of power 1. Let be a whole number, any positive, and greater than 1. Let's look at how to define the power m of a number b:

Power M of a number B is called the product of M factors equal to B.

Thus, the power 5, of the number 8, would be: 8.8.8.8.8 which is written . Here the base (eight) and exponent (five) are already presented, which can also be defined. 8^5

The power m of a number b shall be indicated by the notation: b^m

What does the exponent m indicate? The number of factors in a product of factors equal to the base.

What if the exponent m is equal to 1?

In this case we would have: b^1 and the power could no longer (as in the case where m is a positive integer and greater than 1) be defined as a product of equal factors. A product with one factor will not be admissible.

It will be easy, by extension of concept, to define the power 1 of a number. We wrote: $b^1 = b$. In other words, the power 1 of a number b is defined, not as a product of equal factors (for that would be nonsense), but as the number itself (definition by concept extension). In the same way, that is, by extension of concept, we could define the zero power of b.

Definition by Postulate

Assuming a certain postulate P, we can, as a logical consequence of this postulate, formulate for in the definition for a concept C. We will say, in this case, that it is a definition by postulate.

Example I: Postulating the existence of the plane, the line and the point, we can define "complanar lines": Two lines are said to be complanar when they are located in the same plane.

The truth of the definition derives from the meaning given to the adjective complanar: complanar points, complanar circles, complanar curves, complanar lines, etc.

Example II: Assuming the postulate of Nicholas Lobachevsky (Russian mathematician, 1793-1856), several concepts emerge, surprising to us, that we could define within the new Geometry: non-secant lines, angles of parallelism, etc.

Definitions by postulate, some authors call empirical definitions.

Definition by Classification

When, from a concept A we separate (by classification) a part and, delimiting it, characterize a concept B (contained in A), we say that the definition of concept B was made by classification. Thus, the definition of prime number is taken, by classification, from the concept of integer.

The definition of semi-irregular polyhedron is taken (by classification) from the general concept of polyhedron. Observe this teaching of Rey Pastor and Adam Puig: It is a primordial condition that the characteristic properties of the defined species are compatible with the characteristic properties of the proximate genus. A definition by classification is only logical and perfect when it strictly complies with the Aristotelian principle.

Definition by Generation

In certain cases the definition of a figure (for example) is made by the law of generation of that figure. This is the case (frequent in Geometry) of definition by generation, which many authors, in the case of surfaces, call geometric definition.

Let us cite, as an example, the geometric definition of a conical surface:

A conical surface is the surface generated by a line G (generator), which moves by resting on a fixed curve C (guideline) and always passing through a fixed point V (vertex).

This definition of conical surface (definition by generation) is generic, because even the plane would be, as a particular case, included in it (in case the guideline is a line).

Once a process has been obtained and demonstrated that allows, by a geometrical law, to construct a figure (generated by continuous motion), we will say that this figure is defined by generation.

Negative Definition

Once a concept A has been defined, we can define a new concept B by negating one or more attributes of concept A. We will say, in this case, that the definition of concept B is a negative definition.

An example may shed light on the case.

Let us take as an initial proposition, the following definition (by classification):

A rectilinear surface is said to be developable when, being supposed to be flexible and inextensible, it can be extended over a plane without bending or breaking.

This is what happens with the cylindrical surface and the conical surface.

Starting from this definition of developable surface, we can define (negatively) a reverse surface. The reverse surface is the non-developable rectilinear surface.

Analytical Definition

We can define an entity, or a class of entities, by means of an equation or a formula. We will say, in this case, that the definition is presented in analytical form.

Example:

Bernoulli's lemniscate is the curve defined by the Cartesian equation

$$(x^2+y^2)^2 = a^2(x^2 - y^2)$$

where is a nonzero constant.

The famous algebraic curve, of the 4th degree, was thus defined analytically by means of its Cartesian equation.

