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DIDACTIC SEQUENCE FOR THE TEACHING OF TRIGONOMETRY: CONSTRUCTION AND USE OF THE HOMEMADE THEODOLITE IN HIGH SCHOOL

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ABSTRACT

The article presents a study about Trigonometry in the right triangle through the use of the theodolite. The objective is to investigate the effectiveness of the application of a didactic sequence in the teaching and learning of Trigonometry, with the help of the theodolite, for high school students. As a theoretical foundation, the studies of Zabala (1998), Freitas and Bittar (2004), Pires and Gomes (2004), Silva (2015) and Prudchenko (2021) were adopted. The research is characterized as qualitative, which was applied to 31 students of the 2nd year of high school of a public school in the municipality of Mocajuba, in the state of Pará. A didactic sequence was elaborated, structured with 2 activities, whose results showed that even with some difficulties, the use of the theodolite brought a new way of visualizing the content. It was concluded that the applied didactic sequence can be considered a potential for the teaching and learning process of trigonometric concepts.

Keywords: Trigonometry. Theodolite. Didactic Sequence.

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INTRODUCTION

Teaching mathematics represents a journey that, for many students, can prove to be challenging due to the complexity of the concepts and the traditional approach to transmitting knowledge. In this context, the creation of didactic sequences emerges as an innovative and promising pedagogical strategy, aiming to give mathematics learning a more engaging, meaningful, and accessible dimension. The underlying purpose of this application is to introduce a meticulously designed didactic sequence for the teaching of a specific concept of mathematics, to be selected in line with the level of education and the needs of the students.

The didactic sequence consists of an approach in which it intends to "make learning meaningful for each of the students, enabling knowledge to be given in a pleasant and relaxed way" (Martins; Noble; Chaves, 2014, p. 02). The structure of this learning process is applied in sequential stages, each designed to build on pre-existing knowledge and lead students to a deeper and more autonomous understanding of the subject.

In mathematics, the teaching of Trigonometry, which will be exposed in this work, is a theme that most students are not familiar with its concepts, and with the traditional approach carried out by many schools, it becomes even more complex. So, from this, creating a sequence that makes students more confident tends to be a viable path to learning.

Therefore, this work seeks to explore the students' previous knowledge, with references in reality, using concrete material, taking advantage of the potential of different types of didactic instruments, so that the students themselves can discover trigonometric patterns and properties. This is another opportunity to develop skills in students that allow them to effectively use the tools they have from their environment and culture (Santos; Bisognin, 2007).

The interest in the topic of the study arose from the understanding that the teaching of mathematics is a constant challenge, with the need to make abstract concepts accessible and relevant to students. Trigonometry, an essential part of mathematics, is often seen as complex and difficult to understand. In the context of high school, a solid understanding of Trigonometry is essential for the mastery of more advanced mathematical concepts and their application in various disciplines and situations of daily life.

The adoption of didactic sequences in the teaching of Trigonometry can be an innovative and effective approach to make this topic more accessible and engaging for students. However, despite the potential of didactic sequences, there is still a lack of resources and specific guidelines to develop effective didactic sequences in this context.

The relevance of this research is evident when considering the importance of Trigonometry in several areas, such as physics, engineering, architecture, and social sciences. A solid understanding of trigonometric concepts not only prepares students for future disciplines but also empowers them to deal with real-world situations that involve measurements of angles and distances.

Thus, it seeks to answer the following research question: Can the construction of a didactic sequence help in the teaching and learning of Trigonometry for high school students? With this, the objective is to investigate the effectiveness of the application of a didactic sequence in the teaching and learning of Trigonometry, with the help of the theodolite, for high school students, aiming to improve the understanding of the concepts and the motivation of students in relation to this topic.

To structure this article, the following sections have been organized: Introduction; Theoretical foundation; Methodological procedures; Discussion of the results and final considerations.

