Augustin Cauchy: A mathematical genius who revolutionized science

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ABSTRACT

Augustin-Louis Cauchy was a prominent mathematician of the nineteenth century whose contributions have left an indelible mark on modern mathematics and various areas of science. His pioneering approach to mathematical analysis set a new standard for rigor and clarity, revolutionizing the way mathematical problems were approached. Through his meticulous work, Cauchy laid the groundwork for many branches of mathematics, ensuring that fundamental concepts such as boundaries, continuity, and derivation were accurately understood and defined.

Among his innumerable contributions, his emphasis on function theory and complex analysis stands out remarkably. Theorems such as the Cauchy-Riemann theorem and the residual theorem are essential pillars in this field. In addition, his work on differential equations and infinite series has been crucial for the development of theories that we continue to use today.

Cauchy did not limit himself to advancing pure mathematics; Its impact is also felt in applied areas such as physics and engineering. His research in elasticity theory and partial differential equations has provided vital tools for practical problem-solving in these disciplines.

Keywords: Theory of Functions, Complex Analysis, Cauchy-Riemann, Residue Theorem, Differential Equations.

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INTRODUCTION

The history of mathematics is marked by great figures who have left a lasting legacy in this discipline essential to understanding the world around us. One such big name is Augustin-Louis Cauchy, a 19th-century French mathematician whose contributions have had a profound impact on various fields, from mathematical analysis to physics to engineering.

This article proposes to explore Cauchy's life and works, highlighting his innovative approach and his obsession with precision and clarity. We will investigate their contributions to function theory, calculus, and number theory, as well as their influence on practical areas such as theoretical physics and engineering. Throughout this journey, we will dive into his key theorems and discover how his legacy lives on in modern mathematics, inspiring new generations of mathematicians and scientists.

THEORETICAL OR CONCEPTUAL ELEMENTS

AUGUSTIN CAUCHY "A PRODIGY OF MATHEMATICS"

Augustin-Louis Cauchy (21 August 1789 – 23 May 1857) was an influential French mathematician of the nineteenth century. Coming from an aristocratic family, Cauchy received a privileged education that fostered his talent in mathematics from a young age. At the age of 16, he entered the École Polytechnique, where he studied under renowned mathematicians such as Lagrange and Laplace.

Throughout his life, Cauchy made key contributions in areas such as mathematical analysis, number theory, mechanics, and optics. Among his most notable achievements are the Cauchy-Riemann Theorem, Cauchy's Theorem for integrals, as well as important contributions to the theory of elliptic functions and to the understanding of limits and continuity. He also conducted significant research in number theory and partial differential equations.

Cauchy was known for his energetic personality and his advocacy of mathematical rigor, which sometimes brought him into conflict with his contemporaries. His influence endures in modern mathematics, and his legacy is celebrated by mathematicians and scientists around the world.

From his early years at the École Centrale du Panthéon and the Lycée Louis-le-Grand to his training at the École Polytechnique and the École des Ponts et Chaussées, Cauchy left an indelible mark on the mathematical world. His passion and skills

Exceptional figures were manifested from a young age, and his contributions laid the foundation for his renown as one of the most outstanding mathematicians of his time.

MATHEMATICAL CONTRIBUTIONS MADE.

It introduces the fascinating world of Cauchy's applications, exploring how his theorems and concepts have permeated through different branches of mathematics, from the theory of complex variable functions to the solution of differential equations, Cauchy's legacy manifests itself in a wide range of applications.

THEORY OF COMPLEX VARIABLE FUNCTIONS

Augustyn Louis Cauchy, a prominent French mathematician of the nineteenth century, made significant contributions to the Theory of Complex Variable Functions. His accomplishments in this field include.

CAUCHY-RIEMANN THEOREM

Cauchy formulated the conditions necessary for a complex function to be holomorphic (differentiable) at a point. These conditions are known as the Cauchy-Riemann equations and are fundamental in the study of analytical functions in the complex plane (Clarke, 2004).

RIGOROUS DEFINITION OF BOUNDARIES

Cauchy is known for establishing a precise and rigorous definition of what is now known as the limit of a function. He introduced the idea that a function tends to a limit L when the difference between the function and L becomes arbitrarily small as the independent variable approaches a certain value.

CAUCHY'S THEOREM ON LIMITS.

He proposed Cauchy's theorem on limits, which states that if two functions have the same limit when a variable approaches a given value, then the sum, product, and quotient of these two functions also have the same limit at that point. This theorem is essential for the analysis of limits of complex functions and the application of boundary properties in calculation (Hernández, 1993).

INFINITESIMAL ANALYSIS ACQUIRES SOLID FOUNDATIONS

Infinitesimal analysis acquires solid foundations with Augustyn Louis Cauchy due to his fundamental contributions in the formulation of rigorous definitions and theorems that established a solid mathematical foundation for this field.

FUNDAMENTAL THEORIES

Cauchy formulated and proved a number of fundamental theorems in analysis, such as Cauchy's theorem on limits and the mean value theorem. These theorems provided essential tools for understanding the behavior of functions in the context of limits and derivatives. In addition, it established the foundations for future developments in calculation and analysis (Hernández, 1993)

EMPHASIS ON RIGOROUS DEMONSTRATION

He emphasized the importance of rigorous mathematical demonstrations. His formal and methodical demonstrations helped raise the level of precision and rigor in mathematics. This led to a more rigorous and consistent approach in infinitesimal analysis.