This Bernoulli lemniscate curve could be defined by its polar equation or by its bipolar equation.

Definition of Convention

In order to generalize certain formulas, or to justify the use of certain symbols, the mathematician, in certain cases, for convenience, is forced to admit a convention. From this convention there is generally a definition. We will say, in this case, that it is a definition by convention.

Simple and expressive is the example that occurs with the concept of factorial of a number:

The factorial of an integer and positive n, the product of all integers from 1 to n, is called the factorial.

According to this (nominal) definition, the factorial of 5, for example, would be expressed by the product: 1.2.3.4.5 In this product there are all the integers (in natural succession) from 1 to 5. The factorial of 9 would be expressed in the same way by the product: 1.2.3.4.5.6.7.8.9

Generally speaking: The factorial of n would be: 1.2.3.4. 5....n

We can indicate, in short, the factorial of a number by a very simple notation: We write the number followed by a point of admiration: n! (Read: factorial of n).

When the number is given by an expression we must write that expression in parentheses and then the sign (1) of factorial. Thus the factorial of 5 + 4 + 3 is indicated by the notation:

(5 + 4 + 3)!

where it reads, factorial of 5 + 4 + 3.

According to the definition of factorial, we can immediately write:

2! = 2 3! = 6 4! = 24 5! = 120 ...

Two integers (one and zero) are not reached by the definition of the factorial of ", because the aforementioned definition speaks of the product of the integers and positive numbers from 1 to N, and thus, for both case 1 and case 0, the existence of this product is not verified.

How to define, therefore, the factorial of 1? How to get to the zero factorial?

The problem does not pose any difficulty. Let us consider the number 9, for example; The factorial of 9, as we know, is expressed by the product: 1.2.3.4.5.6.7.8.9

It's easy to go from the factorial of 9 to the factorial of 10. Just multiply the factorial of 9 by 10.

(1.2.3.4.5.6.7.8.9).10 = 10! or better: 9! 10 = 10!

In the same way, we could move from the factorial of 10 to the factorial of 11. Just multiply the factorial of 10 by 11:10! 11 = 11! When we multiply the factorial of one number by the next number, we get the factorial of that next number.

Some authors consider definition by convention to be perfectly equivalent to a simple definition by concept extension. From the didactic point of view, it would be more correct not to confuse the definition by convention with the definition by extension of concept.

About the Factor of Zero, we have to go back to the formula n! (n+1) = (n+1)! To follow up on the definition. Therefore, this formula results: 0! (0+1) = (0+1)! or, again, performing the operations indicated, in brackets: 0! 1 = 1! That is, the factorial of 1. Now, this equality will only be true if we do, by convention: 0! = 1 The factorial of zero is thus defined by convention: The factorial of zero is equal to 1. This definition would be perfectly clarified with the help of Combinatorial Analysis.

DEFINITIONS IN TEXTBOOKS ACCORDING TO MALBA TAHAN

In this chapter of the article, we will perform a comparative analysis of the definitions presented in different high school textbooks with the definitions in the book *The Problem of Definitions in Mathematics*. This approach will allow you to identify the types of definitions most commonly used in each book.

MATHEMATICS: FUNCTIONS AND THEIR APPLICATIONS

This book, *Mathematics: Functions and their Applications*, is a didactic work by Joamir Souza widely adopted by several educational institutions in Brazil. Its main objective is the study and application of mathematical functions, developing topics such as linear, quadratic, exponential, logarithmic and trigonometric functions, illustrated below.

Figure 2: Functions and their applications

Source: Authors' collection, 2024

An approach that balances theory and practice, this book will be useful for high school students as well as others who want to deepen their knowledge of mathematics. More intensely, the work emphasizes how important functions are applied to everyday life, examples applied in various fields – economics, physics and biology, among others.