THEORETICAL FOUNDATION

Constructivism, as an educational theory, emphasizes the active role of the student in the construction of knowledge. In the realm of mathematics, this translates into the conception that students should build their own understandings of mathematical concepts through exploration, experimentation, and problem solving. The content approach, via didactic sequence, aligned with this constructivist perspective, provides a structure that promotes the progressive construction of knowledge, enabling students to develop deeper and more substantial understandings.

This teaching strategy, often permeated by challenging activities and experimentation, is attuned to the belief that mathematical learning thrives when students have the opportunity to apply concepts in real or simulated contexts. The teacher who is interested in improving his teaching methodology can program differentiated classes using concrete material, which is related to theory, to help him in the teaching process, because playfulness is not the main form of teaching, as stated by Freitas and Bittar (2004) "many times, these materials assume the main place in teaching and do not fulfill their function, which is to allow the student, through manipulations of the material, build your knowledge."

The use of didactic sequences in the teaching of mathematics reinforces interactivity, context, activity and the active construction of knowledge by students. This approach seeks to infuse mathematics teaching with greater involvement, significance, and effectiveness, aligning with the needs and development of students. The teacher can, with a differentiated

class, propose contextualized activities experienced in the student's daily life, create a favorable environment in the teaching of mathematics without, however, leaving aside the challenges and difficulties of this practice. To enrich it, it is necessary for the mentor to innovate, yes, but aware, however, that each individual has their specificities with difficulties and different time to learn.

For Pires and Gomes (2004), it is important to provide the student with the experience of mathematization through the manipulation of materials, in this way they develop a playful activity, in addition to providing opportunities for situations that favor the development of abstract thinking. In this context, we show that the employability of manipulable resources in the teaching environment facilitates the apprehension of the activity procedure, the formulation of strategic approaches and the adherence to the prescribed guidelines. Such a conjuncture enables the student to acquire new knowledge in his daily life, consequently enriching the learning framework in mathematical domains.

Silva (2015) addresses this theme and points out in his study that:

> the teaching of Triangle Trigonometry generates motivations, including diversified activities, with problematized situations, which stimulate thinking, investigation and doing, contributing to students constructing the meaning of trigonometric ratios, in addition to favoring argumentation and modifying various misconceptions. (p. 08)

Highlighting the effectiveness of the Trigonometry teaching method in the right triangle through enriched approaches such as diversified activities, challenging situations, and practical applications, a framework emerges in which students are driven by an intrinsic motivation. This approach not only achieves educational goals but also nurtures a substantially deeper understanding of trigonometric concepts, allowing students to establish meaningful connections between theory and practice.

In addition, interaction with problematic situations stimulates not only critical thinking, but also the development of problem-solving skills, giving students the ability to effectively apply concepts in real-world contexts. Engaging practice and active search for solutions encourage autonomous discovery, solidifying learning in a lasting way.

The theodolite is a device of great importance in measurement, used to calculate with high precision both horizontal and vertical angles. This instrument has a broad historical basis, as stated by Prudchenko (2021), whose first historical notions of the existence of the theodolite emerged with the Egyptians, in the creation of the great pyramids. In addition, there are excerpts that this same author affirms the use of this device in Roman society. It is understood, therefore, that many years ago this artifact was already being worked on in civilizations and in their daily lives.

In line with the aforementioned approach regarding the exposure of the usefulness of the theodolite as an addition to the instructional environment, Silva (2015) exposes that this instrument helps in the understanding of the content, because in its study the manipulation of the activity using this instrument, both in construction and in its use in practice, facilitated the understanding of the content about Trigonometry.

The example of the use of the theodolite and its positive influence on the assimilation of Trigonometry concepts exemplifies how tangible approaches can effectively enrich education and provide a solid foundation for the applicability of knowledge. Through this practice, the fundamental importance of adopting pedagogical methodologies that align learning with practical application was perceived, reinforcing the understanding of concepts and promoting the interaction of students with knowledge in the context of their daily experiences.

In this way, to develop the research, the construction of the theodolite was used with the students of the 2nd year of high school of a public school in the city of Mocajuba-PA. For this application, there was a brief introductory class on the theodolite and then, through the materials, they were visualized how to build the theodolite to then apply in the field with examples existing within the school.