INFINITE SERIES THEORY

He made important contributions to the theory of infinite series, including the precise definition of convergence and divergence of series. This was essential for understanding the behavior of functions in terms of infinite series and provided a basis for the theory of analytic functions (Gale, 1999).

CAUCHY'S THEOREM

Cauchy's theorem is a fundamental result in the field of complex analysis that states that if a function is holomorphic (also called analytical) in a simply connected region, then the integral of the function along a closed curve contained in that region is equal to zero. In other words, if a function is holomorphic in a region that has no "holes" and is closed, the integral of that function around any closed curve contained in that region is equal to zero (Ana Lucca, 2018).

CONTRIBUTIONS TO THE ANALYSIS

Louis Cauchy made important contributions to mathematical analysis, but his work did not focus on group permutation theory. Rather, Cauchy is known for his influence in areas such as mathematical analysis, number theory, function theory, and calculus.

DEFINING HOLOMORPHIC FUNCTIONS

Cauchy is credited with making fundamental contributions to the definition and theory of holomorphic functions in the context of complex analysis. His work laid the foundations for the development of this area of mathematics.

Every holomorphic function can be separated into its real and imaginary parts $f(x+iy) = u(x,y) + iv(x,y)$, and each of them is a harmonic function in R2 (each satisfies Laplace's equation.

One of Cauchy's most outstanding contributions is the Cauchy–Riemann theorem, which establishes the conditions necessary for a complex variable function to be holomorphic (also called an analytic function).

DEFINITION OF THE CONVERGENCE AND DIVERGENCE CRITERIA OF THE SERIES

He played a fundamental role in defining the criteria of convergence and divergence of series in mathematics. His contributions in this field helped establish solid foundations in mathematical analysis. One of Cauchy's most notable contributions was his formulation of the "Cauchy Criteria" for series convergence.

MODULAR ARITHMETIC

In the field of modular arithmetic, his contributions are notable and have had important applications in various areas of mathematics and number theory.

When a number is raised to different powers and the remainder modulo m is taken, residue cycles can be obtained. For example, for a given number a and m, cycles such as a0 ≡ 1 (mod m), a1 = a (mod m), a2 = a2 (mod m) ..., ak-1 = ak-1 (mod m), ak = ak (mod m), ak+1 = ak+1 (mod m), where k is the smallest positive integer for which ak = 1 (mod m).

Modular arithmetic is a branch of mathematics that focuses on integers and their properties in relation to certain modules or residuals. Some notable applications of Cauchy's concepts and theorems in modular arithmetic are:

Congruences:

- He conducted research on linear congruences and their solutions in integers. These congruences are essential in modular arithmetic and have applications in number theory and in solving modular equations.
- The linear congruence ax ≡b (mod m) has a solution if and only if the greatest common divisor of a and m divides b. If a solution exists, all solutions are of the form x = x0 (mod m), where x0 is a particular solution.
- Cauchy's contributions in group theory, number theory, and modular arithmetic have had a significant impact on the study of integers and their properties in relation to modules. Its theorems and methods are fundamental in many mathematical applications and in areas such as cryptography, number factorization, and number theory in general.

Residual theory:

Augustyn-Louis Cauchy made important contributions to residual theory in the context of complex analysis. Residual theory focuses on the study of analytical functions at the complex plane and their singularities and is a fundamental part of complex analysis.

DESCRIPTION

The elaboration of this research work is based on a requirement to opt for the degree of Bachelor of Mathematics from the National University of Panama and was prepared with the collaboration of Professor Alcibiades Medina, Professor Eliecer Cedeño and Professor Narciso Galástica, professors of the University of Panama. They are elaborated in chapters that represent the most relevant aspects of the cuneiform legacy in mathematical development, such as its historical evolution, decipherment of mathematical contributions made by Babylonians and a wide relevance to the great scientific contribution and culture of these regions to our society.

The limitations presented in the research of the topic presented was the verification of the source of research, since some of them were very scarce in information, in addition to the little existence in some cases of translations of cuneiform texts that have not been deciphered at present and can represent great contributions to the development of Mathematics as well as other sciences.

FINAL THOUGHTS

Augustin-Louis Cauchy was an exceptional mathematician whose contributions profoundly impacted mathematics and physics, becoming a pillar of the development of modern mathematics. His rigorous approach based on mathematical analysis laid the foundations for greater coherence and precision in various branches of mathematics.

Cauchy excelled especially in function theory and complex analysis, with fundamental theorems such as the Cauchy–Riemann theorem and the Residue Theorem, which remain essential in the study of complex variable functions. In calculus, he formulated clear definitions of key concepts such as limits, continuity, and derivability, thus consolidating the discipline in its current form.

His influence transcended pure mathematics, as his work on elasticity theory and partial differential equations had practical applications in engineering and physics. In addition, his contributions in number theory and functional analysis laid the foundations for future advances in theoretical physics.

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