Figure 3: Vertex equation

Como $f(x_v - p) = y_p$ e $f(x_v + p) = y_p$, temos:

$$f(x_v - p) = f(x_v + p) \Rightarrow a(x_v - p)^2 + b(x_v - p) + c = a(x_v + p)^2 + b(x_v + p) + c \Rightarrow$$

$$\Rightarrow a(x_v^2 - 2x_v p + p^2) + b(x_v - p) + c = a(x_v^2 + 2x_v p + p^2) + b(x_v + p) + c \Rightarrow$$

$$\Rightarrow ax_v^2 - 2ax_v p + ap^2 + bx_v - bp + c = ax_v^2 + 2ax_v p + ap^2 + bx_v + bp + c \Rightarrow$$

$$\Rightarrow -2ax_v p - bp = 2ax_v p + bp \Rightarrow -4ax_v p = 2bp \Rightarrow x_v = -\frac{2bp}{4ap} = -\frac{b}{2a}$$

Para obter $y_v = f(x_v)$:

$$y_v = f\left(-\frac{b}{2a}\right) = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c = a \cdot \frac{b^2}{4a^2} - \frac{b^2}{2a} + c =$$

$$= \frac{b^2 - 2b^2 + 4ac}{4a} = -\frac{b^2 - 4ac}{4a} = -\frac{\Delta}{4a}$$

Dada uma função quadrática $f: \mathbb{R} \rightarrow \mathbb{R}$, definida por $f(x) = ax^2 + bx + c$, e considerando o discriminante $\Delta = b^2 - 4ac$, as coordenadas do vértice da parábola correspondente ao gráfico de f são dadas por:

$$V\left(-\frac{b}{2a}, -\frac{\Delta}{4a}\right)$$

Source: Authors' collection, 2024

This is an excerpt from the book Mathematics Function and Applications, authored by Joamir Souza and published by FTD, where we address the concept of vertex of a parabola in the context of a quadratic function. It is explained in detail how to calculate the coordinates of the vertex using the arithmetic mean of the abscissas of two points symmetrical with respect to the axis of symmetry.

The book provides detailed mathematical arguments that allow us to derive the vertex coordinates of a parabola associated with a quadratic function, from its properties It fits an $f(x) = ax^2 + bx + c$**analytic** definition, as it employs the rigorous use of algebra to prove and establish the vertex x and y formulas.

Figure 4: Definition of exponential function

Função exponencial: características e definição

Você sabe o que é mitose? A mitose é um processo que faz parte da divisão celular e que consiste na divisão do núcleo de uma célula em dois núcleos geneticamente idênticos. As "células-filha", resultantes da mitose, possuem o mesmo número de cromossomos que a célula original.

Agora, observe o esquema ao lado com algumas etapas da mitose.

A partir de uma única célula, podemos estudar a quantidade total de células-filha obtidas por meio de uma função *f*, de acordo com a quantidade *x* de divisões celulares, com $x \in \mathbb{N}$. Acompanhe.

quantidade de divisões celulares

$f(x) = 2^x$

quantidade total de células-filha

Podemos determinar com essa função a quantidade total de células-filha obtidas na 8ª divisão celular calculando *f*(8), por exemplo.

$f(8) = 2^8 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 256$; ou seja, 256 células-filha

Na situação apresentada, é explorada a ideia de **função exponencial**.

Denominamos **função exponencial** toda função $f: \mathbb{R} \rightarrow \mathbb{R}_+^*$ definida pela lei de formação $f(x) = a^x$, em que $a \in \mathbb{R}$, com $a > 0$ e $a \neq 1$.

Observe exemplos de função exponencial.