The methodological procedures adopted in the research are presented below.

METHODOLOGICAL PROCEDURES

As for the methodology adopted, this research follows a qualitative approach of the exploratory type, aimed at the exploration, understanding and description of complex and multifaceted phenomena. In this way, it allows researchers to delve into the perspectives of the participants, in order to obtain contextualized and substantial perceptions, its method has the characteristics of observing, recording, analyzing, describing and correlating facts or phenomena without manipulating them, seeking to discover precisely the frequency in which they occur and their relationship with other factors. Contextualization plays a prominent role in this endeavor, since the research strives to elucidate the context in which the phenomena occur. The incorporation of the daily life of society aimed to provide students with a platform to explore their intellectual capacities in a more natural and immersive way, aiming at a deeper understanding of the topic addressed.

The research was carried out with a 2nd year high school class from a public school in the municipality of Mocajuba, in the state of Pará. The participating students initially received two activities, namely: one containing concepts about right triangles and the other with detailed instructions for the construction of a homemade theodolite.

The application of the Didactic Sequence took place in 1 day during six classes, the class was composed of 31 students, it was proposed that they form groups. Then, in the first stage, the students studied the theoretical contents present in the handouts. The first activity covered the basic principles of right triangles, including definitions, properties, and the application of the tangent formula, followed by a short auxiliary question to help with the application. The second activity provided a step-by-step guide to making a homemade theodolite using simple and accessible materials.

The students, divided into groups, followed the instructions in the workbook to create the measuring instrument. This practical phase was essential to consolidate theoretical learning, providing a concrete experience and stimulating teamwork.

In the third and final stage, each group chose an object outside the school environment to apply the theodolite and measure the height using the tangent formula. With the measurements of the chosen object written down, the students returned to the classroom and made the necessary calculations to find out its height.

In addition, it is worth mentioning that before starting the application of the sequence, the teacher talked to the students and briefly reminded them about what the activity would be, as the class had already seen the content in the previous month. However, some students needed help and assistance with the activity that was carried out in the external area of the school.

After the practical application, the students answered an evaluative questionnaire, which was designed to collect data on the understanding of the concepts covered, the execution of the practical activity and the effectiveness of the teaching method. This methodological approach, based on theoretical-practical learning and the use of accessible didactic resources, aimed to promote a deep and contextualized understanding of trigonometry contents among high school students. The questionnaire was composed of the following questions:

1. What was your experience using the homemade theodolite during this activity?
2. Were you able to understand the basic workings of the homemade theodolite?
3. How did you feel about taking measurements of inaccessible heights with the homemade theodolite?
4. Do you believe that this activity has helped to improve your understanding of trigonometry concepts?
5. Were there any specific challenges you faced when using the homemade theodolite? If so, how did you solve this challenge?

6. Do you think that using the homemade theodolite was an effective way to learn about measuring inaccessible heights? Why?
7. How do you evaluate the importance of this activity for your learning in mathematics?
8. Do you have any suggestions for improvement for this activity?
9. Do you feel more confident now in taking measurements from inaccessible heights after participating in this activity? Why?
   Would you like to participate in similar activities in the future?

The next section will present the discussion of the results obtained in the study.

DISCUSSION OF THE RESULTS

In the discussion of the results, we analyzed the data obtained during the practical activity with the theodolite, focusing on the application of trigonometry concepts in real situations.

In the first moment, the students obtained the sheet with activity 1, containing the definitions of the right triangle and the formulas necessary to perform the calculations. Some students began to recall the classes they had on this subject with statements such as "the right triangle has an angle of ninety degrees" this is the subject where there is cosine, sine and tangent".