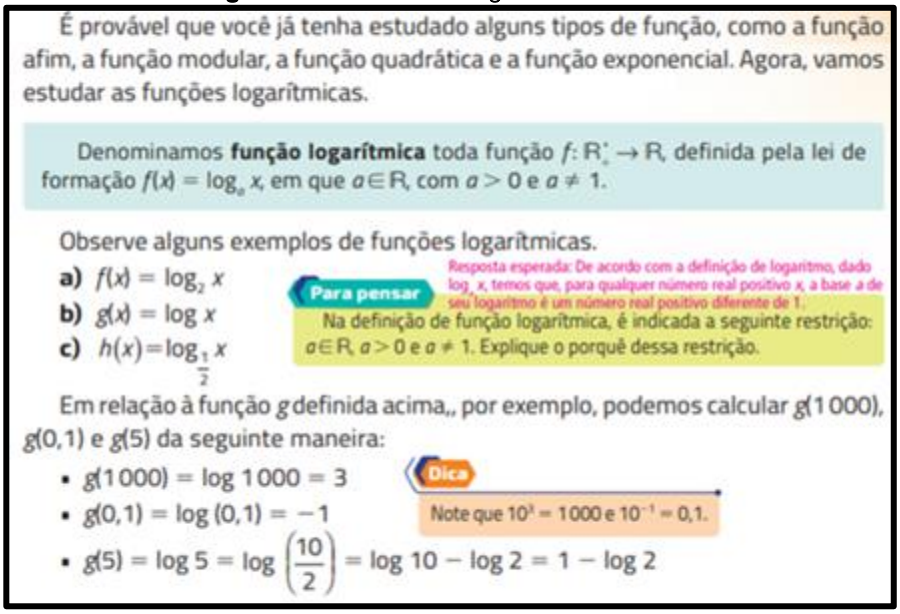
- $f(x) = 6^x$
- $f(x) = (0,2)^x$

Source: Authors' collection, 2024

This part of the book can be analyzed as the sum of several types of definitions according to Malba Tahan. The chapter begins with a descriptive approach. It is also the way the process of mitosis and cell division is done; gives the background of the biological phenomenon in which the exponential function will be applied. So, after bringing the function *f(x) = x2* that represents the number of resulting cells as a function of the number of divisions, we have an **analytic definition**. It is the characterization of this, as it uses a rigorous mathematical language to establish the relationship between the variables. Finally, the generalization of the exponential function of *f(x) = ax*, where > 0 and ≠ 1, broadens the view of the use of growth, giving a view through example application when the concept is applied. Finally, the set of connections between mitosis, a biological phenomenon, and mathematical modeling shows a definition per generation, presenting how the idea of mathematics arises when forming the description of a real process.

This part of the book can be analyzed as the sum of several types of definitions according to Malba Tahan. The chapter begins with a descriptive approach when explaining the process of mitosis and cell division, contextualizing the reader about the biological phenomenon in which the exponential function will be applied. Next, the formalization of the function *f(x) = x2*, representing the number of resulting cells as a function of the number of divisions, is characterized as an **analytical definition**, as it uses rigorous mathematical language to establish the relationship between the variables. In addition, the generalization of the exponential function to (x), with the > 0 and a≠ 1, this definition can also be classified as a $f(x) = a^x$**Nominal or Explicit Definition**, according to Malba Tahan, because it defines the exponential function explicitly, presenting the proximate genus (function) and the specific difference (the law of formation with the conditions for a being a>0 and a≠1. The definition is clear and follows the Aristotelian rule of conceptualizing using gender and distinctive attributes.$f: R \to R^x f(x) = a^x$,

Figure 5: Definition of logarithmic function


É provável que você já tenha estudado alguns tipos de função, como a função afim, a função modular, a função quadrática e a função exponencial. Agora, vamos estudar as funções logarítmicas.

Denominamos **função logarítmica** toda função $f: \mathbb{R}_+^* \to \mathbb{R}$, definida pela lei de formação $f(x) = \log_a x$, em que $a \in \mathbb{R}$, com $a > 0$ e $a \neq 1$.