Still in the first meeting, the question under discussion was at the end of the first workbook, and it is an auxiliary question for students to interpret and visualize how the theodolite is used, how the measurements are linked in each part of the image, and so that they can understand the function of the right triangle in relation to the visualized object. There was no intervention in this part of the activity, the students were able to solve it in a practical way, and showed a lot of interest and curiosity about the theodolite, confirming the thought that practical and playful knowledge is "a set of experiences, knowledge and affections with which the person thinks and acts" (Martins; Noble; Chaves, 2014, p. 02).

In the second and third stages, it took place at the beginning of the second class and went on to the third, the students began assembling the homemade theodolite (figure 1) and many were impressed and questioned if it would really work to measure heights. According to the observations, most were able to assemble the instrument without the need for intervention and were enthusiastic and curious to use the theodolite. In line with the ideas of Santos (2015), the use of playful material helps in the learning process of mathematical content, as well as contributes significantly to interactions with everyday life.

Figure 1 - Students making the theodolite

Source: Authors' collection

In the next stage of the activity, the students had to go to the outside area of the school. It was proposed that they choose an object to measure their height, write down their measurements and return to the room. Using the constructed theodolite and a tape measure, they obtained their measurements. Everyone on the team wanted to use the material to check if the instrument really had functionality, and so they did. One of the students mentioned *"it really works, I can take it to teach my parents",* while another expressed *"I can choose a tree and take the measurements myself".* However, it was explained to the class that the value found is an approximate value, and not the exact value of the object.

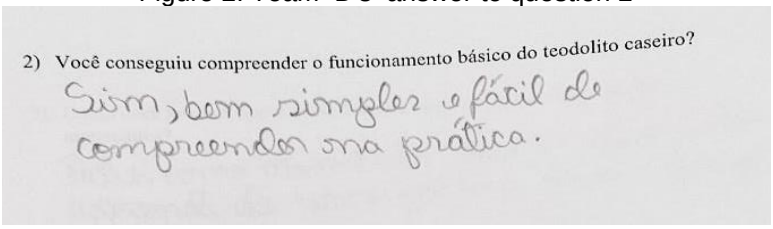
In the fifth class, after returning from the outside area of the school, the students began to perform their calculations, with the measurements they found with the theodolite. It was observed that most drew the right triangle to have a better base from which to place the values. The investigated students used all the formulas that were in the first workbook, and the calculations were made precisely, the only intervention was in a team that was unable to find the value because they forgot to add the height of the observer. With the height found, the students showed excitement for being able to conclude the activity in a positive way, such aspects confirm Martins' assumption; Noble; Chaves (2014) on the importance of carrying out practical activities for the teaching of Trigonometry.

Then, in the sixth class, the questionnaire about the activity developed was presented, each team had a sheet containing ten questions for evaluation and validation of the sequence.

The first question refers to the experience of using the instrument during the activity. The students' response revealed that this type of teaching is not widely used in this environment, since most of the teams answered that they had never done activities in this way.

The second question referred to the students' understanding of the functioning of the theodolite. The response of team D (figure 2) showed that the use of the instrument in practice made it seem that its functionalities were simple and easy to understand.

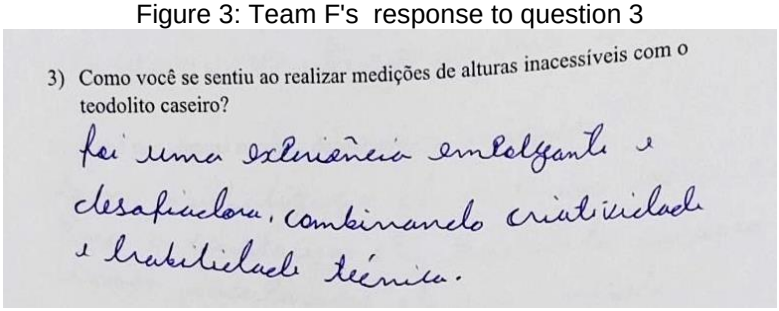
Figure 2: Team D's answer to question 2



Source: Research Protocol (2024)

The third question was intended to understand what each team thought of the experience of using the theodolite. The students portray it as a challenging and exciting experience that combines creativity and skill, as described by team F in figure 3. In this way, it is possible to verify that the students were having something innovative and unique for their teaching.