Observe alguns exemplos de funções logarítmicas.

a) $f(x) = \log_2 x$

b) $g(x) = \log x$

c) $h(x) = \log_{\frac{1}{2}} x$

Para pensar

Resposta esperada: De acordo com a definição de logaritmo, dado $\log_a x$, temos que, para qualquer número real positivo x, a base a de seu logaritmo é um número real positivo diferente de 1.

Na definição de função logarítmica, é indicada a seguinte restrição: $a \in \mathbb{R}$, $a > 0$ e $a \neq 1$. Explique o porquê dessa restrição.

Em relação à função g definida acima,, por exemplo, podemos calcular g(1 000), g(0,1) e g(5) da seguinte maneira:

- $g(1000) = \log 1000 = 3$
- $g(0,1) = \log (0,1) = -1$
- $g(5) = \log 5 = \log \left(\frac{10}{2}\right) = \log 10 - \log 2 = 1 - \log 2$

Dica

Note que $10^3 = 1000$ e $10^{-1} = 0,1$.

Source: Authors' collection, 2024

This definition can be classified as a **Nominal or Explicit Definition**, according to Malba Tahan's criteria. This is because it presents the concept of logarithmic function using close genus and specific difference.

The proximate genus is represented by the term "function", while the specific difference is the way of defining the function by the law of formation *f(x) = loga(x),* with the explicitness of the constraints on the basis . The definition is straightforward and follows the classical Aristotelian structure. The examples and calculations presented are complementary and serve only to illustrate the defined concept, but do not change the main classification. $a\ (a \in R, a > 0, a \neq 1)$

Throughout the analysis of the definitions presented in this book, the prevalence of **the nominal** and **analytical style** in the construction of mathematical concepts is remarkable. This pattern is evidenced by the systematic structure and clarity with which the terms are introduced. For example, when defining exponential inequalities, the book follows a logical approach, presenting the proximate genus ("inequalities") and specifying the distinguishing characteristic ("which present the unknown only in the exponent of a power").

MATHEMATICS: SETS AND FUNCTIONS FOR HIGH SCHOOL

Mathematics: Sets and Functions for High School is a textbook authored by Giovanni Bonjorno Jr. and Paulo Câmara. With this work, the authors propose the formation of high school students within the integral appropriation of basic mathematical concepts, notably from the joint themes and functions. This book from FTD Publishing House can facilitate students' learning, with clear explanations and applications extracted from mathematical theory to real-life situations.

Figure 6: Book Sets and Functions

Source: Authors' collection, 2024

Through problem solving, the work addresses fundamental concepts: operations with sets, relationships between sets, functions, graphs and their applications. It also introduces students to the use of functions in different contexts. The text is very accessible and progressive in the explanations, with ample graphic resources and exercises to make learning dynamic and interactive.

Figure 7: Definition of equality of sets

Igualdade de conjuntos

Analisando os conjuntos A = {vogais da palavra "livro"} e $B = \{i, o\}$, observamos que eles possuem exatamente os mesmos elementos. Nesse caso, dizemos que A e B são iguais.

Agora, observe os conjuntos $X = \{1, 2, 3\}$ e $Y = \{0, 1, 2, 3\}$. Como existe um elemento em Y que não pertence a X, dizemos que X e Y são diferentes.

Dois conjuntos A e B são **iguais** se, e somente se, um deles for subconjunto do outro. Indicamos $A = B$. Em outras palavras, $A = B$ se todo elemento de um conjunto pertence ao outro.

Dois conjuntos A e B são **diferentes** se, pelo menos, um dos elementos de um dos conjuntos não pertence ao outro. Indicamos $A \neq B$.

Source: Authors' collection, 2024

This classification in the image above is called **Nominal Definition**. In this case, the definition still follows the Aristotelian structure of close gender and specific difference. This is because it describes the equality and difference between sets according to clear and straightforward terms, without characterizing the structure or internal properties of sets. His first description is purely the attempt of the concept: "Two sets A and B are equal..." and follows the rule of using a precise formula or condition to characterize the concept of equality and difference of sets, which are comparisons in an explicit way, that is, a concept is defined by its main attribute.

Figure 8: Role Definition

Definição de função

Agora que você já acompanhou algumas situações que envolvem função, vamos conhecer a definição matemática desse tipo de relação e aprofundar o estudo desse conteúdo.

Dados dois conjuntos não vazios, A e B, uma **função** de A em B é uma relação que associa **cada** elemento x de A a um **único** elemento y de B.

Source: Authors' collection, 2024

The proposed definition can be classified as a **nominal definition** as it provides a direct and clear explanation of what a function is, describing the relationship between a set and B. She presents this concept in a simple and objective way using the general term "relation" and details the specific differences that characterize it, namely how each element relates to a single element of B.

Figure 9: Definition of the set of irrational numbers

Esse número tem uma infinidade de casas decimais que não apresentam padrão de repetição; logo, não é uma dízima periódica, e é possível demonstrar que números desse tipo não podem ser escritos na forma de fração de inteiros (com denominador não nulo). Portanto, $\sqrt{2}$ não é um número racional; é um número irracional, que faz parte do conjunto dos números irracionais.

> O **conjunto dos números irracionais**, que indicamos por I, é o conjunto formado pelos números que têm uma representação decimal infinita e não periódica.

Source: Authors' collection, 2024

The definition in Figure 4 can be classified as **descriptive**, as it describes the basic characteristics of irrational numbers and mentions the way they are presented (infinite and aperiodic decimal representation). It emphasizes the properties that distinguish irrational numbers from other sets of numbers.

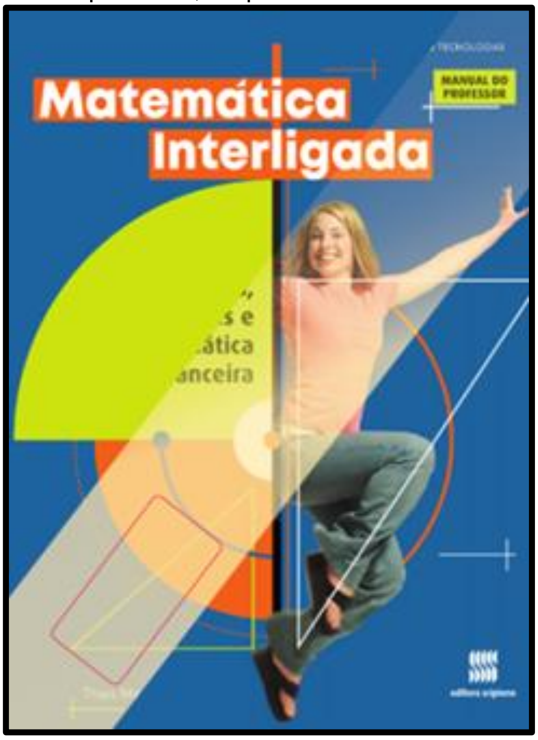
Looking at the last three definitions from the same book, we notice a pattern in the style of the definitions presented, which reveals a strong emphasis on the nominal approach, although we also observe an occasional application of other classifications.

An interesting aspect when looking at these definitions is the predominance of the nominal approach, which reflects the book's focus on structuring concepts with clarity and logical precision. This pattern is critical to building a strong foundation in mathematics, especially in introductory subjects. However, the occasional inclusion of descriptive definitions, such as that of irrational numbers, suggests an attempt to connect the mathematical concept to the reader's intuitive understanding, facilitating practical understanding. This alternation between rigor and description illustrates the material's effort to balance formalism and accessibility.