Figure 3: Team F's response to question 3



Source: Research Protocol (2024)

The fourth question aimed to know if, through the activity, the students were able to understand the concepts of trigonometry worked on in class. The class presented positive answers in their protocols, in which they said that before the didactic sequence they could not have a full understanding of the subject, and that through the activity what they studied about Trigonometry became clearer.

The fifth question was about the team having any challenge with the theodolite, and how they solved that challenge. The students answered that the handling of the instrument was very easy to work with, and their understanding was positive.

The sixth question was intended to verify whether the use of the homemade theodolite was effective in measuring inaccessible heights. The class answered that through the instrument and this new dynamic, understanding became easier.

The seventh question aimed at the students' evaluation of the activity in relation to the teaching of mathematics. The students answered that the activity was important for their understanding of geometry and trigonometry, as indicated by the G team. It is notable that the students were more successful with the activity, as they viewed the subject in a different way, using concrete material.

The eighth question was about suggestions for improvement for the activity, in which we sought to hear indications from the students investigated. The students answered that there is no need for improvement, as they had no difficulty. As recorded by team F, they were able to understand and carry out the activity with the explanation and materials given to them.

The ninth question aimed to find out whether, after the activity, the students felt more confident to measure inaccessible heights. The investigated students indicated positive answers to this question, when team H revealed that one of the reasons for the confidence in height measurements was the issue of application, since they were able to apply their knowledge in a different way.

The tenth question was aimed at knowing if the students would like to participate in activities similar to this one in the future. Their response was very optimistic, and one of the reasons pointed out by team C was the fact that they continued to work on their skills, since they had the opportunity to socialize as a team and set up their own object of study.

Therefore, based on the answers obtained in the evaluative questionnaire, it is possible to infer that the didactic sequence applied was considered a potential for the teaching and learning process of trigonometric concepts. Defined by Zabala (1998), the didactic sequence is as a set of ordered activities aimed at achieving clear educational objectives. Structuring these activities allows students to develop a deeper understanding of content, such as trigonometric concepts.

FINAL CONSIDERATIONS

The main objective of this work is to investigate the effectiveness of the application of a didactic sequence in the teaching and learning of trigonometry, with the aid of the theodolite, for high school students. To this end, concrete material was used for application, accompanied by activity sheets and a questionnaire consisting of questions related to the experience of an activity of this type.

The students were very motivated when they were presented with the activity, how everything would occur and how each stage would be worked on. At first, it is noticeable

that the students were not yet involved with activities of this model, but even with this restriction they were receptive to the didactic sequence and showed interest in participating.

In the first stage, when they were given the workbook with formulas and properties of the right triangle, the students were able to remember what they had studied a few months before, and in some parts of the workbook students asked for a brief explanation, there were no interventions, they studied the workbook and solved the extra question that was at the end of it.

In addition, in the second part of the activity, when the students received the materials to make the theodolite, they were excited about the idea of creating their own study instrument, having the workbook with the step-by-step instructions on how to assemble it, the students successfully managed to build the device.

In the practical application, the students showed that they understood the functioning of the theodolite, and teamwork helped a lot in this part of the activity, since each student had a function, after writing down their measurements, the students return to the classroom and begin to make their calculations using the formulas and then it was verified that this method helped them to understand Trigonometry in a different way.

The application of the questionnaire, which was the final part, shows that the activity was productive and that the students had a unique experience, the response of each team provided satisfaction when verifying the advances that the class had in the subject addressed, even with some difficulties, the didactic sequence brought a new way of viewing the content, The students' excitement to have activities like this in the future only confirmed how positive the work was.

In this way, the didactic sequence can help the teacher to ensure that students are progressing in their knowledge and understanding of a specific topic. However, the conditions of application must be taken into account, due to the time available and mode of application. The sequence must be in accordance with the students' previous knowledge, so that they have a different view from content seen only as a traditional method.

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