CONNECTED MATHEMATICS: QUANTITIES, SEQUENCES AND FINANCIAL MATHEMATICS

The book Connected Mathematics: Magnitudes, Sequences and Financial Mathematics, written by Thaís Marcelle de Andrade and published by Editora Scipione, is recognized for excellence in teaching materials for elementary and high school. This book was written to address the need to teach mathematics in a clear and practical way, promoting connections between mathematical concepts and their applications in everyday life. Divided into several well-organized sections, the book covers essential topics such as ratios, ratios, forward and inverse quantities, arithmetic and geometric series, and basic financial mathematics concepts, including simple and compound interest. The work brings contextualized examples to help students understand the relevance of mathematics in everyday decision-making, whether in financial planning or in the analysis of patterns and patterns, which meets the authors' proposal regarding the type of approach, a fact that facilitates the teaching and learning process of mathematical content.

Figure 10: Book quantities, sequences and financial mathematics



Source: Authors' collection, 2024

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Figure 11: Definition of geometric progress

Essa sequência é um exemplo de progressão geométrica, também chamada PG.

Progressão Geométrica (PG) é uma sequência numérica em que o quociente entre um termo, a partir do 2º, e o termo antecedente é constante. Essa constante chama-se **razão da PG** e é representada pela letra q, com $q \neq 0$.

Dizemos que uma PG é:

- **crescente** quando $a_1 > 0$ e $q > 1$ ou $a_1 < 0$ e $0 < q < 1$.
- **decrescente** quando $a_1 > 0$ e $0 < q < 1$ ou $a_1 < 0$ e $q > 1$.
- **constante** quando $q = 1$.
- **alternante** quando $q < 0$.

Source: Authors' collection, 2024

The definition for Geometric Progression (GP) presented in figure 2 can be classified as the **Nominal Definition type**, since it explains the concept through a clear and objective description, highlighting the close gender (numerical sequence) and the specific difference (the constant quotient between one term and the previous one). In addition, this definition encompasses internal classifications based on the behavior of the sequence, depending on the sign of the first term (a1) and the ratio (q).

Here is the detailed classification of each part:

1. **Main Definition**:
   - Geometric Progression (PG) is a numerical sequence in which the quotient between a term, starting from the 2nd, and the preceding term is constant.". This is a formal definition, which describes what defines PG, describing it in a general and objective way, without giving examples or referring situations.
2. **Additional properties (PG ratio):**
   - "This constant is called the ratio of PG and is represented by the letter q, with q≠0." This complements the nominal definition by specifying a crucial property (the q-ratio), which is essential to the concept of PG.

This definition is well-structured, combining the clarity and rigor of the nominal definition with the practicality of operational definitions. The inclusion of ratings helps the reader to better understand the different behaviors that a PG can present, making the content more didactic and accessible.

Figure 12: Definition of equivalent rates

Um erro muito comum em situações financeiras é achar que 1% ao mês sempre equivale a 12% ao ano. Isso é válido no sistema de juro simples, em que as taxas são proporcionais ao longo do tempo, pois a razão entre elas é igual à razão entre os períodos aos quais elas se referem. Já no sistema de juro composto, uma taxa de 12% ao ano não é equivalente a 1% ao mês.

Taxas equivalentes são taxas de juro que, quando aplicadas ao mesmo capital, num mesmo intervalo de tempo, produzem montantes iguais.

Source: Authors' collection, 2024

This definition seen in Figure 3 on equivalent rates is, of course, a nominal definition. Although devoid of eloquence and detail, it is succinct, direct and precise. According to the definition, equivalent rates are those with which, when levied on the same capital during the same period of time, the same amount is obtained. In view of this, the definition in question is direct and explicit, making use of an order normally employed in the process of introduction and description of an object through its essential attributes. Also, the definition does not support example or context. The definition of the term is sufficient, direct and unambiguous, indicative of a nominal definition.

Figure 13: Definition of affine function

Uma função $f: \mathbb{R} \rightarrow \mathbb{R}$ será uma função afim $f(x) = ax + b$ se, e somente se, para qualquer PA $(x_1, x_2, x_3, ..., x_n, ...)$ de razão r, a sequência das imagens $\left(f(x_1), f(x_2), f(x_3), ..., f(x_n), ...\right)$ for uma PA de razão $a \cdot r$.

Esse resultado pode ser provado, mas não será feito neste livro.

Source: Authors' collection, 2024

In Figure 4, the definitions of related functions can be classified as **concept extension** definitions. This is because it refers to a **known concept**, in this case arithmetic progression (PA), and uses that concept to define the associated function. The definition explains the function converts a sequence of numbers forming an arithmetic sequence into another sequence that will also be an arithmetic sequence, with the same ratio multiplied by $f(x) = ax + b$a. This type of definition extends the concept of arithmetic series to the context of affine functions without having to specify the form of the affine function in more basic terms. Therefore, an affine function is defined in terms of its relationship to another well-established mathematical concept.

The book by Thaís Marcelle de Andrade that we analyzed is characterized by the variety of forms of definition that it uses to present mathematical concepts, which facilitates the understanding of more complex ideas. These definitions involve different strategies for constructing concepts, adapted to the context of each theme.

One such definition involves a geometric series (PG), in which the relationship between the terms in the sequence is characterized by a constant quotient between consecutive terms. The book clearly describes the GP and proposes the basic idea of multiplication sequences, in which constants q (proportion) define the series. This concept is described directly without resorting to other more abstract concepts, constituting a descriptive definition. In addition, examples are provided that vary depending on the value of the ratio, such as when the ratio is greater than 1 or between 0 and 1, which broadens the reader's understanding of how PG behaves in different contexts.

What can be observed when analyzing these definitions is that the book adopts a diversified approach, adapting the type of definition to the specific content and pedagogical objective of each concept. The use of different types of definitions - from descriptive to analytical - helps to build a more detailed understanding of mathematical concepts, applying them in diverse contexts and relationships.

CONCLUSION

Different forms of mathematical definitions extracted from high school textbooks were presented in line with Malba Tahan's definitions contained in her book *The Problem of Definitions in Mathematics that corroborate in answering the question : What types of definitions are presented in the book The Problem of Definitions in Mathematics, by Malba Tahan and how do they appear in mathematics textbooks of current high school*?

From the analysis of high school textbooks, it was observed that they almost always contain the same forms of presentation of definitions, a fact that, compared to the forms presented by Malba Tahan, the possibility of inserting these other forms of definition in the school context to contribute to the understanding of the defined mathematical object is envisioned.

From the different approaches to presenting a definition in mathematics, observing the advantages and limitations inherent to each of them, it is possible to affirm that this book by Malba Tahan can contribute to the training of mathematics teachers, specifically, in the sense of transcending what is contained in mathematics textbooks, and can make classes more attractive and enlightening in relation to the understanding of the mathematical object.

The development of this research provided us with a greater understanding of the ways to present definitions in mathematics, among those identified, many of them were not known to us. In addition, the search for definitions in high school mathematics textbooks contributed to consolidate them. Under the pedagogical primary, we can affirm that clarifications about a mathematical object from its presentation through various ways of defining it can contribute to the understanding of this object.

This work provides teachers in initial or continuing education with a range of ways to define a mathematical object, a fact that pedagogically can be favorable to the improvement of the teaching process and, consequently, the learning of the mathematical contents involved.

Malba Tahan's book addressed in this research transcends the context cut out in this work, that is, we recommend readers to read the full book *The Problem of Definitions in Mathematics*. In addition, it is possible to carry out new research involving the types of definitions in mathematics in other textbooks at different levels of education.

The comparative analysis highlights the importance of integrating these two perspectives in the teaching of mathematics. The balance between Malba Tahan's creativity and the rigor of modern materials enriches teaching practice and promotes more meaningful and comprehensive learning. Valuing and rescuing contributions such as Malba Tahan's, therefore, not only expands the possibilities of education, but also strengthens the role of mathematics as a living science accessible to all.

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